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On the Interaction of Elementary Particles IV

March, 1938.

~~§1. Neutral Heavy Quantum~~

§1. Introduction and Summary.

$$\Psi^{(i)} \tilde{\Psi}^{(j)} \Psi^{(k)} \tilde{\Psi}^{(l)} \Psi^{m}$$

$$= \delta_{ij} \Psi^{(k)} \tilde{\Psi}^{(l)} \Psi^{(m)} - \tilde{\Psi}^{(j)} \Psi^{(i)} \Psi^{(k)} \tilde{\Psi}^{(l)} \Psi^{(m)}$$

$$= \delta_{ij} \delta_{kl} \Psi^{m} + \delta_{ij} \tilde{\Psi}^{(l)} \Psi^{(m)} \Psi^{(k)}$$

$$+ \tilde{\Psi}^{j} \Psi^{(k)} \cdot \delta_{il} \Psi^{m} + \tilde{\Psi}^{(j)} \Psi^{(k)} \tilde{\Psi}^{(l)} \Psi^{(m)} \Psi^{(i)}.$$

$$\Psi^{(i)} \tilde{\Psi}^{j} A_{jk} \Psi^{(k)} \tilde{\Psi}^{(l)} B_{lm} \Psi^{(m)}$$

$$= \boxed{A_{il} B_{lm}} \Psi^{(m)} + A_{ik} \tilde{\Psi}^{l} \Psi^{(m)} \Psi^{(k)} B_{lm}$$

$$+ \tilde{\Psi}^{(j)} A_{jk} \Psi^{(k)} B_{im} \Psi^{(m)}$$

$$\boxed{M + \frac{1+\tau_3}{2} M' = \frac{1-\tau_3}{2} M + \frac{1+\tau_3}{2} (M+M')}$$

$\rho_3 \vec{\sigma}$

$$\frac{4\pi g_1^2}{\varkappa^2} = M_N - M_P$$

$$\frac{4\pi g_1^2}{\frac{M_N - M_P}{\varkappa^2}} = \cancel{\times 4\pi} (1+3) = \frac{16\pi g_1^2}{M_N - M_P} = (M_N - M_P) c^2$$

$$\frac{e^2}{mc^2} \qquad \frac{16\pi g_1^2 \left(\frac{e^2}{mc^2}\right)^2}{\left(\frac{e^2}{mc^2}\right)^2} = (M_N - M_P) c^2$$

$\tilde{\Psi} \Psi$

$$\delta \sim \left(\frac{g_1^2}{Mc^2}\right)^3 \sim \delta$$

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§2. Linear Equations for the V Field ~~of Neutral Heavy Quanta~~

$$\left.\begin{aligned} &\frac{1}{c}\frac{\partial \vec{B}}{\partial t} - \mathrm{curl}\,\vec{D} - \varkappa \vec{V} = 0 \qquad && \mathrm{div}\,\vec{B} + \varkappa V_0 = 0 \\ &\frac{1}{c}\frac{\partial \vec{V}}{\partial t} + \mathrm{grad}\, V_0 + \varkappa \vec{B} = 0 && \mathrm{curl}\,\vec{V} - \varkappa \vec{D} = 0 \end{aligned}\right\}$$

$$\bar{L} = \iiint L\, dv$$

$$L = \frac{1}{8\pi}\left(\vec{B}^2 - \vec{D}^2 + V_0^2 - \vec{V}^2\right)$$

$$V_0^{\dagger} = 0 \qquad \vec{V}^{\dagger} = -\frac{1}{4\pi\varkappa c}\vec{B}$$

$$H_V = 2\pi\varkappa^2 c^2 V^{\dagger 2} - c\, V^{\dagger}\,\mathrm{grad}\, V_0 + \frac{1}{8\pi\varkappa^2}(\mathrm{curl}\, V)^2 + \frac{1}{8\pi}(V^2 - V_0^2)$$

$$V_0 = -\frac{1}{\varkappa}\mathrm{div}\, B = 4\pi c\,\mathrm{div}\, V^{\dagger}$$

$$H_V = 2\pi\varkappa^2 c^2 V^{\dagger 2} \cancel{- c V^{\dagger}\,\mathrm{grad}(\mathrm{div}\, V^{\dagger})} + \frac{1}{8\pi\varkappa^2}(\mathrm{curl}\, V)^2 + 2\pi c^2 (\mathrm{div}\, V^{\dagger})^2 + \frac{1}{8\pi} V^2$$

$$\left.\begin{aligned} \frac{\partial \vec{V}}{\partial t} &= 4\pi\varkappa^2 c^2 \vec{V}^{\dagger} - 4\pi c^2\,\mathrm{grad}\,\mathrm{div}\, V^{\dagger} \\ \frac{\partial \vec{V}^{\dagger}}{\partial t} &= -\frac{1}{4\pi\varkappa^2}\mathrm{curl}(\mathrm{curl}\,\vec{V}) - \frac{1}{4\pi}\vec{V} \end{aligned}\right\}$$

$$\left.\begin{aligned} &\frac{1}{c}\frac{\partial \vec{B}}{\partial t} - \mathrm{curl}\,\vec{D} - \varkappa \vec{V} = -4\pi g_1 \vec{M}', \quad && \mathrm{div}\,\vec{B} + \varkappa V_0 = 4\pi g_1 M_0' \\ &\frac{1}{c}\frac{\partial \vec{V}}{\partial t} + \mathrm{grad}\, V_0 + \varkappa \vec{B} = 4\pi g_2 \vec{T}', && \mathrm{curl}\,\vec{V} - \varkappa \vec{D} = -4\pi g_2 \vec{S}' \end{aligned}\right\}$$

$$\left.\begin{aligned} &\vec{M}' = \tilde{\Psi}(\vec{\alpha}\tau_3)\Psi \qquad && M_0' = \tilde{\Psi}(\tau_3)\Psi \\ &\vec{T}' = -\tilde{\Psi}(\rho_2\vec{\sigma}\tau_3)\Psi, && \vec{S}' = \tilde{\Psi}\rho_3\vec{\sigma}\tau_3\Psi \end{aligned}\right\}$$

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$$L = \frac{1}{8\pi}(B^2 - D^2 + V_0^2 - V^2) + \frac{g_1}{\varkappa}(VM' - V_0 M_0')$$

$$V_0^\dagger = 0 \quad \vec{V}^\dagger = -\frac{\vec{B}}{4\pi\varkappa c}$$

$$\frac{\partial \vec{V}}{\partial t} = 4\pi\varkappa^2 c^2 \vec{V}^\dagger - c \operatorname{grad} V_0 + 4\pi g_2 c \vec{T}'$$

$$H = \vec{V}^\dagger(4\pi\varkappa^2 c^2 \vec{V}^\dagger - c \operatorname{grad} V_0 + 4\pi g_2 c \vec{T}')$$
$$- 2\pi\varkappa^2 c^2 \vec{V}^{\dagger 2} + \frac{1}{8\pi\varkappa^2}(\operatorname{curl} \vec{V} + 4\pi g_2 \vec{S}')^2$$
$$- \frac{1}{8\pi} V_0^2 + \frac{1}{8\pi}\vec{V}^2 - \frac{g_1}{\varkappa}(VM' - V_0 M_0')$$

$$V_0 = 4\pi c \operatorname{div} V^\dagger + \frac{4\pi g_1}{\varkappa} M_0'$$

$$H = H_U + H'$$

$$H_U = 2\pi\varkappa^2 c^2 V^{\dagger 2} + \cancel{2}\pi c^2 (\operatorname{div} V^\dagger)^2 + \frac{1}{8\pi\varkappa^2}(\operatorname{curl} V)^2$$
$$+ \frac{1}{8\pi} V^2$$

$$H' = \frac{4\pi g_1 c}{\varkappa} \operatorname{div} V^\dagger M_0' - \frac{g_1}{\varkappa} \vec{V}\vec{M}'$$
$$+ 4\pi g_2 c \, \vec{V}^\dagger \vec{T}' + \frac{g_2}{\varkappa^2} \operatorname{curl} \vec{V} \cdot \vec{S}'$$
$$+ \frac{2\pi}{\varkappa^2}(g_1^2 M_0'^2 + g_2^2 \vec{S}'^2)$$

$$H = H_V + H' + H_M$$

$$H_M = \tilde{\Psi}\left\{c\vec{\alpha}\vec{p} + \beta\left(\frac{1+\tau_3}{2} M_N c^2 + \frac{1-\tau_3}{2} M_P c^2\right)\right\}\Psi$$

$$i\hbar\frac{\partial \Psi}{\partial t} = \Big[c\vec{\alpha}\vec{p} + \beta\left(\frac{1+\tau_3}{2} M_N c^2 + \frac{1-\tau_3}{2} M_P c^2\right) + \frac{4\pi g_1 c}{\varkappa} \operatorname{div} V^\dagger \cdot \tau_3$$
$$- \frac{g_1}{\varkappa}\vec{V}\vec{\alpha}\tau_3 + 4\pi g_2 c \, \vec{V}^\dagger \rho_2 \vec{\sigma} \tau_3 + \frac{g_2}{\varkappa^2} \operatorname{curl} \vec{V} \cdot \rho_3 \vec{\sigma} \tau_3$$
$$+ \frac{4\pi g_1^2}{\varkappa^2} M_0' \tau_3 \Big]\Psi$$
$$+ \frac{4\pi g_2^2}{\varkappa^2} S' \tau_3 \rho_3 \vec{\sigma}$$

