On the Interaction of Elemnetary Particles. III.

By Hideki Yukawa, Shoichi Sakata and Mitsuo Taketani.

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§6. Interaction of the U-Field with the Heavy Particle.

In the preceeding paper (II, §3), we found that the sclar U-field was

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§1. Introduction and Summary.

In two previous papers,[1] the interaction of elementary particles was discussed by introducing a new field of force. On quantizing this field, we obtained new quanta, obeying Bose statistics, each with the elementary charge either positive or negative and the mass m_u, which is connected with the range $\frac{1}{\kappa}$ of the nuclear force by the relation $\kappa = \frac{m_u c}{\hbar}$. This field was described by two four vector functions complex conjugate to each other in I, whereas it was described by two scalar functions in II. These formulations were adopted for the sake of simplicity, but neither of them was ample enough for the derivation of complete expressions for the interaction of the heavy particles and their anomalous magnetic moments.

In this paper, we begin with the construction of the linear equations for the new field, which can be considered as a generalization of Maxwell's equations for the electromagnetic field. The field is thus described by two four vectors and two six vectors, which are complex conjugate to each other respectively. (§2.) It is interesting that this system of equations written

1) Yukawa, Proc. Phys.-Math. Soc. Japan 17, 48, 1935; Yukawa and Sakata, ibid. 19, 1084, 1937. These papers will be referred to as I and II respectively. See also Yukawa, ibid. 19, 712, 1937.

in spinor form reduces to a special case of Dirac's wave equations for the particle with the spin larger than 1/2.[2] Meanwhile, it came to our notice that our formulation was equivalent to a method of linearization of wave equations for the electron, which had been developed by Proca[3] as an extension of the scalar theory of Pauli and Weisskopf.[4] Very recently, Kemmer and Bhabha[5] also discussed the nature of nuclear force, anomalous magnetic moment of the heavy particle[6] and cosmic ray by using Proca's scheme.

The new field equations can be derived from the Lagrangian, so that the canonical variables and the Hamiltonian can be determined in the usual way. We can then go over into the quantum theory by constructing the commutation relations and the equations of motion for these variables. (§2.) We can decompose the field variables into Fourier components, each of which consists, in turn, of three components, indicating that the quanta accompanying this field have each spin 1. (§3.)

In the presence of the electromagnetic field, the Lagrangian for the U-field is transformed in the usual way, corresponding to the fact that the U-quantum has the charge either positive or negative. It follows, further, that the quantum has the magnetic moment of the magnitude $e\hbar/2m_u c$ in nonrelativistic approximation. (§4.) It is likely that the anomalous magnetic

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2) Dirac, Proc. Roy. Soc. A. 155, 447, 1936. See further Sakata and Yukawa, Proc. Phys.-Math. Soc. Japan 19, 91, 1937. Detailed discussions of more formal problems will be made elsewhere.
3) Proca, Jour. d. Phys. 7, 347, 1936. See further Durandin and Erschow, Phys. Zeits. d. Sowj. 12, 466, 1937.
4) Pauli and Weisskopf, Helv. Phys. 7, 709, 1934.
5) Kemmer, Nature 141, 116, 1938; Bhabha, ibid. 141, 117, 1938.
6) The problem of the magnetic moment of the heavy particle was discussed especially by Taketani, Kagaku 7, 532, 1937 and by Frohlich and Heitler, Nature 141, 37, 1938.
~~Nature 141, 37, 1938.~~

moments of the neutron and the proton can be attributed to the virtual presence of the heavy quanta in the intermediate states. ~~(§5.)~~ The order of magnitude and the sign of the additional moment thus obtained is in agreement with the experiment. (§5.)

Finally, the possible forms of the interaction between the U-field and the heavy ~~quanta~~ particle is considered and the force between the neutron and the proton is deduced. It is found that the combination of the exchange forces of Majorana and Heisenberg types, which is in accord with the result of the current theory both in sign and magnitude, can be obtained in a natural way. It should be noticed, however, that the force is not strictly central, so that considerable departures from the results of the current theory are expected in some cases. (§6. §7.)

The problem of possible existence of the neutral heavy quanta, which seems to be important in connection with the problem of the forces between two neutrons and between two ~~neutron~~ protons, as well as that of the ordinary force between the neutron and the proton, is not considered in detail in this paper.

Among various phenomena, which are related with the interaction of the heavy quanta with the light particles, only the process of annihilation of a heavy quantum in free space by emitting a positive or negative electron and a neutrino or an antineutrino simultaneously, the possibility of which was pointed out by Bhabha,[7] is dealed with quantitatively and its bearing on the problem of the hard component of the cosmic ray is discussed. (§8.)

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7) Bhabha, loc. cit.

§2. Linear Equations for the U-Field in Vacuum.

We consider two three dimensional vectors $\vec{F}$ and $\vec{G}$ forming a six vector in four dimensional space in analogy with the electric and magnetic vectors in electrodynamics and assume them to satisfy linear equations of Maxwell type, which however, result in the quadratic equations

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \kappa^2\right)\begin{pmatrix}\vec{F}\\ \vec{G}\end{pmatrix} = 0 \qquad (1)$$

by iteration, instead of the D'Alembert equations, where $\kappa = m_u c/\hbar$. Such a system of linear equations can be constructed only by introducing, further, a four vector with the time components U_0 and the space components $\vec{U}$. Thus, we obtain ten equations

$$\left.\begin{aligned} &\frac{1}{c}\frac{\partial\vec{F}}{\partial t} - \operatorname{curl}\vec{G} - \kappa\vec{U} = 0 \qquad && \operatorname{div}\vec{F} + \kappa U_0 = 0 \\ &\frac{1}{c}\frac{\partial\vec{U}}{\partial t} + \operatorname{grad} U_0 + \kappa\vec{F} = 0 && \operatorname{curl}\vec{U} - \kappa\vec{G} = 0, \end{aligned}\right\} \qquad (2)$$

from which we can easily derive the quadratic equations (1) for $\vec{F}$, $\vec{G}$ and also for U_0, $\vec{U}$. Furthermore, it can be shown ~~we can easily show~~ that five relations

$$\frac{1}{c}\frac{\partial\vec{G}}{\partial t} + \operatorname{curl}\vec{F} = 0 \qquad \operatorname{div}\vec{G} = 0 \qquad \frac{1}{c}\frac{\partial U_0}{\partial t} + \operatorname{div}\vec{U} = 0 \qquad (3)$$

are satisfied, which, together with (2), are equivalent to Dirac's generalized wave equations with $k = 1/2$ and $\ell = 1$.[8] On the other hand, the equations (2) are the same with those proposed by Proca[9], if we take $\kappa = mc/\hbar$.

8) Dirac, loc. cit.
9) Proca, loc. cit. (9) and (21).

Hereafter, the arrow $\longrightarrow$ denoting the space vector will often be omitted in order to avoid the unnecessary complication of the formulae.

Exactly the same set of equations is assumed for the six vector $(\tilde{F}, \tilde{G})$ and the four vector $(\tilde{U}_0, \tilde{U})$, which are complex conjugate to (F, G) and (U_0, U) respectively. First four equations in (2) and the equations complex conjugate to them can be derived from the Lagrangian

$$\bar{L} = \iiint L \, dv \qquad (4)$$

with ~~L = FF - GG + U₀U₀ - UU,~~ $$L = \frac{1}{4\pi}(\tilde{F}F - \tilde{G}G + \tilde{U}_0 U_0 - \tilde{U}U), \qquad (5)$$

where U_0, U, $\tilde{U}_0$ and $\tilde{U}$ are considered as independent variables and F, G, $\tilde{F}$ and $\tilde{G}$ are defined by the remaining six equations in (2) and the equations complex conjugate to them.

