

E 04030P14

DEPARTMENT OF PHYSICS
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第六回 理論212中のα粒子
(1229+2295)の土佐

DATE April May

On the Interaction of Elementary Particles. IV
g-p.

§3. Theory of the deuteron.

Neutron & Proton とか system への非相対論的 wave equation ψ , 中心の運動を separate して,

$$\left\{ \frac{\hbar^2}{M} \Delta + E - V \right\} \psi = 0$$

ポテンシャル V は,

$$V = \epsilon \left\{ g_1^2 + g_2^2 (\vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)}) - g_2^2 \frac{(\sigma^{(1)} \text{grad})(\sigma^{(2)} \text{grad})}{x^2} \right\} \frac{e^{-\kappa r}}{r}$$

$\epsilon_{\pm} = \mp 1$ for $\left\{ \begin{array}{l} \text{symmetric state } ^3S, ^1P, ^3D, \dots \\ \text{antisymmetric state } ^1S, ^3P, ^1D, \dots \end{array} \right.$

V を考慮して

$$V = \epsilon_{\pm} \left[g_1^2 + \frac{2}{3} g_2^2 (\vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)}) + \frac{g_2^2}{3} (\vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)}) \left(1 + \frac{3}{\kappa r} + \frac{3}{\kappa^2 r^2} \right) \right] \frac{e^{-\kappa r}}{r}$$

triplet state $\kappa r \ll 1$
 $(\vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)}) = +1$

singlet state $\kappa r \ll 1$
 -3

$$\text{Spin } (\vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)}) = \begin{pmatrix} z & x-iy \\ x+iy & -z \end{pmatrix}_1 \begin{pmatrix} z & x-iy \\ x+iy & -z \end{pmatrix}_2$$

V は $\vec{r} \times \vec{p}$ である orbital angular momentum とは commute
total angular momentum
 $(\vec{r} \times \vec{p}) + \frac{\hbar}{2} (\vec{\sigma}^{(1)} + \vec{\sigma}^{(2)})$

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Triplet state,

${}^3S_1, {}^3D_1$

Wigner, Gruppentheorie, S. 208, Braunschweig, 1931

$$\chi_{-1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 \quad \chi_0 = \frac{1}{\sqrt{2}} \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 + \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \right\}$$

$$\chi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2$$

$${}^3S_1 : \frac{u(r)}{\sqrt{4\pi} r} \chi_{-1}, \left(\frac{u(r)}{\sqrt{4\pi} r} \chi_0, \frac{u(r)}{\sqrt{4\pi} r} \chi_1 \right)$$

$${}^3D_1 : \Psi_{-1} = \sqrt{\frac{1}{10}} (\sqrt{3} \Psi_0 \chi_{-1} - \sqrt{3} \Psi_{-1} \chi_0 + \sqrt{6} \Psi_{-2} \chi_1)$$

$$\Psi_0 = \sqrt{\frac{1}{10}} (\sqrt{3} \Psi_1 \chi_{-1} - 2 \Psi_0 \chi_0 + \sqrt{3} \Psi_{-1} \chi_1)$$

$$\Psi_1 = \sqrt{\frac{1}{10}} (\sqrt{6} \Psi_2 \chi_{-1} - \sqrt{3} \Psi_1 \chi_0 + \Psi_0 \chi_1)$$

$$\Psi_0 = \sqrt{\frac{5}{4\pi}} \left(\frac{1}{2} \cos^2 \theta - \frac{1}{2} \right) \left. \begin{array}{l} = \sqrt{\frac{5}{4\pi}} \cdot \frac{3}{2} \left(\cos^2 \theta - \frac{1}{3} \right) \end{array} \right\}$$

$$\Psi_{\pm 1} = \sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{\pm i\varphi} \left. \begin{array}{l} = \sqrt{\frac{5}{4\pi}} \cdot \sqrt{\frac{3}{4}} \sin \theta \cos \theta e^{\pm i\varphi} \end{array} \right\}$$

$$\Psi_{\pm 2} = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{\pm 2i\varphi} \left. \begin{array}{l} = \sqrt{\frac{5}{4\pi}} \cdot \sqrt{\frac{3}{8}} \sin^2 \theta e^{\pm 2i\varphi} \end{array} \right\}$$