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$$\dot{V} = 4\pi\varkappa^2 c\, V^{\dagger} - 4\pi c\, \mathrm{grad\,div}\, V^{\dagger} + 4\pi g_1 c \vec{T}' - \frac{4\pi g_1 c}{\varkappa}\mathrm{grad}\, M_0'$$

$$-\dot{V}^{\dagger} = \frac{1}{4\pi\varkappa^2}\mathrm{curl\,curl}\, V + \frac{1}{4\pi} V - \frac{g_1}{\varkappa} M' + \frac{g_2}{\varkappa^2}\mathrm{curl}\, \vec{S}'$$

$$\frac{1}{c^2}\ddot{V} = -\mathrm{curl\,curl}\, V - \varkappa^2 V + \mathrm{grad\,div}\, V$$

$$-\frac{4\pi g_1}{\varkappa c}\frac{\partial}{\partial t}\mathrm{div}\, M_0' + \frac{4\pi g_2}{c}\frac{\partial \vec{T}'}{\partial t} - \frac{4\pi g_1}{\varkappa}\mathrm{grad\,div}\, M'$$

$$+ 4\pi\varkappa g_1 M' - 4\pi g_2\, \mathrm{curl}\, S' \qquad \frac{g_1}{\varkappa c}\frac{\partial M'}{\partial t}$$

$$\frac{1}{c^2}\ddot{V}^{\dagger} = -\mathrm{curl\,curl}\, V^{\dagger} - \frac{g_2}{\varkappa^2 c}\mathrm{curl\,curl}\, T'$$

$$-\varkappa^2 V^{\dagger} + \mathrm{grad\,div}\, V^{\dagger} + \frac{g_1}{\varkappa c}\mathrm{grad}\, M_0'$$

$$-\frac{g_2}{c}\frac{\partial T'}{\partial t} + \frac{g_1}{\varkappa c^2}\frac{\partial M'}{\partial t} - \frac{g_2}{\varkappa^2 c^2}\frac{\partial}{\partial t}\mathrm{curl}\, S'$$

$$\left\{\frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} - \Delta V + \varkappa^2 V\right\} = -\frac{4\pi g_1}{\varkappa c}\frac{\partial}{\partial t}\mathrm{div}\, M_0' + \frac{4\pi g_2}{c}\frac{\partial T'}{\partial t}$$

$$+ 4\pi\varkappa g_1 M' - 4\pi g_2\, \mathrm{curl}\, S' - \frac{4\pi g_1}{\varkappa}\mathrm{grad\,div}\, M'$$

$$\left\{\frac{1}{c^2}\frac{\partial^2 V^{\dagger}}{\partial t^2} - \Delta V^{\dagger} + \varkappa^2 V^{\dagger}\right\} = -\frac{g_2}{\varkappa^2 c}\mathrm{curl\,curl}\, T' + \frac{g_1}{\varkappa c}\mathrm{grad}\, M_0$$

$$-\frac{g_2}{c} T'$$

Non-relativistic Approximation.

$$\left.\begin{aligned} -\Delta V + \varkappa^2 V &= -4\pi g_2\, \mathrm{curl}\, S' \\ -\Delta V^{\dagger} + \varkappa^2 V^{\dagger} &= \frac{g_1}{\varkappa c}\mathrm{grad}\, M_0' \end{aligned}\right\}$$

Interaction Potential.

$$\frac{4\pi g_1 c}{\varkappa}\mathrm{div}\, V^{\dagger}\cdot\tau_3 + \frac{c g_2}{3\varkappa^2}\mathrm{curl}\, V\cdot\vec{\sigma}\tau_3 + \frac{4\pi g_1^2}{\varkappa^2}(\tilde{\Psi}\tau_3\Psi)\cdot\tau_3$$

$$+ \frac{4\pi g_2^2}{\varkappa^2}(\tilde{\Psi}\vec{\sigma}\tau_3\Psi)\cdot\vec{\sigma}\tau_3$$

$$-\frac{g_1^2\tau_3}{\varkappa^2}\left(\frac{e^{-\varkappa r}}{r}\mathrm{div\,grad}\, M_0'\right) - \frac{g_2^2\tau_3}{\varkappa^2}\left(\vec{\sigma}\frac{e^{-\varkappa r}}{r}\mathrm{curl\,curl}\, S'\right)$$

$$+ \frac{4\pi g_1^2\tau_3}{\varkappa^2} M_0' + \frac{4\pi g_2^2}{\varkappa^2} S'\cdot\vec{\sigma}\tau_3$$

(5) §3, Deduction of Forces between Heavy Particles

$$\frac{g_1^2\tau_3^{(1)}\tau_3^{(2)}}{\varkappa^2}\Delta\frac{e^{-\varkappa r}}{r}+\frac{g_2^2\tau_3^{(1)}\tau_3^{(2)}}{\varkappa^2}\vec{\sigma}^{(2)}\operatorname{curl}\left(\operatorname{curl}\vec{\sigma}^{(1)}\frac{e^{-\varkappa r}}{r}\right)\Big\}$$

$$+\frac{4\pi g_1^2\tau_3}{\varkappa^2}\delta(\vec{r})+\frac{4\pi g_2^2}{\varkappa^2}\delta(\vec{r})\vec{\sigma}^{(1)}\tau_3^{(1)}\vec{\sigma}^{(2)}\tau_3^{(2)},$$

$$\vec{\sigma}^{(2)}[\operatorname{grad}\times\cancel{\sigma^{(2)}}(\operatorname{grad}\times\frac{\vec{\sigma}^{(1)}e^{-\varkappa r}}{r})]]$$

$$=\{(\sigma^{(1)}\operatorname{grad})(\sigma^{(2)}\operatorname{grad})-\Delta\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}\}\frac{e^{-\varkappa r}}{r}$$

$$A\times[B\times C]=(A\cdot C)B-(A\cdot B)C$$

$$=(\operatorname{grad}\ \operatorname{grad}\ \vec{\sigma}^{(1)})\frac{e^{-\varkappa r}}{r}-\Delta\vec{\sigma}^{(1)}\frac{e^{-\varkappa r}}{r}$$

$$\frac{e^{-\varkappa r}}{r}$$

$$=g_1^2\tau_3^{(1)}\tau_3^{(2)}\frac{e^{-\varkappa r}}{r}+g_2^2\tau_3^{(1)}\tau_3^{(2)}\left(\vec{\sigma}^{(1)}\sigma^{(2)}-\frac{(\sigma^{(1)}\operatorname{grad})(\sigma^{(2)}\operatorname{grad})}{\varkappa^2}\right)$$

$$=\frac{1+\tau_3^{(1)}\tau_3^{(2)}}{2}(\qquad)$$

$$-\frac{1-\tau_3^{(1)}\tau_3^{(2)}}{2}(\qquad)$$

Exchange Interaction との Resultant をとると

$$\left\{\frac{1+\tau_3^{(1)}\tau_3^{(2)}}{2}-\frac{1-\tau_3^{(1)}\tau_3^{(2)}}{2}+\frac{\tau_1^{(1)}\tau_1^{(2)}+\tau_2^{(1)}\tau_2^{(2)}}{2}\right\}(\qquad)$$

もっと一般に τ_3 のかわりに $a+b\tau_3$ を入れると,

$$\tau_3^{(1)}\tau_3^{(2)}\to\frac{(\tilde{a}+\tilde{b}\tau_3^{(1)})(a+b\tau_3^{(2)})+(\tilde{a}+\tilde{b}\tau_3^{(2)})(a+b\tau_3^{(1)})}{2}$$

$$=2\{|a|^2+|b|^2\tau_3^{(1)}\tau_3^{(2)}\}+(\tilde{a}b+a\tilde{b})(\tau_3^{(1)}+\tau_3^{(2)})$$

neutron 同志と proton 同志 の force が等しいとすれば

$$\tilde{a}b+a\tilde{b}=0\qquad \left.\begin{array}{l}a=|a|e^{i\theta}\\ b=|b|e^{i\phi}\end{array}\right\}\qquad |a||b|\cdot\{e^{i(\phi-\theta)}+e^{i(\theta-\phi)}\}=0$$

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故に、 $|a| = 0$ or $|b| = 0$ or $\phi - \theta = \pm\pi$.