The variables, which are canonically conjugate to U, U_0, $\tilde{U}$ and $\tilde{U}_0$ respectively, can be defined in the usual way by the relations

$$\left.\begin{aligned} U_0^{\dagger} &= \frac{\partial L}{\partial \frac{\partial U_0}{\partial t}} = 0, \qquad U_x^{\dagger} = \frac{\partial L}{\partial \frac{\partial U_x}{\partial t}} = -\frac{1}{4\pi\kappa c}\tilde{F}_x \quad \text{etc.} \\ \tilde{U}_0^{\dagger} &= \frac{\partial L}{\partial \frac{\partial \tilde{U}_0}{\partial t}} = 0, \qquad \tilde{U}_x^{\dagger} = \frac{\partial L}{\partial \frac{\partial \tilde{U}_x}{\partial t}} = -\frac{1}{4\pi\kappa c}F_x \quad \text{etc.} \end{aligned}\right\} \qquad (6)$$

The Hamiltonian for the U-field in vacuum becomes thus

$$\bar{H}_U = \iiint H_U \, dv \qquad (7)$$

with
$$\begin{aligned} H_U &= U_0^{\dagger}\frac{\partial U_0}{\partial t} + U^{\dagger}\frac{\partial U}{\partial t} + \text{comp. conj.} - L \\ &= 4\pi\kappa^2 c^2\, \tilde{U}^{\dagger} U^{\dagger} - c\,(U^{\dagger}\,\mathrm{grad}\, U_0 + \tilde{U}^{\dagger}\,\mathrm{grad}\,\tilde{U}_0) \\ &\quad + \frac{1}{4\pi\kappa^2}\,\mathrm{curl}\,\tilde{U}\cdot\mathrm{curl}\,U + \frac{1}{4\pi}(\tilde{U}U - \tilde{U}_0 U_0). \end{aligned} \qquad (8)$$

$$\cancel{= c^2 \tilde{U} U - c(\tilde{U} \operatorname{grad} U_0 + U \operatorname{grad} \tilde{U}_0) + \operatorname{curl} \tilde{U} \operatorname{curl} U}$$
$$\cancel{+ \tilde{U}U - U_0 \tilde{U}_0}.$$

In the quantum theory, the canonical variables U_x, $U_x^\dagger$, etc. should satisfy the commutation relations

$$\left.\begin{aligned} &U_x(\vec{r}, t)\, U_x^\dagger(\vec{r}', t) - U_x^\dagger(\vec{r}', t)\, U_x(\vec{r}, t) = i\hbar\, \delta(\vec{r}, \vec{r}') \\ &U_x(\vec{r}, t)\, U_y^\dagger(\vec{r}', t) - U_y^\dagger(\vec{r}', t)\, U(\vec{r}, t) = 0 \\ &U_x(\vec{r}, t)\, \tilde{U}_x^\dagger(\vec{r}', t) - \tilde{U}_x^\dagger(\vec{r}', t)\, U_x(\vec{r}, t) = 0 \\ &\qquad \text{etc.,} \end{aligned}\right\} \quad (9)$$

corresponding to the Bose statistics. The variables $U_0^\dagger$ and $\tilde{U}_0^\dagger$, however, vanish identically, so that they can not be taken into the quantum theory as canonically conjugate to U_0 and $\tilde{U}_0$ respectively. Hence, we have to eliminate U_0 and $\tilde{U}_0$ themselves by using the conditions

$$\operatorname{div} F + \kappa U_0 = 0 \qquad \operatorname{div} \tilde{F} + \kappa \tilde{U}_0 = 0$$

or

$$U_0 = 4\pi c \operatorname{div} \tilde{U}^\dagger \qquad \tilde{U}_0 = 4\pi c \operatorname{div} U^\dagger. \quad (10)$$

The Hamiltonian (8) now takes the form

$$H_U = 4\pi\kappa^2 c^2 \tilde{U}^\dagger U^\dagger - 4\pi c^2 (U^\dagger \operatorname{grad} \operatorname{div} \tilde{U}^\dagger + \tilde{U}^\dagger \operatorname{grad} \operatorname{div} U^\dagger)$$
$$+ \frac{1}{4\pi\kappa^2} \operatorname{curl} \tilde{U} \operatorname{curl} U + \frac{1}{4\pi} \tilde{U}U - 4\pi c^2 \operatorname{div} U^\dagger \operatorname{div} \tilde{U}^\dagger,$$

so that we obtain

$$\bar{H}_U = \iiint \left(4\pi\kappa^2 c^2 \tilde{U}^\dagger U^\dagger + 4\pi c^2 \operatorname{div} U^\dagger \operatorname{div} \tilde{U}^\dagger + \frac{1}{4\pi\kappa^2} \operatorname{curl} \tilde{U} \operatorname{curl} U + \frac{1}{4\pi} \tilde{U}U\right) dv \quad (11)$$

by partial integration. The above procedure is not satisfactory from relativistic point of view. A more symmetrical method of quantization will be discussed elsewhere.

The field equations are, now, expressed in the form of the equations of motion

$$\left.\begin{aligned} i\hbar\frac{\partial U_x}{\partial t} &= U_x H - H U_x \qquad \text{etc.} \\ i\hbar\frac{\partial \tilde{U}_x^\dagger}{\partial t} &= \tilde{U}_x^\dagger H - H\tilde{U}_x^\dagger \qquad \text{etc.,} \end{aligned}\right\} \tag{12}$$

which reduce to the form

$$\left.\begin{aligned} \frac{\partial U_x}{\partial t} &= 4\pi\kappa^2 c^2 \tilde{U}_x^\dagger - 4\pi c^2 \frac{\partial}{\partial x}\operatorname{div}\tilde{U}^\dagger, \ \text{etc.} \\ \frac{\partial \tilde{U}_x^\dagger}{\partial t} &= -\frac{1}{4\pi\kappa^2}\operatorname{curl}_x(\operatorname{curl} U) - \frac{1}{4\pi}U_x, \ \text{etc.} \end{aligned}\right\} \tag{13}$$

by the help of the commutation relations (9). These equations, together with the conditions (10) and the relations (6) and

$$\operatorname{curl} U - \kappa G = 0, \qquad \operatorname{curl}\tilde{U} - \kappa\tilde{G} = 0, \tag{14}$$

are equivalent ~~with~~ to the equations (2).

§3. Representation of the U-Field in Vacuum by Normal Coordinates.

If we consider the field in a unit cube, we can change the field variables into new ones by Fourier transformation

$$\left.\begin{aligned} \vec{U} &= \sum_k \sum_{j=1,2,3} (u_{jk} + \tilde{v}_{jk})\,\vec{e}_{jk}\, e^{i\vec{k}\vec{r}} \\ \vec{U}^\dagger &= \sum_k \sum_{j=1,2,3} (u_{jk}^\dagger + \tilde{v}_{jk}^\dagger)\,\vec{e}_{jk}\, e^{-i\vec{k}\vec{r}}, \end{aligned}\right\} \tag{15}$$

where the suffix k stands for the vector $\vec{k}$ with the integer components multiplied by 2π and $\vec{e}_{1k}$ is a unit vector parallel to $\vec{k}$, while $\vec{e}_{2k}$ and $\vec{e}_{3k}$ are those perpendicular to $\vec{e}_{1k}$ and to each other, forming a right handed system. By inserting (15) into the field equations (13), we obtain the equations of motion for the new variables u_{jk}, $u^{\dagger}_{jk}$ etc.