χ_{-1} & combine to form Ψ_{-1} , etc,
 $j_2 = 0, 1$ or $j_2 = 1, 0, -1$

$$\Psi = \frac{u(r)}{r} \frac{\chi_{-1}}{\sqrt{4\pi}} + \frac{v(r)}{r} \Psi_{-1}$$

It is a solution to the Schrödinger equation.

$$\frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} = \begin{pmatrix} \cos \theta & \sin \theta e^{-i\varphi} \\ \sin \theta e^{i\varphi} & -\cos \theta \end{pmatrix}_1 \begin{pmatrix} \cos \theta & \sin \theta e^{-i\varphi} \\ \sin \theta e^{i\varphi} & -\cos \theta \end{pmatrix}_2$$

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$$\frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} \chi_{-1} = \cos^2 \theta \chi_{-1} - \sqrt{2} \cos \theta \sin \theta e^{-i\varphi} \chi_0 + \sin^2 \theta e^{2i\varphi} \chi_1$$

$$'' \chi_0 = -\sqrt{2} \cos \theta \sin \theta e^{i\varphi} \chi_{-1} + (1 - 2 \cos^2 \theta) \chi_0 + \sqrt{2} \sin^2 \theta e^{-i\varphi} \chi_1$$

$$'' \chi_{+1} = \sin^2 \theta e^{2i\varphi} \chi_{-1} + \sqrt{2} \cos \theta \sin \theta e^{i\varphi} \chi_0 + \cos^2 \theta \chi_1$$

$$\left\{ \frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{3} \right\} \begin{pmatrix} \chi_{-1} \\ \chi_0 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} (\cos^2 \theta - \frac{1}{3}) \chi_{-1} - \sqrt{2} \cos \theta \sin \theta e^{-i\varphi} \chi_0 + \sin^2 \theta e^{2i\varphi} \chi_1 \\ -\sqrt{2} \cos \theta \sin \theta e^{i\varphi} \chi_{-1} + 2(\cos^2 \theta - \frac{1}{3}) \chi_0 + \sqrt{2} \sin^2 \theta e^{-i\varphi} \chi_1 \\ \sin^2 \theta e^{2i\varphi} \chi_{-1} + \sqrt{2} \cos \theta \sin \theta e^{i\varphi} \chi_0 + (\cos^2 \theta - \frac{1}{3}) \chi_1 \end{pmatrix}$$

$$\chi_{-1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \chi_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \chi_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{3} = \begin{pmatrix} \cos^2 \theta - \frac{1}{3} & -\sqrt{2} \cos \theta \sin \theta e^{+i\varphi} & \sin^2 \theta e^{2i\varphi} \\ -\sqrt{2} \cos \theta \sin \theta e^{-i\varphi} & -2(\cos^2 \theta - \frac{1}{3}) & \sqrt{2} \sin^2 \theta e^{-i\varphi} \\ \sin^2 \theta e^{2i\varphi} & \sqrt{2} \cos \theta \sin \theta e^{i\varphi} & \cos^2 \theta - \frac{1}{3} \end{pmatrix}$$

$$\Psi_{-1} = \frac{1}{\sqrt{10}} \begin{pmatrix} \sqrt{4} \chi_0 \\ \sqrt{5} \chi_{-1} \\ \sqrt{6} \chi_{-2} \end{pmatrix} = \frac{1}{\sqrt{10}} \cdot \frac{1}{\sqrt{4\pi}} \begin{pmatrix} \sqrt{\frac{3}{4}} (\cos^2 \theta - \frac{1}{3}) \\ \sqrt{\frac{3}{4}} \sqrt{2} \sin \theta \cos \theta e^{\mp i\varphi} \\ \sqrt{\frac{3}{4}} \sin^2 \theta e^{\mp 2i\varphi} \end{pmatrix}$$

$$= \frac{1}{\sqrt{10}} \frac{1}{\sqrt{4\pi}} \frac{3}{2} \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} = \frac{1}{\sqrt{4\pi}} \frac{1}{\sqrt{2}} \dots$$