$\therefore \ \tau_3 \to a + b i \tau_3$ a; b: real でなければならぬ,

例へば $a = b = \frac{1}{2}$ とすると

$$\tau_3^{(1)}\tau_3^{(2)} \to \frac{1+\tau_3^{(1)}\tau_3^{(2)}}{2}$$; like particle force を与へる,

$a = 1 \quad b = 0$ なら,

$$\tau_3^{(1)}\tau_3^{(2)} \to 1, = \frac{1+\tau_3^{(1)}\tau_3^{(2)}}{2} + \frac{1-\tau_3^{(1)}\tau_3^{(2)}}{2} + \cdots$$

故に, 最も一般な場合として,

$$\left\{\frac{1+\tau_3^{(1)}\tau_3^{(2)}}{2} - \varepsilon\frac{1-\tau_3^{(1)}\tau_3^{(2)}}{2} + \frac{\tau_1^{(1)}\tau_1^{(2)}+\tau_2^{(1)}\tau_2^{(2)}}{2}\right\}$$

$$\times\left\{g_1^2 + g_2^2(\sigma^{(1)}\sigma^{(2)}) - g_2^2\frac{(\sigma^{(1)}\mathrm{grad})(\sigma^{(2)}\mathrm{grad})}{\varkappa^2}\right\}\frac{e^{-\varkappa r}}{r}$$

但し, $\varepsilon = 0, \pm 1$.

の三種が考へられる.

$$N-P:\ {}^3S:\ -(1+\varepsilon)\cdot\left(g_1^2 + \frac{2}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r}$$

$$\ {}^1S:\ (1-\varepsilon)\left(g_1^2 - 2g_2^2\right)\frac{e^{-\varkappa r}}{r}.$$

$$\left.\begin{matrix}N-N\\P-P\end{matrix}\right\}\ {}^1S:\ (g_1^2 - 2g_2^2)\frac{e^{-\varkappa r}}{r}$$

$\varepsilon = 0$, (~~$a = b = 1$~~ $\frac{1+i\tau_3}{2}$ or $\frac{\tau_3+i}{2}$)

$$g_1 = g_2:\ \left.\begin{matrix}\boxed{N-P}\left\{\begin{matrix}{}^3S:\ -\frac{5}{3}g^2\frac{e^{-\varkappa r}}{r}\\ {}^1S:\ -g^2\frac{e^{-\varkappa r}}{r}\end{matrix}\right.\\ \left.\begin{matrix}N-N\\P-P\end{matrix}\right\}\ {}^1S:\ -g^2\frac{e^{-\varkappa r}}{r}\end{matrix}\right\}$$

Kemmer, The charge-dependence of Nuclear Forces.
(Proc. Camb. Phil. Soc. , 1938)

$$V = \frac{1}{2}\left\{ \left(g'\frac{1+\tau_3^{(1)}}{2} + g''\frac{1-\tau_3^{(1)}}{2}\right)\times\left(g'^*\frac{1+\tau_3^{(2)}}{2}+g''^*\frac{1-\tau_3^{(2)}}{2}\right)\right.$$

$$\left. + \left(g'\frac{1+\tau_3^{(2)}}{2}+g''\frac{1-\tau_3^{(2)}}{2}\right)\times\left(g'^*\frac{1+\tau_3^{(1)}}{2}+g''^*\frac{1-\tau_3^{(2)}}{2}\right)\right\}$$

$$= \frac{1}{2}\left\{|g'+g''|^2 + |g'-g''|^2(\tau_3^{(1)}\tau_3^{(2)})\right.$$

$$+ \cancel{(|g'|^2-|g''|^2)}\{(g'+g'')(g'^*-g''^*)$$

$$\left. + (g'-g'')(g'^*+g''^*)\}(\tau_3^{(1)}+\tau_3^{(2)}) \right\}$$

$$\| \quad |g'|^2-|g''|^2$$

$$|g'-g''|^2 = |g|^2.$$

$$(1+i\tau_2)(1-i\tau_2)$$

$$\rightarrow 1+\tau_3^{(1)}\tau_3^{(2)}$$

$$\boxed{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)} + \tau_3^{(1)}\tau_3^{(2)}}$$

P-N	symm.	$-2-1=-3$
	anti,	$2-1=1$
N-N P-N	~~symm~~ anti.	1

$$\boxed{1+\vec{\tau}^{(1)}\cdot\vec{\tau}^{(2)}}$$

P-N	symm	$1-2-1=-2,$
	anti	$1+2-1=2,$
N-N P-N	~~symm~~ anti	$1+1=2.$

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§4. Theory of the Deuteron

$$\left\{\frac{\hbar^2}{M}\Delta + E - V\right\}\psi = 0$$

$$V = \left\{\frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}\right\}\left\{g_1^2 + g_2^2(\sigma^{(1)}\sigma^{(2)}) - g_2^2\frac{(\sigma^{(1)}\mathrm{grad})(\sigma^{(2)}\mathrm{grad})}{\varkappa^2}\right\} \times \frac{e^{-\varkappa r}}{r}$$

Symmetric States

(α) $\boxed{{}^3S,\ {}^1P,\ {}^3D,\ \dots}$

$$V = -\left\{g_1^2 + g_2^2(\sigma^{(1)}\sigma^{(2)}) - g_2^2\frac{(\sigma^{(1)}\mathrm{grad})(\sigma^{(2)}\mathrm{grad})}{\varkappa^2}\right\}\frac{e^{-\varkappa r}}{r}$$

i) Even. (Triplet) ${}^3S,\ {}^3D,\ \dots$

$$V = -\left\{g_1^2 + g_2^2 - g_2^2\frac{\quad}{\varkappa^2}\right.$$

$$(\sigma^{(1)}\mathrm{grad})(\sigma^{(2)}\mathrm{grad}) = \begin{pmatrix}\frac{\partial}{\partial z} & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & -\frac{\partial}{\partial z}\end{pmatrix}_1 \begin{pmatrix}0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial x} & 0\end{pmatrix}_2 + \begin{pmatrix}0 & -i\frac{\partial}{\partial y} \\ i\frac{\partial}{\partial y} & 0\end{pmatrix}_1 \begin{pmatrix}0 & -i\frac{\partial}{\partial y} \\ i\frac{\partial}{\partial y} & 0\end{pmatrix}_2$$

$$+ \begin{pmatrix}\frac{\partial}{\partial z} & 0 \\ 0 & -\frac{\partial}{\partial z}\end{pmatrix}_1 \begin{pmatrix}\frac{\partial}{\partial z} & 0 \\ 0 & -\frac{\partial}{\partial z}\end{pmatrix}_2$$

Triplet. $\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2$: $\quad \begin{pmatrix}0\\ \frac{\partial}{\partial x}\end{pmatrix}_1\begin{pmatrix}0\\ \frac{\partial}{\partial x}\end{pmatrix}_2 + \begin{pmatrix}0\\ i\frac{\partial}{\partial y}\end{pmatrix}_1\begin{pmatrix}0\\ i\frac{\partial}{\partial y}\end{pmatrix}_2 + \begin{pmatrix}\frac{\partial}{\partial z}\\ 0\end{pmatrix}_1\begin{pmatrix}\frac{\partial}{\partial z}\\ 0\end{pmatrix}_2$