$$\left.\begin{aligned} \dot{u}_{jk} &= 4\pi\kappa^2c^2\,\tilde{u}^{\dagger}_{jk} + 4\pi k^2c^2\,\delta_{j1}\,\tilde{u}^{\dagger}_{1k} \\ 4\pi\kappa^2\,\dot{\tilde{u}}^{\dagger}_{jk} &= -(k^2+\kappa^2)\,u_{jk} + k^2\,\delta_{j1}\,u_{1k}, \quad \text{etc.} \end{aligned}\right\} \qquad (16)$$

and similar equations for v_{jk}, $v^{\dagger}_{jk}$ etc. These equations show that each ~~//////~~ variable has the time factor either e^{-ik_0ct} or e^{ik_0ct} with $k_0 = +\sqrt{k^2+\kappa^2}$, so that we denote the variables with the former factor by u_{jk}, $\tilde{u}^{\dagger}_{jk}$, v_{jk}, $\tilde{v}^{\dagger}_{jk}$ and those with the latter factor by $\tilde{u}_{jk}$, $u^{\dagger}_{jk}$, $\tilde{v}_{jk}$, $v^{\dagger}_{jk}$. The commutation relations (9) are transformed ~~in/to the form~~ into

$$\left.\begin{aligned} u_{jk}u^{\dagger}_{lk'} - u^{\dagger}_{lk'}u_{jk} &= \frac{i\hbar}{2}\,\delta_{jl}\,\delta(\vec{k},\vec{k}'), \\ u_{jk}u_{lk'} - u_{lk'}u_{jk} &= 0, \quad \text{etc.} \end{aligned}\right\} \qquad (17)$$

and similar relations for v's, u's and v's being commutative with one another. Correspondingly, the Hamiltonian (11) becomes

$$\bar{H}_u = \sum_k\sum_j\left\{4\pi c^2(\kappa^2+k^2\delta_{j1})(\tilde{u}^{\dagger}_{jk}+v^{\dagger}_{jk})(u^{\dagger}_{jk}+\tilde{v}^{\dagger}_{jk}) + \frac{1}{4\pi\kappa^2}(k_0^2-k^2\delta_{j1})(\tilde{u}_{jk}+v_{jk})(u_{jk}+\tilde{v}_{jk})\right\} \qquad (18)$$

From (16), we obtain the relations

$$-4\pi i k_0\kappa^2 c\,u^{\dagger}_{jk} = (k_0^2 - k^2\delta_{j1})\,\tilde{u}_{jk} \qquad (16)'$$

and consequently, (17) takes the form

$$u_{jk}\tilde{u}_{lk'} - \tilde{u}_{lk'}u_{jk} = \frac{2\pi\kappa^2k_0\hbar c}{k_0^2-k^2\delta_{j1}}\,\delta_{jl}\,\delta(\vec{k},\vec{k}'), \ \text{etc.} \qquad (17)'$$

Hence, if we introduce the new variables

$$\left.\begin{aligned} N_{jk} &= \frac{(k_0^2-k^2\delta_{j1})}{2\pi\kappa^2k_0\hbar c}\,\tilde{u}_{jk}u_{jk} \\ M_{jk} &= \frac{(k_0^2-k^2\delta_{j1})}{2\pi\kappa^2k_0\hbar c}\,\tilde{v}_{jk}v_{jk} \end{aligned}\right\} \qquad (19)$$

which are all commutative with one another and each has the eigenvalues 0, 1, 2, ..., the Hamiltonian reduces to

$$\bar{H}_u = \sum_k\sum_j k_0\hbar c\,(N_{jk} + M_{jk} + 1) \qquad (20)$$

The variables N_{jk} denotes the number of the heavy quanta with the positive charge in the state of energy $E_k = k_0\hbar c = \hbar c\sqrt{k^2+\kappa^2}$ and momentum ~~$\hbar ck$~~ $\hbar\vec{k}$, while

M_{jk} that with the negative charge in the state of energy E_k and momentum $-\hbar c\vec{k}$. The suffix j = 1 denotes the state represented by the longitudinal wave, while j = 2 or 3 that represented by the transverse wave polarized in a definite direction.

Thus, the quantized U-field, which is described by two four vectors and two six vectors complex conjugate with each other respectively, is accompanied by the quanta obeying Bose statistics[10] with the mass $m_U = \frac{\hbar\varkappa}{c}$ and the charge either positive or negative, as in the case of the scalar field discussed in II. The suffix j, which can take three values, indicates the extra degree of freedom for the quanta corresponding to the spin 1, in contrast to the spin 0 in the case of the scalar field.

§4. Interaction of the Heavy Quanta with the Electromagnetic Field.

In the presence of the electromagnetic field with the scalar and the vector potentials A_0 and $\vec{A}$, grad and $\frac{1}{c}\frac{\partial}{\partial t}$ operating on the variables F, G, U_0 and U should be replaced by

$$\mathrm{grad} - \frac{ie}{\hbar c}\vec{A} \qquad \text{and} \qquad \frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0$$

respectively, since these variables involve the operator, which ~~in~~decreases the ~~t~~ number of the positively charged quanta by one and ~~de~~increases that of the negatively charged by one. Similarly, grad and $\frac{1}{c}\frac{\partial}{\partial t}$ operating on $\tilde{F}$, $\tilde{G}$, $\tilde{U}_0$ and $\tilde{U}$ should be replaced by

$$\mathrm{grad} + \frac{ie}{\hbar c}\vec{A} \qquad \text{and} \qquad \frac{1}{c}\frac{\partial}{\partial t} - \frac{ie}{\hbar c}A_0$$

respectively.

Thus, the field equations (2) are replaced by

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10) The impossibility of quantization of the scalar field corresponding to Fermi ~~statistics~~ statistics, which was shown by Pauli and Weisskopf, loc. cit., can easily be extended to ~~of~~ our case, as already pointed out by Durandin and Erschow, loc. cit.

$$\left.\begin{aligned}&\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)\vec{F}-\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\times\vec{G}-\varkappa\vec{U}=0\\&\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\vec{F}+\varkappa U_0=0\end{aligned}\right\}\qquad(21)$$

$$\left.\begin{aligned}&\left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{ie}{\hbar c}A_0\right)\vec{U}+\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)U_0+\varkappa\vec{F}=0\\&\left(\mathrm{grad}-\frac{ie}{\hbar c}\vec{A}\right)\times\vec{U}-\varkappa\vec{G}=0\end{aligned}\right\}\qquad(22)$$

where the symbol $\times$ denotes the vector product.

The Lagrangian for the system consisting of the U-field and the electromagnetic field can be written in the form

$$\bar{L}=\iiint L\,dv \qquad (23)\ \cancel{(22)}$$

with

$$\left.\begin{aligned}L&=L_U+L_E\\L_U&=\frac{1}{4\pi}(\tilde{F}F-\tilde{G}G-\tilde{U}U+\tilde{U}_0U_0)\\L_E&=\frac{1}{8\pi}(E^2-H^2),\end{aligned}\right\}\qquad(24)$$

so that the equations ~~(22)~~ (21) and those complex conjugate to them can be derived from this Lagrangian by performing the variation with respect to U, U_0, $\tilde{U}$ and $\tilde{U}_0$ and by using the relations (22). On the other hand, by performing the variation with respect to A and A_0 and by using the relations

$$E=-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}-\mathrm{grad}\,A_0,\qquad \vec{H}=\mathrm{curl}\,\vec{A}, \qquad (25)$$

we obtain Maxwell's equations for the electromagnetic field

$$\mathrm{curl}\,\vec{H}=\frac{1}{c}\frac{\partial\vec{E}}{\partial t}+\frac{4\pi}{c}\vec{I}\qquad \mathrm{div}\,\vec{E}=4\pi\rho, \qquad (26)$$

where

$$\left.\begin{aligned}\rho &= \frac{ie}{4\pi\kappa^2\hbar c}\Big\{\tilde{U}\frac{1}{c}\frac{\partial U}{\partial t} - \frac{1}{c}\frac{\partial \tilde{U}}{\partial t}U - \frac{2ie}{\hbar c}A_0\tilde{U}U \\ &\quad - \tilde{U}\,\mathrm{grad}\,U_0 + \mathrm{grad}\,\tilde{U}_0\cdot U + \frac{ie}{\hbar c}(\tilde{U}A)U_0 + \frac{ie}{\hbar c}\tilde{U}_0(AU)\Big\} \\ I_x &= -\frac{ie}{4\pi\kappa^2\hbar}\Big\{\tilde{U}\frac{\partial U}{\partial x} - \frac{\partial \tilde{U}}{\partial x}U - \frac{2ie}{\hbar c}A_x\tilde{U}U \\ &\quad - (\tilde{U}\,\mathrm{grad})U_x - (U\,\mathrm{grad})\tilde{U}_x + \frac{ie}{\hbar c}(\tilde{U}A)U_x + \frac{ie}{\hbar c}\tilde{U}_x(AU)\Big\}, \text{ etc.}\end{aligned}\right\} \quad (27)$$

are the ~~ch~~ charge and current densities due to the presence of the heavy quanta.