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$$\sqrt{2} \left(\cos^2 \theta - \frac{1}{3} \right) \sin \theta \cos \theta + \sin^2 \theta \cos \theta$$

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$$\sqrt{2} - \frac{\sqrt{2}}{3} \quad \left(1 - \frac{1}{3}\right)$$

$$\cos^4 \theta - \frac{2}{3} \cos^2 \theta + \frac{1}{9}$$

$$\frac{10}{9} - \frac{2}{3} \cos^2 \theta = \frac{1}{3} \quad \frac{2}{9}$$

$$\left(\frac{(\vec{\sigma}^{(1)} \vec{\gamma}) (\vec{\sigma}^{(2)} \vec{\gamma})}{r^2} - \frac{(\vec{\sigma}^{(1)} \vec{\sigma}^{(2)})}{3} \right) \Psi_{-1}$$

$$= \frac{\sqrt{10}}{\sqrt{4\pi}} \left(\frac{8}{9} (\cos^2 \theta - \frac{1}{3})^2 + \frac{2}{3} \cdot 2 \sin^2 \theta \cos^2 \theta + \sin^4 \theta \right)$$

$$\frac{1}{2} \left(-\sqrt{2} \cos \theta \sin \theta (\cos^2 \theta - \frac{1}{3}) e^{-i\varphi} + 2\sqrt{2} (\cos^2 \theta - \frac{1}{3}) \sin \theta \cos \theta e^{-i\varphi} + \sqrt{2} \sin^2 \theta e^{i\varphi} \right)$$

$$\left(\sin^2 \theta (\cos^2 \theta - \frac{1}{3}) + 2 \sin^2 \theta \cos^2 \theta + \sin^2 \theta (\cos^2 \theta - \frac{1}{3}) \right) e^{-2i\varphi}$$

$$= \frac{\sqrt{10}}{\sqrt{4\pi}} \frac{3}{2} \left(\frac{8}{9} - \frac{2}{3} (\cos^2 \theta - \frac{1}{3}) + \frac{2\sqrt{2}}{3} \sin \theta \cos \theta e^{-i\varphi} - \frac{2}{3} \sin^2 \theta e^{-2i\varphi} \right)$$

$$= -\frac{2}{3} \Psi_{-1} + \frac{\sqrt{10}}{\sqrt{4\pi}} \cdot \frac{4}{3} \chi_{-1} = -\frac{2}{3} \Psi_{-1} + \frac{1}{\sqrt{2}} \cdot \frac{4}{3} \frac{\chi_{-1}}{\sqrt{4\pi}}$$

$$\left(\frac{(\vec{\sigma}^{(1)} \vec{\gamma}) (\vec{\sigma}^{(2)} \vec{\gamma})}{r^2} - \frac{(\vec{\sigma}^{(1)} \vec{\sigma}^{(2)})}{3} \right) \frac{\chi_{-1}}{\sqrt{4\pi}} = \begin{pmatrix} \cos^2 \theta - \frac{1}{3} \\ -\sqrt{2} \cos \theta \sin \theta e^{-i\varphi} \\ \sin^2 \theta e^{-2i\varphi} \end{pmatrix} \frac{1}{\sqrt{4\pi}}$$

$$= \frac{\Psi_{-1}}{\sqrt{4\pi} \frac{1}{\sqrt{10}} \frac{3}{\sqrt{4\pi}} \frac{3}{2}} = \frac{\frac{2}{3} \Psi_{-1}}{\frac{2}{3} \sqrt{2} \Psi_{-1}}$$