$\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2$

$\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2$

$$(\sigma^{(1)}\mathrm{grad})(\sigma^{(2)}\mathrm{grad}) = \begin{pmatrix}\frac{\partial}{\partial z} & \frac{\partial}{\partial x} - i\frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} + i\frac{\partial}{\partial y} & -\frac{\partial}{\partial z}\end{pmatrix}_1 \begin{pmatrix}\frac{\partial}{\partial z} & \frac{\partial}{\partial x} - i\frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} + i\frac{\partial}{\partial y} & -\frac{\partial}{\partial z}\end{pmatrix}_2$$

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$$V = \frac{\tau_1^{(1)}\tau_1^{(2)}+\tau_2^{(1)}\tau_2^{(2)}}{2}\Big[g_1 + g_2\{(\sigma^{(1)}\sigma^{(2)}) - \frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2}\}$$

$$+ g_2^2\{(\sigma^{(1)}\sigma^{(2)}) - \frac{3(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2}\}\left(\frac{1}{\varkappa r} - \frac{1}{\varkappa^2 r^2}\right)\Big]\frac{e^{-\varkappa r}}{r}$$

a) Symmetric State $({}^3S, {}^1P, {}^3D, \cdots)$

$$V = -\Big[g_1^2 + g_2^2\{(\sigma^{(1)}\sigma^{(2)}) - \frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2}\} + g_2^2\{(\sigma^{(1)}\sigma^{(2)}) - \frac{3(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2}\}$$

$$\times\left(\frac{1}{\varkappa r} - \frac{1}{\varkappa^2 r^2}\right)\Big]\frac{e^{-\varkappa r}}{r}$$

~~i) Triplet state (^3S, ...)~~

$$\sigma^{(1)}\sigma^{(2)} = \begin{pmatrix}0&1\\1&0\end{pmatrix}_1\begin{pmatrix}0&1\\1&0\end{pmatrix}_2 + \begin{pmatrix}0&-i\\i&0\end{pmatrix}_1\begin{pmatrix}0&-i\\i&0\end{pmatrix}_2 + \begin{pmatrix}1&0\\0&-1\end{pmatrix}_1\begin{pmatrix}1&0\\0&-1\end{pmatrix}_2$$

$\sigma^{(1)}\sigma^{(2)}$

$$\left\{\begin{aligned}
&\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 = \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\\
&\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 = 2\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 - \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2\\
&\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 = 2\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\\
&\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 = \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2
\end{aligned}\right.$$

$\sigma^{(1)}\sigma^{(2)}$ (triplet) = 1. (triplet)

$$(\sigma^{(1)}r)(\sigma^{(2)}r) = \begin{pmatrix}z & x-iy\\ x+iy & -z\end{pmatrix}_1\begin{pmatrix}z & x-iy\\ x+iy & -z\end{pmatrix}_2$$

$$(\sigma^{(1)}r)(\sigma^{(2)}r)\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 = \begin{pmatrix}z\\x+iy\end{pmatrix}_1\begin{pmatrix}z\\x+iy\end{pmatrix}_2$$

$$(\sigma^{(1)}r)(\sigma^{(2)}r)\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 = \begin{pmatrix}z\\x+iy\end{pmatrix}_1\begin{pmatrix}x-iy\\-z\end{pmatrix}_2$$

$$\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 = \begin{pmatrix}x-iy\\-z\end{pmatrix}_1\begin{pmatrix}z\\x+iy\end{pmatrix}_2$$

$$\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 = \begin{pmatrix}x-iy\\-z\end{pmatrix}_1\begin{pmatrix}x-iy\\-z\end{pmatrix}_2$$

Triplet, Diagonal (9)

$\binom{1}{0}_1\binom{1}{0}_2(\sigma^{(1)}r)(\sigma^{(1)}r)\binom{1}{0}_1\binom{1}{0}_2$

$\frac{1}{2}\left\{\binom{1}{0}_1\binom{0}{1}_2+\binom{0}{1}_1\binom{1}{0}_2\right\}(\sigma r)(\sigma r)\frac{1}{2}\left\{\binom{1}{0}_1\binom{0}{1}_2+\binom{0}{1}_1\binom{1}{0}_2\right\}=-z^2+(x^2+y^2)=r^2(\sin^2\theta-\cos^2\theta)$ (3)

$\binom{0}{1}_1\binom{0}{1}_2(\ )(\ )\binom{0}{1}_1\binom{0}{1}_2=z^2=r^2\cos^2\theta$

Singlet.

$\frac{1}{2}\left\{\binom{1}{0}_1\binom{0}{1}_2-\binom{0}{1}_1\binom{1}{0}_2\right\}(\sigma r)_1(\sigma r)_2\left\{\binom{1}{0}_1\binom{0}{1}_2-\binom{0}{1}_1\binom{1}{0}_2\right\}=-(z^2+y^2+x^2)$ (1)

Triplet, Non-Diagonal (6)

$\binom{0}{1}_1\binom{0}{1}_2(\ )(\ )\binom{1}{0}_1\binom{1}{0}_2=(x+iy)^2=x^2-y^2+2ixy$

$\binom{1}{0}_1\binom{1}{0}_2(\ )(\ )\binom{0}{1}_1\binom{0}{1}_2=x^2-y^2-2ixy,$

$\frac{1}{\sqrt{2}}\left\{\binom{1}{0}_1\binom{0}{1}_2+\binom{0}{1}_1\binom{1}{0}_2\right\}(\sigma r)_1(\sigma r)_2\left\{\binom{1}{0}_1\binom{1}{0}_2\right\}=\sqrt{2}\,zx+\sqrt{2}\,iyz,$

$\frac{1}{\sqrt{2}}\left\{\binom{1}{0}_1\binom{0}{1}_2+\binom{0}{1}_1\binom{1}{0}_2\right\}(\sigma r)_1(\sigma r)_2\left\{\binom{0}{1}_1\binom{0}{1}_2\right\}=-\sqrt{2}\,zx+\sqrt{2}\,iyz,$

Triplet - Singlet. (6)

$\frac{1}{\sqrt{2}}\left\{\binom{1}{0}_1\binom{0}{1}_2-\binom{0}{1}_1\binom{1}{0}_2\right\}(\sigma r)_1(\sigma r)_2\binom{1}{0}_1\binom{1}{0}_2=0$

$\frac{1}{2}\left\{\binom{1}{0}_1\binom{0}{1}_2-\binom{0}{1}_1\binom{1}{0}_2\right\}(\sigma r)_1(\sigma r)_2\left\{\binom{1}{0}_1\binom{0}{1}_2+\binom{0}{1}_1\binom{1}{0}_2\right\}=0$

$[(\vec\sigma^{(1)}\vec\sigma^{(2)}),(\vec\sigma^{(1)}\vec r)]=[(\sigma_x^{(1)}\sigma_x^{(2)}+\sigma_y^{(1)}\sigma_y^{(2)}+\sigma_z^{(1)}\sigma_z^{(2)}),(\sigma_x^{(1)}x+\sigma_y^{(1)}y+\sigma_z^{(1)}z)]$

$=2i(\sigma_z^{(1)}\sigma_x^{(2)}y-\sigma_y^{(1)}\sigma_x^{(2)}z+\ \ )$

$[(\vec\sigma^{(1)}\vec\sigma^{(2)}),(\vec\sigma^{(1)}\vec r)(\vec\sigma^{(2)}\vec r)]=0.$

Total Angular Mom...

$$(\vec{r}\times\vec{p})(\vec{\sigma}^{(1)}\vec{r}) - (\vec{\sigma}^{(1)}\vec{r})(\vec{r}\times\vec{p}) = -i\hbar\,\vec{r}\times\vec{\sigma}^{(1)}$$

$$\Big(\because\ (xp_y - yp_x)\{\sigma_x^{(1)}x + \sigma_y^{(1)}y + \sigma_z^{(1)}z\} - \{\ \ \}(xp_y - yp_x)$$

$$= -i\hbar(x\sigma_y^{(1)} - y\sigma_x^{(1)}) = -i\hbar(\vec{r}\times\vec{\sigma}^{(1)})_z\Big)$$

$$[(\vec{r}\times\vec{p}),(\vec{\sigma}^{(1)}\vec{r})(\vec{\sigma}^{(2)}\vec{r})] = -i\hbar(\vec{r}\times\vec{\sigma}^{(1)})(\sigma^{(2)}r)$$
$$-i\hbar(\sigma^{(1)}r)(r\times\sigma^{(2)})$$

$$\vec{\sigma}^{(1)},(\vec{\sigma}^{(1)}\vec{r}) - (\vec{\sigma}^{(1)}\vec{r}),\vec{\sigma}^{(1)} = i(\vec{r}\times\vec{\sigma}^{(1)})$$

$$\Big(\because\ \sigma_x(\sigma_x x + \sigma_y y + \sigma_z z) - (\ \ )\sigma_x = 2i(\sigma_z y - \sigma_y z)\Big)$$

$$\Big\{(\vec{r}\times\vec{p}) + \frac{\hbar}{2}(\sigma^{(1)} + \sigma^{(2)})\Big\}(\sigma^{(1)}r)(\sigma^{(2)}r) - (\ )(\ )\{\ \ \} = 0$$