If we introduce the variables canonically conjugate to U_0, U etc. defined by

$$\left.\begin{aligned} U_x^{+} &= \frac{\partial L}{\partial \frac{\partial U_x}{\partial t}} = -\frac{1}{4\pi\kappa c}\tilde{F}_x \\ &= \frac{1}{4\pi\kappa c}\Big\{\Big(\frac{1}{c}\frac{\partial}{\partial t} - \frac{ie}{\hbar c}A_0\Big)\tilde{U} + \Big(\mathrm{grad} + \frac{ie}{\hbar c}A\Big)\tilde{U}_0\Big\}, \text{ etc.} \\ U_0^{+} &= 0, \quad \text{etc.}\end{aligned}\right\} \quad (28)$$

in addition to the canonical variables for the ~~electric~~ electromagnetic field, the Hamiltonian for the total system becomes

$$\bar{H} = \iiint H\,dv \qquad (29)$$

with

$$\left.\begin{aligned} H &= H_U + H_E \\ H_U &= U^{+}\frac{\partial U}{\partial t} + \tilde{U}^{+}\frac{\partial \tilde{U}}{\partial t} - L_U \\ &= 4\pi\kappa^2c^2\tilde{U}^{+}U^{+} - \frac{ie}{\hbar}U^{+}A_0U - cU^{+}\Big(\mathrm{grad} - \frac{ie}{\hbar c}A\Big)U_0 - \frac{ie}{\hbar}\tilde{U}^{+}A_0\tilde{U} \\ &\quad - c\tilde{U}^{+}\Big(\mathrm{grad} + \frac{ie}{\hbar c}A\Big)U_0 + \frac{1}{4\pi\kappa^2}\Big\{\Big(\mathrm{grad} + \frac{ie}{\hbar c}A\Big)\times\tilde{U}\Big\}\Big\{\Big(\mathrm{grad} - \frac{ie}{\hbar c}A\Big)\times U\Big\} \\ &\quad + \frac{1}{4\pi}\tilde{U}U - 4\pi c^2\Big\{\Big(\mathrm{grad} + \frac{ie}{\hbar c}A\Big)U^{+}\Big\}\Big\{\Big(\mathrm{grad} - \frac{ie}{\hbar c}A\Big)\tilde{U}^{+}\Big\} \\ &\quad + \frac{ie}{\hbar}\tilde{U}^{+}A_0\tilde{U}^{+} \\ H_E &= \frac{1}{8\pi}(E^2 + H^2)\end{aligned}\right\} \quad (30)$$

In the quantum theory, the canonical variables U, $U^\dagger$, $\tilde{U}$ and $\tilde{U}^\dagger$ should satisfy the commutation relations (9) as in §2 and U_0 and $\tilde{U}_0$ should be eliminated by using the relations (28) and

$$\left(\mathrm{grad} - \frac{ie}{\hbar c}\vec{A}\right)F + \kappa U_0 = 0, \quad \left(\mathrm{grad} + \frac{ie}{\hbar c}\vec{A}\right)\tilde{F} + \kappa \tilde{U}_0 = 0 \qquad (31)$$

Thus, the field equations are obtained by constructing the equations of the ~~typ~~e (12).

If we change the variables for the U-field by Fourier transformation (15), the new variables satisfy again the commutation relations (17) and the Hamiltonian takes the form similar to (20). In this way, we can deal with the phenomena due to the interaction of the U-field with the electromagnetic field in any detail, but we consider, here, only the problem of the magnetic moment of the heavy quantum.

Now, we can deduce second order equations such as

$$\left\{\left(\frac{1}{c}\frac{\partial}{\partial t} + \frac{ie}{\hbar c}A_0\right)^2 - \left(\mathrm{grad} - \frac{ie}{\hbar c}\vec{A}\right)^2 + \kappa^2\right\}\vec{U}$$
$$- \frac{ie}{\hbar c}\left\{\vec{H}\times\vec{U} \mp \vec{E}U_0 + \underbrace{(\vec{E}\vec{F} - \vec{H}\vec{G})}_{\frac{1}{\kappa}(\mathrm{grad} - \frac{ie}{\hbar c}A)}\right\} = 0 \qquad (32)$$

from (21) and (22) by iteration. Thus, when a heavy quantum with the positive charge is present in a state of the energy $m_U c^2 + W$, where W is small compared with the proper energy $m_U c^2$, the equations (32) reduce to

$$\left\{-W + eA_0 - \frac{\hbar^2}{2m_U}\left(\mathrm{grad} - \frac{ie}{\hbar c}\vec{A}\right)^2\right\}\vec{U} - \frac{ie\hbar}{2m_U c}(\vec{H}\times\vec{U} \cancel{\mp \vec{E}\vec{F}}) = 0 \qquad (33)$$

in the first approximation, because U_0, ~~and~~ $\vec{G}$ $\overbrace{\text{are}}^{\text{and } \frac{1}{\kappa}\,\mathrm{div}\,F}$ small compared with ~~and~~ $\vec{U}$ in this case.

The equations (33) show that the heavy quantum has the magnetic moment $e\hbar/2m_u c \cdot \vec{\sigma}$, where $\vec{\sigma}$ is a vector with the components given by the matrices

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (34)$$

These matrices have each eigenvalues 1, 0 and -1, so that the magnitude of the magnetic moment becomes $e\hbar/2m_u c$, which is smaller than that of the ele~~c~~tron by the factor $m/m_u \cong 1/100$.

§5. Anomalous Magnetic Moments of the Neutron and the Proton.

As well known, the magnetic moments of the neutron and the proton have the values, which are in apparent condradiction with the assumption that they satisfy the relativistic wave equations of Dirac type. Wick[11] suggested that this anomaly might be attributed to the presence of the electrons in the intermediate states, which was expected from the theory of β-disintegration of Fermi and others, but the extra magnetic moments thus calculated was found to be far too small compared ~~with~~ with the actual values owing to the small probability of the virtual presence of the light particles.

Now, as shown in the preceeding section, the heavy quantum has the magnetic moment of amount $eh/2m_u c$ in nonrelativistic approximation. This is small by itself compared with the magnetic moment of the electron, but the ~~presence~~ probability of the virtual presence of the heavy quanta in the neighborhood of the heavy particle is very much larger than that of the electrons, on account of the strong interaction of the former with the heavy parti~~c~~cle, which ~~was~~ is considered to be the origin of the nuclear force in our theory.

11) Wick, Rend. Lincei 21, 170, 1935.

In fact, such a suggestion was proposed recently by one of the present authors, by Fröhlich and Heitler and by Bhabha[12].

~~Although~~ Since this problem can be solved exhaustively, only if the form of the interaction of the heavy particle with the U-field is determined in detail, as will be discussed in §6 and §7, we want to content ourselves, for the time being, with the rough estimation as follows.

~~When a neutron is in a state with the energy E,~~ The fraction of time, during which the neutron is splitting up into a proton and a heavy quantum with the negative charge virtually, is roughly given by $g^2/\hbar c$, where g is the constant characterizing the strength of the interaction of the heavy particle with the heavy quantum. Thus, the contribution of the heavy quanta to the magnetic moment of the neutron has the negative sign and the magnitude of the order

$$\frac{g^2}{\hbar c}\cdot\frac{e\hbar}{2m_U c} = \frac{g^2}{\hbar c}\cdot\frac{M}{m_U}\cdot\mu_n \tag{35}$$

where μ_n is the nuclear magneton and M is the mass of the heavy particle. Now, $g^2/\hbar c$ and M/m_U have the orders of magnitude 1/10 and 10 respectively, so that (35) is equal to the nuclear magneton multiplied by a number of the order of 1. It is clear from the symmetry considerations that the extra magnetic moment of the proton due to the virtual presence of the heavy quantum with the positive charge has approximately the same magnitude as that of the neutron, but the sign is positive. These results agree with the experiment both in sign and in the order of magnitude.