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$$V = - \left(g_1^2 + \frac{2}{3} g_2^2 \right) \frac{1}{r} + g_2^2 \left(\frac{\sigma^{(1)} r}{r^2} - \frac{\sigma^{(2)}}{3} \right) \left(1 + \frac{3}{\kappa r} + \frac{3}{\kappa^2 r^2} \right) \frac{e^{-\kappa r}}{r}$$

$$V \Psi_{+1} \frac{\chi_{-1}}{\sqrt{4\pi}} = \frac{2}{3} \sqrt{2} \Psi_{+1} - \left(g_1^2 + \frac{2}{3} g_2^2 \right) \frac{e^{-\kappa r}}{r} \frac{\chi_{-1}}{\sqrt{4\pi}} + g_2^2 \left(1 + \frac{3}{\kappa r} + \frac{3}{\kappa^2 r^2} \right) \frac{e^{-\kappa r}}{r} \cdot \frac{2}{3} \sqrt{2} \Psi_{-1}$$

$$V \Psi_{-1} = - \left(g_1^2 + \frac{2}{3} g_2^2 \right) \frac{e^{-\kappa r}}{r} \Psi_{-1} + \frac{2\sqrt{2}}{3} \frac{\chi_{-1}}{\sqrt{4\pi}} * - \frac{2}{3} g_2^2 \left(1 + \frac{3}{\kappa r} + \frac{3}{\kappa^2 r^2} \right) \frac{e^{-\kappa r}}{r} \Psi_{-1}$$

$$\Psi = \frac{u(r)}{r} \frac{\chi_{-1}}{\sqrt{4\pi}} + \frac{v(r)}{r} \Psi_{-1}$$

wave equation ψ

$$\frac{\hbar^2}{M} \frac{d^2 u}{dr^2} + \left(E + \left(g_1^2 + \frac{2}{3} g_2^2 \right) \frac{e^{-\kappa r}}{r} \right) u = 0$$

$$- \frac{2\sqrt{2}}{3} g_2^2 \left(1 + \frac{3}{\kappa r} + \frac{3}{\kappa^2 r^2} \right) \frac{e^{-\kappa r}}{r} \cdot \Psi_{-1} \quad v = 0$$

$$\frac{\hbar^2}{M} \left(\frac{d^2 v}{dr^2} + \frac{2}{r} \frac{dv}{dr} \right) + \left\{ E + \left(g_1^2 + \frac{2}{3} g_2^2 \right) \frac{e^{-\kappa r}}{r} - \frac{2\sqrt{2}}{3} g_2^2 \left(1 + \frac{3}{\kappa r} + \frac{3}{\kappa^2 r^2} \right) \frac{e^{-\kappa r}}{r} \right\} v = 0$$

$$- \frac{2\sqrt{2}}{3} g_2^2 \left(1 + \frac{3}{\kappa r} + \frac{3}{\kappa^2 r^2} \right) \frac{e^{-\kappa r}}{r} u = 0$$

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neutron-proton force

Ansatz in \vec{r} $\tau_1^{(1)} \tau_1^{(2)} + \tau_2^{(1)} \tau_2^{(2)}$ $\tau_3^{(1)} \tau_3^{(2)}$

$$\frac{\tau_1^{(1)} \tau_1^{(2)} + \tau_2^{(1)} \tau_2^{(2)} + \tau_3^{(1)} \tau_3^{(2)}}{2} = \begin{cases} -3/2 \\ +1/2 \end{cases}$$

$$3g_1^2 : 5g_2^2 = \frac{3}{2}(g_1^2 + \frac{2}{3}g_2^2) : \frac{1}{2}(g_1^2 - 2g_2^2)$$

$$= 3g_1^2 + 2g_2^2 : 2g_2^2 - g_1^2$$

$= 2:1$ となる

$$3g_1^2 + 2g_2^2 = 4g_2^2 - 2g_1^2$$

$$5g_1^2 = 2g_2^2$$

Ansatz \vec{r} $g_1^2 + \frac{2}{3}g_2^2 : 2g_2^2 - g_1^2 = 2:1$

$$3g_1^2 + 2g_2^2 = 12g_2^2 - 6g_1^2$$

$$\frac{5g_1^2 - 2g_2^2}{9g_1^2} = 10g_2^2$$