$j = l+s, \ldots |l-s|$

		sym	anti	sym	anti
		^3S	^3P	^3D	^3F
triplet, $s=1$,	$l=$	0,	1,	2	3,
	$j=$	1 $(^3S_1)$	2, 1, 0 $\begin{pmatrix}^3P_2\\ ^3P_1\\ ^3P_0\end{pmatrix}$	3, 2, 1 $\begin{pmatrix}^3D_3\\ ^3D_2\\ ^3D_1\end{pmatrix}$	4, 3, 2 $\begin{pmatrix}^3F_4\\ 3\\ 2\end{pmatrix}$

^3S と combine するのは $^3D_1$ 丈。

$\boxed{3S - {}^3D_1 \text{- state}}$

$$V = -\left[g_1^2 + g_2^2 \left(1 - \frac{\begin{pmatrix} z, & x-iy \\ x+iy, & -z \end{pmatrix}_1 \begin{pmatrix} z, & x-iy \\ x+iy, & -z \end{pmatrix}_2}{r^2}\right) + g_2^2 \left(1 - \frac{3\begin{pmatrix} z, & x-iy \\ x+iy, & -z \end{pmatrix}_1 \begin{pmatrix} z, & x-iy \\ x+iy, & -z \end{pmatrix}_2}{r^2}\right)\left(\frac{1}{\kappa r} + \frac{1}{\kappa^2 r^2}\right)\right]\frac{e^{-\kappa r}}{r}$$

$$\frac{1}{r}(\vec{\sigma}^{(1)}\vec{r}) = \sigma_r^{(1)} \qquad {\sigma_r^{(1)}}^2 = 1$$
$$\frac{1}{r}(\vec{\sigma}^{(2)}\vec{r}) = \sigma_r^{(2)} \qquad {\sigma_r^{(2)}}^2 = 1$$

$$V = -\left[g_1^2 + g_2^2 \left(1 - \sigma_r^{(1)}\sigma_r^{(2)}\right) + g_2^2\left(1 - 3\sigma_r^{(1)}\sigma_r^{(2)}\right)\left(\frac{1}{\kappa r} + \frac{1}{\kappa^2 r^2}\right)\right]\frac{e^{-\kappa r}}{r}$$

$\frac{2}{3}$

$$\left\{\frac{\hbar^2}{M}\Delta + E + \left[\qquad \right]\frac{e^{-\kappa r}}{r}\right\}\psi = 0$$

S-state $\quad u_s(r)$.

D-state $\quad m=0 \quad v_0 \quad \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$

$m=1 \quad v_1 \quad \sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\varphi}$

$m=-1 \quad v_{-1} \quad \ldots\; e^{-i\varphi}$

$m=2 \quad v_2 \quad \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\varphi}$

$m=-2 \quad v_{-2} \quad \ldots\; e^{-2i\varphi}$

${}^3D_1$ $\quad m_s = 1$, $\quad m_s = 0$, $\quad m_s = -1$

$\left(\frac{3}{2}x^2 - \frac{1}{2}\right)^2 = \frac{9}{4}x^4 - \frac{3}{2}x^2 + \frac{1}{4}$ $\quad \frac{2\sqrt{2}}{}$ $\quad \frac{9}{5} - 1 + \frac{1}{4}$ $\quad \frac{9}{5}x^5 \quad \frac{x^3}{2} \quad \frac{x}{4}$ $\quad \frac{4}{5} \quad \frac{8}{15}$

$j=1$.

$\boxed{\begin{matrix}\sqrt{\frac{3}{4\pi}}\sin\theta\, e^{i\varphi} \\ \sqrt{\frac{3}{4\pi}}\cos\theta \\ \sqrt{\frac{3}{4\pi}}\sin\theta\, e^{-i\varphi}\end{matrix}}$ $\quad \begin{matrix}\frac{1}{\sqrt{2}}(x+iy) \\ z \\ \frac{1}{\sqrt{2}}(x-iy)\end{matrix}$

$\frac{\sqrt{3}}{2\sqrt{2}}(x+iy)^2$

$\sqrt{\frac{3}{2}}(x+iy)z$

$(3z^2 - r^2) = (2z^2 - x^2 - y^2)$

$\sqrt{\frac{3}{2}}(x-iy)z$

$\frac{1}{2}\sqrt{\frac{3}{2}}(x+iy)^2$

$l=2$, $s=1$ $\quad j = 3, 2, 1 \quad 7, 5, \boxed{3}$

$5 \times 3 = 15$.

$s=1$

$$\left.\begin{aligned} \chi_1 &= \tfrac{1}{\sqrt2}(x+iy) \\ \chi_0 &= z \\ \chi_{-1} &= \tfrac{1}{\sqrt2}(x-iy) \end{aligned}\right\} \quad \left.\begin{aligned} v_2 &= \tfrac12\sqrt{\tfrac32}(x+iy)^2 \\ v_1 &= \sqrt{\tfrac32}(x+iy)z \\ v_0 &= 2z^2-x^2-y^2 \\ v_{-1} &= \sqrt{\tfrac32}(x-iy)z \\ v_{-2} &= \tfrac12\sqrt{\tfrac32}(x-iy)^2 \end{aligned}\right\}$$

$$\begin{aligned} &(x+iy)^3 &&= \chi_1 v_2 \\ &(x+iy)^2 z &&= \chi_1 v_1,\ \chi_0 v_2 \\ &(x+iy)(4z^2-x^2-y^2) &&= \tfrac{2}{\sqrt3}\chi_0 v_1 + \chi_1 v_0 \\ &(4z^2-3x^2-3y^2)z &&= \\ &(x-iy)(4z^2-x^2-y^2) &&= \\ &(x-iy)^2 z && \\ &(x-iy)^3 && \end{aligned}$$

$\sqrt3 z^2$ +

$\frac{2}{\sqrt3}$

$$j=1 \quad \begin{cases} a_{11}\chi_1 v_0 + a_{12}\chi_0 v_1 + a_{13}\chi_{-1} v_2 \propto \frac{1}{\sqrt2}(x+iy) f(r) \\ a_{21}\chi_1 v_{-1} + a_{22}\chi_0 v_0 + a_{23}\chi_{-1} v_1 \propto z\, f(r) \\ a_{31}\chi_1 v_{-2} + a_{32}\chi_0 v_{-1} + a_{33}\chi_{-1} v_0 \propto \frac{1}{\sqrt2}(x-iy) f(r) \end{cases}$$

$$a_{11}(\underbrace{2z^2-x^2-y^2}_{-r^2+z^2}) + a_{12}(\sqrt3 z^2) + a_{13}\tfrac12\sqrt{\tfrac32}(\underbrace{x^2+y^2}_{(r^2-z^2)}) = f(r)$$

$$= \left(a_{13}\tfrac12\sqrt{\tfrac32} - 2a_{11}\right) r^2 + (\underbrace{2a_{11} + \sqrt3 a_{12} - a_{13}\tfrac12\sqrt{\tfrac32}}_{=0}) z^2$$

$$a_{11}\tfrac{1}{\sqrt2}(x+iy)(2z'^2 - x'^2 - y'^2) + a_{12}\sqrt{\tfrac32}\, z\, (x'+iy')z' + a_{13}\tfrac{\sqrt3}{4}(x-iy)(x'+iy')^2$$

$$\Psi_m^L = \sum_{\mu\nu} S^*_{Lm;\mu\nu}\, \psi_\mu \bar\psi_\nu$$ Wigner, Gruppentheorie XVII.