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12) Taketani, loc. cit.; Frohlich and Heitler, loc. cit.; Bhabha, loc. cit.

* on page 1087, II, ~~are not correct~~

§6. Interaction of the U-Field with the Heavy Particle.

In the preceeding paper (II, §3), we found that the scalar U-field was not adequate for describing the interaction between the neutron and the proton, since the exchange force of Heisenberg type thus obtained had the sign opposite to the one required from the experimental result. Now, the U-field considered in this paper consists of two four vectors and two six vectors, so that there ~~are~~ is ample space for various types of interaction between the U-field and the heavy particle, which in turn result in various types of forces between the neutron and the proton as second order effects.

If we ignore the electromagnetic field altogether, the most general equations for the U-field in the presence of the heavy particle may have the form

$$\frac{1}{c}\frac{\partial \vec{F}}{\partial t} - \mathrm{curl}\,\vec{G} - \kappa\vec{U} = -4\pi g_1 \vec{M}, \quad \mathrm{div}\,\vec{F} + \kappa U_0 = 4\pi g_1 M_0, \tag{36}$$

$$\frac{1}{c}\frac{\partial \vec{U}}{\partial t} + \mathrm{grad}\,U_0 + \kappa\vec{F} = 4\pi g_2 \vec{T}, \quad \mathrm{curl}\,\vec{U} - \kappa\vec{G} = -4\pi g_2 \vec{S}, \tag{37}$$

where $\vec{M}$ and M_0 are the space and time components of a four vector, which can be expressed each by a ~~linear~~ bilinear form of the wave functions ~~of the~~ for the heavy particle. If we assume the heavy particle to satisfy the wave equations of Dirac's type, the simplest expressions for $\vec{M}$, M_0 become

以下 $\tilde{Q}$ を全部 Q^* にかえること.

$$\vec{M} = \tilde{\Psi}(\vec{\alpha}\,\tilde{Q}^*)\Psi, \quad M_0 = \tilde{\Psi}\tilde{Q}^*\Psi, \tag{38}$$

where Ψ, $\tilde{\Psi}$ are the wave functions for the heavy particle, which have each eight components and satisfy the same commutation relations as in II. $\vec{\alpha}$ is the Dirac's spin vector and $\tilde{Q}$ is the operator changing the heavy particle ~~from~~ from the proton to the neutron state.* $\vec{T}$ and $\vec{S}$ are two space vectors forming a six vector, which can be constructed in the way similar to $\vec{M}$, M_0. The simplest possible expressions for them are

$$\vec{T} = -\tilde{\Psi}\rho_2\vec{\sigma}\,\tilde{Q}\Psi, \quad \vec{S} = \tilde{\Psi}\rho_3\vec{\sigma}\,\tilde{Q}\Psi, \tag{39}$$

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*) ~~The operator changing the~~

) Q^* can be ~~represented by the~~ written in the form $Q^* = \frac{\tau_1 + i\tau_2}{2}$. If we use the isotopic spin matrices τ_1, τ_2, τ_3, ~~The si-~~ ~~It~~ should be noticed that in the relations ~~on~~ page immediately below the eq. (40) there ~~are~~ mistakes in ~~the~~ ~~signs~~;

corresponding to (38), where ρ's and $\vec{\sigma}$ ~~has~~ have the same meanings as those employed in Dirac's theory of the electron usually. g_1 and g_2 are the constants with the dimension of the electric charge as the constant g in I and II and characterize the strengths of two types of interactions above considered.

The field equations (36) can be derived from the Lagrangian

$$\bar{L} = \iiint L\, dv \tag{40}$$

with

$$L = \frac{1}{4\pi}(\tilde{F}F - \tilde{G}G + \tilde{U}_0 U_0 - \tilde{U}U) + \frac{g_1}{\kappa}(\tilde{U}M - \tilde{U}_0 M_0 + U\tilde{M} - U_0\tilde{M}_0), \tag{41}$$

where U, U_0, $\tilde{U}$ and $\tilde{U}_0$ are considered as independent variables and F, G, $\tilde{F}$ and $\tilde{G}$ as defined by (37) and the equations complex conjugate to (37). Hence, the variables canonically conjugate to U_0, U etc. become

$$U_0^+ = \frac{\partial L}{\partial \frac{\partial U_0}{\partial t}} = 0, \quad U_x^+ = \frac{\partial L}{\partial \frac{\partial U_x}{\partial t}} = -\frac{\tilde{F}_x}{4\pi\kappa c}, \quad \text{etc.}, \tag{42}$$

so that we obtain conversely

$$\frac{\partial \tilde{U}}{\partial t} = 4\pi\kappa^2 c^2 U^+ - c \operatorname{grad} \tilde{U}_0 + 4\pi g_2 c \tilde{T}, \quad \text{etc.} \tag{43}$$

conversely by the help of the relations (37).

The Hamiltonian, thus, takes the form

$$\bar{H} = \iiint H\, dv \tag{44}$$

with

$$\left.\begin{aligned} H = {} & U^+ (4\pi\kappa^2 c^2 \tilde{U}^+ - c \operatorname{grad} U_0 + 4\pi g_2 c T) \\ & + \tilde{U}^+ (4\pi\kappa^2 c^2 U^+ - c \operatorname{grad} \tilde{U}_0 + 4\pi g_2 c \tilde{T}) \\ & - 4\pi\kappa^2 c^2 U^+ \tilde{U}^+ + \frac{1}{4\pi\kappa^2}(\operatorname{curl} \tilde{U} + 4\pi g_2 \tilde{S})(\operatorname{curl} U + 4\pi g_2 S) \\ & - \frac{1}{4\pi}(\tilde{U}_0 U_0 - \tilde{U}U) + \frac{g_1}{\kappa}(\tilde{U}_0 M_0 - \tilde{U}M + U_0 \tilde{M}_0 - U\tilde{M}) \end{aligned}\right\} \tag{45}$$

In the quantum theory, U_x, $U_x^\dagger$ etc. should satisfy the commutation relations of the type (9), and U_0 and $\tilde{U}_0$ should be eliminated by using the relations

$$U_0 = 4\pi c \operatorname{div} \tilde{U}^\dagger + \frac{4\pi g_1}{\kappa} M_0, \quad \tilde{U}_0 = 4\pi c \operatorname{div} U^\dagger + \frac{4\pi g_1}{\kappa} \tilde{M}_0 \tag{46}$$

The Hamiltonian becomes thus

$$\bar{H} = \bar{H}_U + \bar{H}' = \iiint H_U \, dv + \iiint H' \, dv$$

with

$$\left.\begin{aligned} H_U &= 4\pi\kappa^2 c^2 \tilde{U}^\dagger U^\dagger + 4\pi c^2 \operatorname{div} U^\dagger \operatorname{div} \tilde{U}^\dagger + \frac{1}{4\pi\kappa^2} \operatorname{curl} \tilde{U} \cdot \operatorname{curl} U \\ &\quad + \frac{1}{4\pi} \tilde{U} U . \\ H' &= \frac{4\pi g_1 c}{\kappa} (\operatorname{div} U^\dagger \cdot M_0 + \operatorname{div} \tilde{U}^\dagger \cdot \tilde{M}_0) - \frac{g_1}{\kappa} (\tilde{U} M + U \tilde{M}) \\ &\quad + 4\pi g_2 c (U^\dagger T + \tilde{U}^\dagger \tilde{T}) + \frac{g_2}{\kappa^2} (\operatorname{curl} U \cdot \tilde{S} + \operatorname{curl} \tilde{U} \cdot S) \\ &\quad + \frac{4\pi}{\kappa^2} (g_1^2 \tilde{M}_0 M_0 + g_2^2 \tilde{S} S). \end{aligned}\right\} \tag{47}$$

In these expressions, $\bar{H}_U$ denotes the energy of the U-field in vacuum as in §2 and $\bar{H}'$ the energy of interaction between the heavy particle and the U-field. The last two terms in H' involving g_1^2 and g_2^2 respectively depend only on the wave functions of the heavy particle and can be considered as due to ~~corresponding to~~ the zero range force between the neutron and the proton. Such terms appear aslo in the calculation of the second order effects and ~~they~~ compensate with ~~one another partly~~ the terms above considered, as will be shown in §7.