$$S_{L,\mu+\nu;\mu,\nu} = S_{L\mu\nu}$$

$$\Psi_m^L = \sum_\mu S_{L\mu\, m-\mu}\, \psi_\mu \bar\psi_{m-\mu}$$

$$S_{L\mu\nu} = \frac{\sqrt{(L+l-\bar l)!\,(L-l+\bar l)!\,(l+\bar l-L)!\,(L+\mu+\nu)!\,(L-\mu-\nu)!}}{\sqrt{(L+l+\bar l+1)!\,(l-\mu)!\,(l+\mu)!\,(\bar l-\nu)!\,(\bar l+\nu)!}} \times \sum_\kappa \frac{(-1)^{\kappa+\bar l+\nu}\sqrt{2L+1}\,(L+\bar l+\mu-\kappa)!\,(l-\mu+\kappa)!}{(L-l+\bar l-\kappa)!\,(L+\mu+\nu-\kappa)!\,\kappa!\,(\kappa+l-\bar l-\mu-\nu)!}$$

$L = 1, \quad l = 2$ $\quad \binom{0}{\mu+\nu-1} < \varkappa < \binom{1+\mu+\nu}{0}$ (13)

$$S_{*\mu\nu} = \frac{2\cancel{\sqrt{2}}\sqrt{\cdot(1+\mu+\nu)!\,(1-\mu-\nu)!}}{\sqrt{5!\,(2-\mu)!\,(2+\mu)!\,(1-\nu)!\,(1+\nu)!}}$$

$$\times \sum_{\varkappa} \frac{(-1)^{\varkappa+1+\nu}\sqrt{3}\,(2+\mu-\varkappa)!\,(2-\mu+\varkappa)!}{(-\varkappa)!\,(1+\mu+\nu-\varkappa)!\,\varkappa!\,(\varkappa-\mu-\nu+1)!}$$

$m = 1$:

$$\Psi_1 = \sum_{\mu} S_{\mu,1-\mu}\,\psi_\mu\,\overline{\varphi}_{1-\mu} = S_{2,-1}\psi_2\overline{\varphi}_{-1} + S_{1,0}\psi_1\overline{\varphi}_0 + S_{0,1}\psi_0\overline{\varphi}_1$$

$$\Psi_0 = \sum \qquad = S_{1,-1}\psi_1\overline{\varphi}_{-1} + S_{0,0}\psi_0\overline{\varphi}_0 + S_{-1,1}\psi_{-1}\overline{\varphi}_1$$

$$\Psi_{-1} = \qquad = S_{0,-1}\psi_0\overline{\varphi}_{-1} + S_{-1,0}\psi_{-1}\overline{\varphi}_0 + S_{-2,1}\psi_{-2}\overline{\varphi}_1$$

$$S_{2,-1} = \frac{2\sqrt{2}}{\sqrt{5!\,4!\,2!}}\;\frac{\sqrt{3}\,4!}{2!} = \sqrt{\frac{1}{5}}$$

$$S_{1,0} = \frac{2\sqrt{2}}{\sqrt{5!\,3!}}\;\frac{(-1)\sqrt{3}\,3!}{2!} = \cancel{-\sqrt{\frac{4!}{5}}} = -\sqrt{\frac{24}{5}} = -\sqrt{\frac{3}{10}}$$

$S_{0,1} =$

$\bar{l} = s = 1, \quad L = l \quad S_{l-1,\mu,\nu}$

L	$\nu = -1$	0	$\nu = 1$
$l-1$	$\frac{\sqrt{l+\mu}}{\sqrt{2l}}\frac{\sqrt{l+\mu-1}}{\sqrt{2l+1}}$	$-\frac{\sqrt{l-\mu}\sqrt{l+\mu}}{\sqrt{l}\sqrt{2l+1}}$	$\frac{\sqrt{l-\mu-1}\sqrt{l-\mu}}{\sqrt{2l}\sqrt{2l+1}}$
$l=2$, $L=1$	$\frac{\sqrt{2+\mu}\sqrt{1+\mu}}{2\cdot\sqrt{5}}$	$-\frac{\sqrt{2-\mu}\sqrt{2+\mu}}{\sqrt{2}\sqrt{5}}$	$\frac{\sqrt{1-\mu}\sqrt{2-\mu}}{2\sqrt{5}}$

$S_{0,-1} = \frac{\sqrt{2}\cdot}{2\sqrt{5}} = \sqrt{\frac{1}{10}}$ $\qquad S_{-1,0} = -\frac{\sqrt{3}}{\sqrt{10}} = -\sqrt{\frac{3}{10}}$ $\qquad S_{-2,1} = \frac{\sqrt{3}\cdot\cancel{2}}{2\sqrt{5}} = \sqrt{\frac{3}{5}}$

$S_{1,-1} = \frac{\sqrt{3}\cdot\sqrt{2}}{2\sqrt{5}} = \sqrt{\frac{3}{10}}$ $\qquad S_{0,0} = -\sqrt{\frac{2}{5}}$ $\qquad S_{-1,1} = \frac{\sqrt{2}\cdot\sqrt{3}}{2\sqrt{5}} = \sqrt{\frac{3}{10}}$

$S_{2,-1} = \frac{2\cdot\sqrt{3}}{2\sqrt{5}} = \sqrt{\frac{3}{5}}$ $\qquad S_{1,0} = -\sqrt{\frac{3}{10}}$ $\qquad S_{0,1} = \sqrt{\frac{1}{10}}$

$$\Psi_{-1} = \sqrt{\frac{1}{10}}\cdot\left(v_0 \chi_{-1} - \sqrt{3}\, v_{-1}\chi_0 + \sqrt{6}\, v_{-2}\chi_1\right)$$

$$\Psi_0 = \sqrt{\frac{1}{10}}\left(\sqrt{3}\, v_{+1}\chi_{-1} - 2 v_0 \chi_0 + \sqrt{3}\, v_{-1}\chi_1\right)$$

$$\Psi_{+1} = \sqrt{\frac{1}{10}}\left(\sqrt{6}\, v_2 \chi_{-1} - \sqrt{3}\, v_1 \chi_0 + v_0 \chi_1\right)$$

$$\left.\begin{aligned} \chi_{-1} &= \beta(1)\beta(2) \\ \chi_0 &= \tfrac{1}{\sqrt{2}}\left(\alpha(1)\beta(2) + \alpha(2)\beta(1)\right) \\ \chi_1 &= \alpha(1)\alpha(2) \end{aligned}\right\} \qquad \left.\begin{aligned} v_0 &= \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) \\ v_{\pm 1} &= \sqrt{\frac{15}{8\pi}} \sin\theta\cos\theta\, e^{\pm i\varphi} \\ v_{\pm 2} &= \frac{1}{4}\sqrt{\frac{15}{2\pi}} \sin^2\theta\, e^{\pm 2i\varphi} \end{aligned}\right\}$$

$$\Psi_1 = \sqrt{\frac{1}{10}}\left\{\sqrt{\frac{5}{4\pi}}(\quad)\chi_{-1} - \sqrt{\frac{5}{8\pi}}\; 3\sin\theta\cos\theta\right.$$

$$\frac{1}{4}\sqrt{\frac{3}{\pi}}\,.$$

$$(\vec\sigma_1\vec r)(\vec\sigma_2\vec r) = \begin{pmatrix} z & x-iy \\ x+iy & -z\end{pmatrix}_1 \begin{pmatrix} z & x-iy \\ x+iy & -z\end{pmatrix}_2$$

$$\begin{pmatrix} z & x-iy \\ x+iy & -z\end{pmatrix}_1 \begin{pmatrix} z & x-iy \\ x+iy & -z\end{pmatrix}_2 \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 = \begin{pmatrix}x-iy\\-z\end{pmatrix}_1\begin{pmatrix}x-iy\\-z\end{pmatrix}_2$$

$$= z^2\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 - \sqrt2 z(x-iy)\frac{1}{\sqrt2}\left\{\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right\}$$

$$+ (x-iy)^2\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2$$

$$\boxed{(\sigma_1 r)(\sigma_2 r)\chi_{-1} = z^2\chi_{-1} - \sqrt2 z(x-iy)\chi_0 + (x-iy)^2\chi_1}$$

$$\begin{pmatrix} z & x-iy \\ x+iy & -z\end{pmatrix}_1 \begin{pmatrix} z & x-iy \\ x+iy & -z\end{pmatrix}_2 \frac{1}{\sqrt2}\left\{\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right\}$$

$$= \frac{1}{\sqrt2}\left\{\begin{pmatrix}z\\x+iy\end{pmatrix}_1\begin{pmatrix}x-iy\\-z\end{pmatrix}_2 + \begin{pmatrix}x-iy\\-z\end{pmatrix}_1\begin{pmatrix}z\\x+iy\end{pmatrix}_2\right\}$$

$$= \sqrt2 z(x-iy)\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 + \frac{1}{\sqrt2}(x^2+y^2-z^2)\left\{\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\right\}$$

$$- \sqrt2 z(x+iy)\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2$$

$$\boxed{(\sigma_1 r)(\sigma_2 r)\chi_0 = -\sqrt2 z(x+iy)\chi_{-1} + (x^2+y^2-z^2)\chi_0 + \sqrt2 z(x-iy)\chi_1}$$

$$\begin{pmatrix} z & x-iy \\ x+iy & -z\end{pmatrix}_1 \begin{pmatrix} z & x-iy \\ x+iy & -z\end{pmatrix}_2 \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 = \begin{pmatrix}z\\x+iy\end{pmatrix}_1\begin{pmatrix}z\\x+iy\end{pmatrix}_2$$

$$= z^2\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 + \sqrt2 z(x+iy)\frac{1}{\sqrt2}\left\{\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right\} + (x+iy)^2\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2$$

$$\boxed{(\sigma_1 r)(\sigma_2 r)\chi_1 = (x+iy)^2\chi_{-1} + \sqrt2 z(x+iy)\chi_0 + z^2\chi_1}$$

$$\begin{pmatrix} z^2 & -\sqrt2 z(x+iy) & (x+iy)^2 \\ -\sqrt2 z(x-iy) & -z^2+(x^2+y^2) & \sqrt2 z(x+iy) \\ (x-iy)^2 & \sqrt2 z(x-iy) & z^2 \end{pmatrix} = r^2\begin{pmatrix} \cos^2\theta & -\sqrt2\cos\theta\sin\theta e^{+i\varphi} & \sin^2\theta e^{+2i\varphi} \\ -\sqrt2\cos\theta\sin\theta e^{-i\varphi} & -\cos^2\theta+\sin^2\theta & \sqrt2\cos\theta\sin\theta e^{+i\varphi} \\ \sin^2\theta e^{-2i\varphi} & \sqrt2\cos\theta\sin\theta e^{-i\varphi} & \cos^2\theta \end{pmatrix}$$

$$\left\{\frac{(\sigma_1 r)(\sigma_2 r)}{r^2}\right\}_{ij} = \sum \chi_i \left\{\frac{(\sigma_1 r)(\sigma_2 r)}{r^2}\right\} \Sigma_j$$

$$\sum \chi_{-1}\left\{\frac{(\sigma_1 r)(\sigma_2 r)}{r^2}\right\} = \chi_{-1}\Sigma_{-1} = \chi_{-1}\begin{pmatrix}1\\0\\0\end{pmatrix}\sqrt{\frac{1}{10}}\begin{pmatrix} v_0 \cos^2\theta \\ +\sqrt{6}\, v_{-1}\cos\theta\sin\theta\, e^{+i\varphi} \\ +\sqrt{6}\sin^2\theta\, e^{\mp 2i\varphi} \end{pmatrix}$$

$$\{\ \}_{-1-1} = \sqrt{\frac{1}{10}}\{v_0\cos^2\theta + \sqrt{6}\, v_{-1}\cos\theta\sin\theta\, e^{i\varphi} + \sqrt{6}\, v_{-2}\sin^2\theta\, e^{2i\varphi}\}$$

$$\{\ \}_{-1\,0} = 0$$

$$\{\ \}_{-1\,1} = 0$$

$$\left\{\frac{(\sigma_1 r)(\sigma_2 r)}{r^2}\right\}_{ij} = 0 \quad \text{for } i \neq j$$

(∵ total angular momentum z-comp is $\left\{\frac{(\sigma_1 r)(\sigma_2 r)}{r^2}\right\}$ commutative.)

$$\left\{\frac{(\sigma_1 r)(\sigma_2 r)}{r^2}\right\}_{-1-1} = \sqrt{\frac{1}{10}}\{v_0\cos^2\theta - \sqrt{6}\, v_{-1}\cos\theta\sin\theta\, e^{i\varphi} + \sqrt{6}\, v_{-2}\sin^2\theta\, e^{2i\varphi}\}$$

$$= \sqrt{\frac{1}{10}}\left\{\sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)\cos^2\theta - \sqrt{6}\sqrt{\frac{15}{8\pi}}\sin^2\theta\cos^2\theta + \frac{\sqrt{6}}{4}\sqrt{\frac{15}{2\pi}}\sin^4\theta\right\}$$

$$= \sqrt{\frac{1}{10}}\sqrt{\frac{5}{4\pi}}\frac{1}{r^4}\left\{\frac{3}{2}\frac{z^4}{r^4} - \frac{1}{2}\frac{z^2}{r^2} + 3\frac{z^2(x^2+y^2)}{r^4} + \frac{3\sqrt{2}}{24}\frac{(x^2+y^2)^2}{r^4}\right\}$$

$$= \sqrt{\frac{1}{10}}\sqrt{\frac{5}{4\pi}}\left\{\frac{3}{2} - \frac{1}{6}\right\} = \sqrt{\frac{1}{10}}\sqrt{\frac{5}{4\pi}}\,\frac{8}{6} = \frac{1}{3}\sqrt{\frac{16}{8\pi}}$$

$$\frac{\sqrt{6}}{4}\sqrt{3} = \frac{\sqrt{2}\,3}{4}$$

$$\frac{\{z^2+(x^2+y^2)\}^2}{r^4}$$

$$= \frac{1}{3}\sqrt{\frac{2}{\pi}}\cdot r$$

$$= \{\ \}_{00} = \{\ \}_{+1+1}$$

$$\psi = \frac{u(r)}{r}\chi_{-1} + \frac{v(r)}{r}\sqrt{\frac{1}{10}}\left(Y_0\chi_{-1} - \sqrt{3}\,Y_{-1}\chi_0 + \sqrt{6}\,Y_{-2}\chi_1\right)$$

$$Y_0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$
$$Y_{-1} = \sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{-i\varphi}$$
$$Y_{-2} = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{-2i\varphi}$$

$$\left.\begin{matrix}\sqrt{\frac{1}{10}}Y_0 \\ -\sqrt{\frac{3}{10}}Y_{-1} \\ \sqrt{\frac{6}{10}}Y_{-2}\end{matrix}\right\} = \left\{\begin{matrix}\frac{3}{2}\sqrt{\frac{1}{8\pi}}\left(\cos^2\theta - \frac{1}{3}\right) \\ -\sqrt{\frac{1}{8\pi}}\frac{3}{\sqrt{2}}\cos\theta\sin\theta\, e^{-i\varphi} \\ \sqrt{\frac{1}{8\pi}}\frac{3}{2}\sin^2\theta\, e^{-2i\varphi}\end{matrix}\right.$$

$$\left\{\frac{\hbar^2}{M}\Delta + E + \left[g_1^2 + g_2^2\left(1 - \frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2}\right)\right.\right.$$
$$\left.\left. + g_2^2\left(1 - \frac{3(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2}\right)\left(\frac{1}{\kappa r} \pm \frac{1}{\kappa^2 r^2}\right)\right]\frac{e^{-\kappa r}}{r}\right\}\psi = 0$$

$$\psi = \begin{pmatrix} \frac{u(r)}{r} + \sqrt{\frac{1}{10}}\frac{v(r)}{r}Y_0 \\ -\sqrt{\frac{3}{10}}\frac{v(r)}{r}Y_{-1} \\ \sqrt{\frac{6}{10}}\frac{v(r)}{r}Y_{-2} \end{pmatrix}$$

$$\frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2} = \begin{pmatrix} \cos^2\theta & -\sqrt{2}\cos\theta\sin\theta\, e^{i\varphi} & \sin^2\theta\, e^{2i\varphi} \\ -\sqrt{2}\cos\theta\sin\theta\, e^{-i\varphi} & -\cos^2\theta + \sin^2\theta & \sqrt{2}\cos\theta\sin\theta\, e^{i\varphi} \\ \sin^2\theta\, e^{-2i\varphi} & \sqrt{2}\cos\theta\sin\theta\, e^{-i\varphi} & \cos^2\theta \end{pmatrix}$$