Now, the equations of motion for U, $U^\dagger$ etc., which are equivalent to the field equations (36), (37), are derived from the above Hamiltonian in the usual manner and become

$$\left.\begin{aligned} \dot{U} &= 4\pi\kappa^2 c^2 \tilde{U}^\dagger - 4\pi c^2 \operatorname{grad} \operatorname{div} \tilde{U}^\dagger + 4\pi g_2 c T \\ &\quad - \frac{4\pi g_1 c}{\kappa} \operatorname{grad} M_0 \\ -\dot{\tilde{U}}^\dagger &= \frac{1}{4\pi\kappa^2} \operatorname{curl} \operatorname{curl} U + \frac{1}{4\pi} U + \frac{g_2}{\kappa^2} \operatorname{curl} S - \frac{g_1}{\kappa} M \end{aligned}\right\} \tag{48}$$

From these equations, we obtain at once the quadratic ~~d~~ equations

$$\left.\begin{aligned}
&\frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} - \Delta U + \kappa^2 U = 4\pi g_1\left(\kappa M - \frac{1}{\kappa}\,\mathrm{grad}\,\mathrm{div}\,M - \frac{1}{\kappa}\,\mathrm{grad}\,\frac{1}{c}\frac{\partial M_0}{\partial t}\right)\\
&\qquad\qquad + 4\pi g_2\left(\frac{1}{c}\frac{\partial T}{\partial t} - \mathrm{curl}\,S\right)\\
&\frac{1}{c^2}\frac{\partial^2 \tilde{U}^+}{\partial t^2} - \Delta \tilde{U}^+ + \kappa^2 \tilde{U}^+ = \frac{g_1}{\kappa c}\left(\mathrm{grad}\,M_0 + \frac{1}{c}\frac{\partial M}{\partial t}\right)\\
&\qquad\qquad - \frac{g_2}{c}\left(T + \frac{1}{\kappa^2}\,\mathrm{curl}\,\mathrm{curl}\,T + \frac{1}{\kappa^2}\,\mathrm{curl}\,\frac{1}{c}\frac{\partial S}{\partial t}\right)
\end{aligned}\right\} \tag{49}$$

The complete Hamiltonian for the system consisting of the U-field and the heavy particle is

$$\bar{H} = \iiint (H_U + H' + H_M)\,dv \tag{50}$$

with H_U, H' given by (47) and

$$H_M = \tilde{\Psi}\left\{c\vec{\alpha}\vec{p} + \beta\left(\frac{1+\tau_3}{2}M_N c^2 + \frac{1-\tau_3}{2}M_P c^2\right)\right\}\Psi, \tag{51}$$

if we ignore the electromagnetic interaction between the heavy quantum ~~and~~ and the proton, where $\vec{p} = -i\hbar\,\mathrm{grad}$.

By using this Hamiltonian and the commutation relations

$$\left.\begin{aligned}
&\Psi^{(i)}(r,t)\tilde{\Psi}^{(j)}(r',t) + \tilde{\Psi}^{(j)}(r',t)\Psi^{(i)}(r,t) = \delta_{ij}\,\delta(r,r')\\
&\Psi^{(i)}(r,t)\Psi^{(j)}(r',t) + \Psi^{(j)}(r',t)\Psi^{(i)}(r,t) = 0\\
&\tilde{\Psi}^{(i)}(r,t)\tilde{\Psi}^{(j)}(r',t) + \tilde{\Psi}^{(j)}(r',t)\tilde{\Psi}^{(i)}(r,t) = 0
\end{aligned}\right\} \tag{52}$$

for i, j = 1, 2, ... 8, we obtain the quantized wave equations for the heavy particle

Mc^2

$$\begin{aligned}
i\hbar\frac{\partial\Psi}{\partial t} = \Big[&c\vec{\alpha}\vec{p} + \beta\left(\frac{1+\tau_3}{2}M_N c^2 + \frac{1-\tau_3}{2}M_P c^2\right)\\
&+ \frac{4\pi g_1 c}{\kappa}\left(\mathrm{div}\,U^+\cdot\tilde{Q} + \mathrm{div}\,\tilde{U}^+\cdot Q\right) - \frac{g_1}{\kappa}\left\{(\tilde{U}\alpha)\tilde{Q} + (U\alpha)Q\right\}\\
&- 4\pi g_2 c\left\{(U^+\sigma)\rho_2\tilde{Q} + (\tilde{U}^+\sigma)\rho_2 Q\right\} + \frac{g_2}{\kappa^2}\left\{\mathrm{curl}\,\tilde{U}\cdot\rho_3\vec{\sigma}\tilde{Q} + \mathrm{curl}\,U\cdot\rho_3\vec{\sigma}Q\right\}\\
&+ \frac{4\pi g_1^2}{\kappa^2}\left(\tilde{M}_0\tilde{Q} + M_0 Q\right) + \frac{4\pi g_2^2}{\kappa^2}\left\{(\tilde{S}\sigma)\rho_3\tilde{Q} + (S\sigma)\rho_3 Q\right\}\Big]\Psi
\end{aligned} \tag{53}$$

provided that M, M_0, $\vec{T}$, $\vec{S}$ in H' is given by the simplest expressions (38) and (39).

$\left(+\frac{4\pi g_1^2}{\kappa^2}\frac{1+\tau_3}{2}\delta.\right.$　　$\left(+\frac{12\pi g_2^2}{\kappa^2}\frac{1+\tau_3}{2}\delta.\right.$

§7. Deduction of Exchange Forces between the Neutron and the Proton.

The interaction between the neutron and the proton, which is caused by virtual absorption and emission of the heavy quanta, can be calculated from the Hamiltonian (50) as second order effect by straightforward application of the perturbation theory as in §3, II. Namely, we first transform the 一字アケ variables for the unperturbed system with the Hamiltonian $\bar{H}_0 = \bar{H}_U + \bar{H}_M$ into the normal coordinates, then express the perturbation energy $\bar{H}'$ in terms of these coordinates and perform the calculation ~~of~~ to the second order, until we arrive at the required formula. An alternative method, which is similar to that used in §2, I, leads to the final result more quickly, so that only this method will be mentioned in the following. We verified that the results of these ~~to~~ two methods are identical.

We consider two heavy particles with the coordinates $\vec{r}_j$, the momenta $\vec{p}_j$, the spin matrices $(\rho_1^{(j)}, \rho_2^{(j)}, \rho_3^{(j)})$, $\vec{\sigma}^{(j)}$ and the isotopic spin matrices $\tau_1^{(i)}, \tau_2^{(i)}, \tau_3^{(i)}$ where j = 1, 2 denote the first and the second particles respectively. When the kinetic energies of two particles are both ~~sm~~ small compared with $m_U c^2$, $\rho_1^{(j)} \vec{\sigma}^{(j)}$, $\rho_2^{(j)} \vec{\sigma}^{(j)}$ are negligibly small and $\rho_3^{(j)} \vec{\sigma}^{(j)}$ is approximately equal to $\vec{\sigma}^{(j)}$. In this approximation, the wave equations (53) reduce to

$$i\hbar \frac{\partial \Psi}{\partial t} = \left\{ c\vec{\alpha}\vec{p} + \beta\left(\frac{1+\tau_3}{2} M_N c^2 + \frac{1-\tau_3}{2} M_P c^2 \right) + H' \right\} \Psi \tag{54}$$

where
$$H' = \frac{4\pi g_1 c}{\kappa}\left(\mathrm{div}\, \mathrm{U}^+ \cdot \tilde{Q} + \mathrm{div}\, \tilde{\mathrm{U}}^+ \cdot Q \right) + \frac{g_2}{\kappa^2}\left(\mathrm{curl}\, \tilde{\mathrm{U}} \cdot \vec{\sigma} \tilde{Q} + \mathrm{curl}\, \mathrm{U} \cdot \vec{\sigma} Q \right) + \frac{4\pi g_1^2}{\kappa^2}\left(\tilde{M}_0 \tilde{Q} + M_0 Q \right) + \frac{4\pi g_2^2}{\kappa^2}\left\{ (\tilde{S}\sigma)\tilde{Q} + (S\sigma)Q \right\}$$

~~Similarly,~~ Moreover, the terms in (49), ~~whet~~ which involve the time derivatives can be neglected, so that we obtain

$$\left.\begin{aligned} -\Delta \mathrm{U} + \kappa^2 \mathrm{U} &= 4\pi g_2\, \mathrm{curl}\, S \\ -\Delta \tilde{\mathrm{U}}^+ + \kappa^2 \tilde{\mathrm{U}}^+ &= \frac{g_1}{\kappa e}\, \mathrm{grad}\, M_0 \end{aligned}\right\} \tag{55}$$

where

$$M_0 = \tilde{\Psi}\tilde{Q}\Psi, \qquad \vec{S} = \tilde{\Psi}\vec{\sigma}\tilde{Q}\Psi$$

These equations show that the U-field at $\vec{r}_1$ due to the presence of a heavy particle at $\vec{r}_2$ is given by

$$\left.\begin{aligned} U(\vec{r}_1) &= -g_2 \,\mathrm{curl}\left(\vec{\sigma}^{(2)} \frac{e^{-\kappa r}}{r} Q_2\right) \\ \tilde{U}^{+}(\vec{r}_1) &= \frac{g_1}{4\pi\kappa c} \,\mathrm{grad}\, \frac{e^{-\kappa r}}{r} \tilde{Q}_2, \end{aligned}\right\} \tag{56}$$

where $\vec{r} = \vec{r}_1 - \vec{r}_2$.

Inserting these expressions into (55), we obtain the energy of interaction between two particles at $\vec{r}_1$ and $\vec{r}_2$ respectively

$$\begin{aligned} H_{12} = (Q_1\tilde{Q}_2 + \tilde{Q}_1 Q_2)\Big\{ &\frac{g_1^2}{\kappa^2} \,\mathrm{div}\,\mathrm{grad}\, \frac{e^{-\kappa r}}{r} + \frac{4\pi g_1^2}{\kappa^2}\delta(\vec{r}) \\ &- \frac{g_2^2}{\kappa^2} \vec{\sigma}^{(1)} \,\mathrm{curl}\,\mathrm{curl}\, \frac{\vec{\sigma}^{(2)} e^{-\kappa r}}{r} + \frac{4\pi g_2^2}{\kappa^2}(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})\delta(\vec{r})\Big\} \end{aligned} \tag{57}$$

By the help of the relation

$$\Delta \frac{e^{-\kappa r}}{r} = \kappa^2 \frac{e^{-\kappa r}}{r} - 4\pi\delta(\vec{r}), \tag{58}$$

the terms containing δ functions disappear and (57) reduces to the simple form

$$H_{12} = \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}\left\{ g_1^2 + g_2^2(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) - g_2^2 \frac{(\vec{\sigma}^{(1)}\mathrm{grad})(\vec{\sigma}^{(2)}\mathrm{grad})}{\kappa^2} \right\} \frac{e^{-\kappa r}}{r}$$

or

$$\begin{aligned} = \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}\Big[&g_1^2 + g_2^2\left\{(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) - \frac{(\vec{\sigma}^{(1)}\vec{r})(\vec{\sigma}^{(2)}\vec{r})}{r^2}\right\} \\ &+ g_2^2\left\{(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) - \frac{3(\vec{\sigma}^{(1)}\vec{r})(\vec{\sigma}^{(2)}\vec{r})}{r^2}\right\}\left(\frac{1}{\kappa r} + \frac{1}{\kappa^2 r^2}\right)\Big] \frac{e^{-\kappa r}}{r} \end{aligned} \tag{59}$$

As well known, this is a combination of exchange forces of Majorana and Heisenberg types between the neutron and the proton.[13] It should be remarked, however, that the force thus obtained is not strictly central, so that we can separate S state, P state etc. only in the first approximation.

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13) Similar formula was obtained by Kemmer, loc. cit., which had three arbitrary constants A, B, C instead of g_1 and g_2.

Especially, when the system consisting of a neutron and a proton is approximately in the S state as in the case of the deuteron, the wave function for the system is spherically symmetric with respect to $\vec{r}$, so that we have the average value

$$\overline{\frac{(\vec{\sigma}^{(1)}\vec{r})(\vec{\sigma}^{(2)}\vec{r})}{r^2}} = \frac{1}{3}\vec{\sigma}^{(1)}\vec{\sigma}^{(2)} \tag{60}$$

Thus, in S state, the interaction energy reduces to

$$H_{12} = \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}\left\{ g_1^2 + \frac{2g_2^2}{3}(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})\right\}\frac{e^{-\varkappa r}}{r}$$

$$= P^H\left(g_1^2 - \frac{2}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r} + P^M\frac{4g_2^2}{3}\frac{e^{-\varkappa r}}{r} \tag{61}$$

where

$$P^H = \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}, \quad P^M = \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}\frac{(1+\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})}{2}$$

are the exchange operators of Heisenberg and Majorana respectively. The expression (61) shows that the Majorana force is always attractive in the S state, while the Heisenberg force can be made attractive for ^3S state and repulsive for ^1S state by taking the constants g_1 larger than ~~$g_2/\sqrt{3}$~~ $\sqrt{\frac{2}{3}}g_2$, in conformity with the assumptions of the current theory. The relative magnitude of the Heisenberg and Majorana forces is $3g_1^2 - 2g_2^2 : 4g_2^2$ and already the simplest assumption $g_1 = g_2$ gives the ratio 1:4, which is not far from that accepted in the current theory. Thus, our theory can deduce, in a natural way, the exchange force between the neutron and the proton, which is correct in range, sign and magnitude. In the higher states such as P, D etc., the expression H_{12} for the exchange force becomes more complicated, which can not be separated from the S state, ~~and will be considered in detail elsewhere.~~ so that we can expect considerable change in the theory of the deuteron. This problem will be dealed with in detail elsewhere.

In order to obtain the short range forces between two neutrons or two protons, as well as the ordinary forces of Wigner and Bartlett types between the neutron and the proton, we have to calculate the perturbation energy of the fourth order. It was shown, however, in II, that these forces was smaller by 一字アケ afactor 10 than the exchange force of the second order, so that the introduction of the neutral heavy quanta was felt necessary in order to reproduce the approximate equality of the like particle and unlike particle forces assumed in the current theory. These conclusions seem to be true in the present case and indeed, ~~is~~ it is not difficult to consider the field accompanied by the neutral heavy quanta and described by the linear equations similar to those considered above. The detailed discussions of these subjects will be made in the next paper.

§8. Creation and Annihilation of the Heavy Quanta.

In the preceeding sections, we considered the ~~contribution of the~~ effects due to the virtual presence of the heavy quanta in the intermediate states. Now, if the energy greater than $m_u c^2$ is supplied, a heavy quantum can be created, but it will soon be annihilated by collision with matter as discussed in §5, II. It should be noticed, furtehr, that a heavy quantum disappears even in the free space by emitting a positive ~~and~~ or negative electron and a neutrino or an antineutrino simultaneously according as the charge of the heavy quantum is positive or negative, as already pointed out by Bhabha[14]. In this case, the conservation laws are guaranteed by the proper energy of the heavy quantum. Owing to the small interaction of the heavy quantum with the

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14) Bhabha, loc. cit.

light particle, the probability of occurrence of the above process is so small that the mean free path of the high speed heavy quantum in the free space is large compared with the dimension of the measuring apparatus, but is not small enough, in many cases, to make the mean free path larger than the height of the whole atmosphere, as will be shown presently. Hence, if we consider the heavy quanta as the main constituent of the hard component of the cosmic ray, those which are observed on sea level should have been created, for the most part, in the atmosphere. These conclusions are in 一字アケ accord with recent arguments of Bowen, Millikan and Neher[15] and Bhabha[16]. In connection with this, the problem of the creation of the heavy quanta becomes very significant, ~~so~~ so that it will also be briefly discussed in the following.

We consider a heavy quantum with the negative charge, for example, which is at rest with respect to a certain coordinate system. The annihilation of it is accompanied by the transition of the light particle from the neutrino ~~to~~ state of the negative energy to the electron state of the positive energy. In the above coordinate system, the conservation laws require that the magnitudes of the energies of both states are equal to $m_u c^2/2$ and the momenta are equal, to ~~be~~ each other, $\hbar k$ say, with the magnitude ~~.....~~ $\hbar k = m_u c/2$.

According to ordinary perturbation theory, the probability per unit time of annihilation by emitting the electron in the direction within the solid angle $d\Omega$ is given by

$$dw_0 = \frac{2\pi}{\hbar} \sum |V|^2 \frac{k^2 dk}{(2\pi)^3 dE} d\Omega , \qquad (62)$$

15) Bowen, Millikan and Neher, Phys. Rev. 52, 80, 1937; ibid. 53, 217, 1938. See also Blackett, Proc. Roy. Soc. A. 164, S 7, 1938.
16) Bhabha, loc. cit. and further Bhabha, Proc. Roy. Soc. A. 164, 257, 1938.

if we consider the whole system to be in the unit ~~volume~~ cube, where E is the total energy of the final ~~system~~ state and is equal to $m_u c^2$ in this case, so that (62) reduces at once to

$$dw_0 = \frac{m_u c}{32\pi^2\hbar^4}\sum |V|^2 d\Omega \tag{63}$$

V is the matrix element of the energy of interaction of the U-field with the light particle corresponding to the above transition, and $\sum$ means the summation with respect to the spins of the electron and the neutrino.

Now, if we assume the interaction in this case has the same form as that between the U-field and the heavy particle, we can use the formula for H'~~s~~ as given by (47) with the modification that the constants g_1 and g_2 are replaced by smaller constants, g_1' and g_2' say, and the wave functions, spin matrices etc. for the heavy particle by the corresponding quantities for the light particle. As the heavy quantum is at rest in our case, the derivatives of U, U^+ etc. vanish, so that the energy of interaction becomes simply

$$\overline{H'} = -\frac{g_1'}{\kappa}\iiint \tilde{U}\tilde{\psi}\sigma\varphi\, dv - 4\pi g_2' c\iiint U^+\tilde{\psi}\rho_2\sigma\varphi\, dv \tag{64}$$

where ψ, φ are the wave functions for the electron and the neutrino respectively. If we denote the direction of polarization of the heavy quantum by the unit ~~a~~ vector $\vec{e}$, the matrix element takes the form

$$V = -\sqrt{\frac{2\pi\hbar c}{\kappa}}\left\{g_1'(\tilde{u}\rho_1\vec{\sigma}\vec{e}v) + i g_2'(\tilde{u}\rho_2\vec{\sigma}\vec{e}v)\right\} \tag{65}$$

where u, v are the constant spinors representing the amplitudes of the wave functions of the electron and the neutrino respectively.

By inserting (65) in (63) and by performing the summation ~~and~~ with respect to the spins and the integration with respect to the direction of emission, we obtain the total probability per unit time of annihilation

$$w_0 = \iint dw_0 = \frac{2g_1'^2 + g_2'^2}{6\hbar c}\frac{m_u c^2}{\hbar}. \tag{66}$$

The coressponding probability, when the heavy quantum is moving with the velocity v and energy E, is reduced to

$$w = w_0 \cdot \sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{2g_1'^2 + g_2'^2}{6\hbar c} \cdot \frac{m_u c^2}{\hbar} \cdot \frac{m_u c^2}{E} \qquad (67)$$

owing to the change of time scale under Lorentz transformation. The mean life time τ and the mean free path λ of the heavy quantum with the energy E can be defined by the relations

$$\tau = \frac{1}{w}, \qquad \lambda = v\tau \qquad (68)$$

If we take $g_1' = g_2' = g' = 4\times 10^{-17}$, a value which was determined from the ~~pp~~ probability of β-disintegration in §4, I, and $m_u = 100\, m$,[17] we obtain

$$w = 2\times 10^{8}\, \frac{mc^2}{E} \qquad (69)$$

The numerical values of τ and λ corresponding to this w are shown in Table 1 for several values of the energy.

Table 1.

Kinetic Energy	$E - m_u c^2$	0	10^9	10^{10}	10^{11}	10^{12}	eV
Mean Life Time	τ	5×10^{-7}	10^{-5}	10^{-4}	10^{-3}	10^{-2}	sec
Mean Free Path	λ	——	3	30	300	3000	km

Thus, even the heavy quantum with the energy 10^{12}eV can travel only a distance smaller than the radius of the earth, before it changes into an electron and an antineutrino. The bearing of this conclusion on the interpretation of the hard component of the cosmic ray was already discussed above.

On the other hand, there are many processes, which are connected with the creation of the heavy quanta, such as the creation of a pair of positive and negative quanta by γ-ray of energy larger than $2m_u c^2$, the emission of one or more heavy quanta by nuclear disintegration caused by high energy radiations of various sorts, etc. Among them, the creation of a pair by γ-ray has the

17) The mass of the new particle in cosmic ray, which is considered to be identified with the heavy quantum in our theory, is not yet known accurately. Very recently, a value 120 m was obtained by Ruhlig and Crane, Phys. Rev. 53, 266, which is in close agreement with the value 130 m of Street and Stevenson, but is considerably smaller than those of Nishina, Takeuchi and Ichimiya and Corson and Brode.

cross section, which is roughly about $(m/m_U)^2$ times that of the creation by γ-ray of a pair of electrons. Hence, if the primary cosmic ray consists exclusively of the positive and negative electrons and consequently, is accompanied by about equal number of light quanta produced by nuclear collision, the number of the secondary heavy quanta per one primary electron should have the order of magnitude 10^{-4}, which seems to be 一字アケ a little too small to account for the experimental result of Bowen, Millikan and Neher[18] as pointed out by Bhabha.[19] In this connection, it is important to determine how large are the contributions of other processes to the creation of heavy quanta.

In conclusion, it should be remarked that the interaction of the U-field with the light particle, as well as that with the heavy particle, has rather complicated form in the present theory, so that the force between the light and heavy particles, which is responsible for the β-disintegration, can be identified neither with the force of Fermi type nor with that of Konopinski-Uhlenbeck type, but will be a combination of various types of forces. The detailed discussion of these subjects, however, will be made in the next paper.

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18) Bowen, Millikan and Neher, loc. cit.

19) Bhabha, Proc. Roy. Soc. A. 164, 257, 1938.

§7. Deduction of Exchange Forces between the Neutron and the Proton.

The interaction between the neutron and the proton, which is caused by virtual emission and absorption of the heavy quanta, can be calculat~~e~~ed from the Hamiltonian (50) as second order effect by straightforward application of the perturbation theory, as in §3, II. Namely, we first transform the variables for the unperturbed system with the Hamiltonian $\overline{H_o}^{1} = H + H$