$$\left\{\frac{\hbar^2}{M}\Delta + E + \left[g_1^2 + g_2^2\left(1 + \frac{1}{\kappa r} \pm \frac{1}{\kappa^2 r^2}\right)\right]\frac{e^{-\kappa r}}{r}\right\}\begin{pmatrix} u(r) + \sqrt{\frac{1}{10}}v(r)Y_0 \\ -\sqrt{\frac{3}{10}}v(r)Y_{-1} \\ \sqrt{\frac{6}{10}}v(r)Y_{-2} \end{pmatrix}$$

$$-\left(1 + \frac{3}{\kappa r} + \frac{3}{\kappa^2 r^2}\right)\begin{pmatrix} \left(u + \sqrt{\frac{1}{10}}vY_0\right)\cos^2\theta + \sqrt{\frac{6}{10}}vY_{-1}\cos\theta\sin\theta\, e^{i\varphi} + \sqrt{\frac{6}{10}}vY_{-2}\sin^2\theta\, e^{2i\varphi} \\ -\left(u + \sqrt{\frac{1}{10}}vY_0\right)\sqrt{2}\cos\theta\sin\theta\, e^{-i\varphi} + \frac{\sqrt{3}}{\sqrt{10}}vY_{-1}(\cos^2\theta - \sin^2\theta) + 2\frac{\sqrt{3}}{\sqrt{10}}vY_{-2}\cos\theta\sin\theta\, e^{i\varphi} \\ \left(u + \sqrt{\frac{1}{10}}vY_0\right)\sin^2\theta\, e^{-2i\varphi} - \sqrt{\frac{6}{10}}vY_{-1}\cos\theta\sin\theta\, e^{-i\varphi} + \sqrt{\frac{6}{10}}vY_{-2}\cos^2\theta \end{pmatrix}$$

$$\left\{\frac{\hbar^2}{M}\frac{d^2u}{dr^2} + Eu + \left[\frac{g^2}{3g}\left(1+\frac{3}{\kappa r}+\frac{3}{\kappa^2 r^2}\right)\right]\frac{e^{-\kappa r}}{r}u\right\}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$+\left\{\frac{\hbar^2}{M}\frac{d^2v}{dr^2} - \frac{l(l+1)}{r^2}v + Ev + [\quad]\frac{e^{-\kappa r}}{r}v\right\}\begin{pmatrix}\sqrt{\frac{1}{10}}Y_0\\ -\sqrt{\frac{3}{10}}Y_{-1}\\ \sqrt{\frac{6}{10}}Y_{-2}\end{pmatrix}$$

$$-\left(1+\frac{3}{\kappa r}+\frac{3}{\kappa^2 r^2}\right)u\begin{pmatrix}\cos^2\theta-\frac{1}{3}\\ -\sqrt{2}\cos\theta\sin\theta e^{-i\varphi}\\ \sin^2\theta\, e^{-2i\varphi}\end{pmatrix} \qquad \frac{3}{4}\sqrt{\frac{1}{2\pi}}\begin{pmatrix}\cos^2\theta-\frac{1}{3}\\ -\sqrt{2}\cos\theta\sin\theta e^{-i\varphi}\\ \sin^2\theta\, e^{-2i\varphi}\end{pmatrix}$$

$$-\left(1+\frac{3}{\kappa r}+\frac{3}{\kappa^2 r^2}\right)v\,\frac{3}{4}\sqrt{\frac{1}{2\pi}}\begin{pmatrix}(\cos^2\theta-\frac{1}{3})\cos^2\theta + 2\cos^2\theta\sin^2\theta + \sin^4\theta \\ (\cos^2\theta+\sin^2\theta)^2 \\ \sqrt{2}(\cos^2\theta-\frac{1}{3})\cos\theta\sin\theta e^{-i\varphi} \pm \sqrt{2}\cos\theta\sin\theta(\cos^2\theta-\sin^2\theta)e^{-i\varphi} \\ \pm\sqrt{2}\sin^2\theta\cos\theta\sin\theta e^{-i\varphi} \\ (\cos^2\theta-\frac{1}{3})\sin^2\theta e^{-2i\varphi} \mp 2\cos^2\theta\sin^2\theta e^{-2i\varphi} \\ +\sin^2\theta\cos^2\theta e^{-2i\varphi}\end{pmatrix} = 0$$

$2\cos^2\theta - 2\sin^2\theta - \frac{1}{3}$

$$\begin{pmatrix}1-\frac{1}{3}\cos^2\theta \\ \sqrt{2} + \frac{\sqrt{2}}{3}\cos\theta\sin\theta e^{-i\varphi} \\ -\frac{1}{3}\sin^2\theta\, e^{-2i\varphi}\end{pmatrix}$$

$$\left\{\frac{\hbar^2}{M}\frac{d^2u}{dr^2}+Eu+[\quad]\frac{e^{-\kappa r}}{r}u\right\}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$+\left\{\frac{\hbar^2}{M}\frac{d^2v}{dr^2}-\frac{l(l+1)}{r^2}v+Ev+[\quad]\frac{e^{-\kappa r}}{r}v\right\}\frac{3}{4}\sqrt{\frac{1}{2\pi}}\begin{pmatrix}\cos^2\theta-\frac{1}{3}\\-\sqrt{2}\cos\theta\sin\theta e^{-i\varphi}\\\sin^2\theta e^{-2i\varphi}\end{pmatrix}$$

$$=\left\{\begin{pmatrix}-\frac{1}{3}\\0\\0\end{pmatrix}+\begin{pmatrix}\cos^2\theta\\-\sqrt{2}\cos\theta\\\cdots\end{pmatrix}\right\}$$

$$-\left(1+\frac{3}{\kappa r}+\frac{3}{\kappa^2r^2}\right)u\begin{pmatrix}\cos^2\theta\\-\sqrt{2}\cos\theta\sin\theta e^{-i\varphi}\\\sin^2\theta e^{-2i\varphi}\end{pmatrix}$$

$$-\left(1+\frac{3}{\kappa r}+\frac{3}{\kappa^2r^2}\right)\frac{e^{\kappa r}}{r}v\,\frac{3}{4}\sqrt{\frac{1}{2\pi}}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$+\left(1+\frac{3}{\kappa r}+\frac{3}{\kappa^2r^2}\right)\frac{e^{-\kappa r}}{r}v\,\frac{1}{4}\sqrt{\frac{1}{2\pi}}\begin{pmatrix}\cos^2\theta\\\cdots\\\cdots\end{pmatrix}=0$$

$$\frac{\frac{3}{2}\frac{x^2}{3r^2}}{2}$$

$$(x^2+y^2)^2$$

$$3\left\{\frac{\hbar^2}{M}\frac{d^2u}{dr^2}+Eu+[\quad]\frac{e^{-\kappa r}}{r}u\right\}-\frac{1}{4}\sqrt{\frac{1}{2\pi}}\left\{\frac{\hbar^2}{M}\frac{d^2v}{dr^2}-\frac{l(l+1)}{r^2}v\right.$$

$$\left.+Ev+[\quad]\frac{e^{-\kappa r}}{r}v-3\left(1+\frac{3}{\kappa r}+\frac{3}{\kappa^2r^2}\right)\frac{e^{-\kappa r}}{r}v\right\}=0$$

$$\frac{3}{4}\sqrt{\frac{1}{2\pi}}\left\{\frac{\hbar^2}{M}\frac{d^2v}{dr^2}-\frac{l(l+1)}{r^2}v+Ev+[\quad]\frac{e^{-\kappa r}}{r}v\right\}-\left(1+\frac{3}{\kappa r}+\frac{3}{\kappa^2r^2}\right)u=0$$

$$+\frac{1}{3}\left(1+\frac{3}{\kappa r}+\frac{3}{\kappa^2r^2}\right)\frac{e^{-\kappa r}}{r}v$$

$$3\left\{\frac{\hbar^2}{M}\frac{d^2u}{dr^2}+Eu+[\quad]\frac{e^{-\kappa r}}{r}u\right\}-\frac{g^2}{3}\left(1+\frac{3}{\kappa r}+\frac{3}{\kappa^2r^2}\right)\frac{e^{-\kappa r}}{r}u$$

$$+\frac{1}{4}\sqrt{\frac{1}{2\pi}}\,\cancel{\hbar\ g^2}\left(3-\frac{1}{3}\right)\left(1+\frac{3}{\kappa r}+\frac{3}{\kappa^2r^2}\right)\frac{e^{-\kappa r}}{r}v=0$$

$$\frac{8}{3}\sqrt{\frac{1}{2\pi}}$$

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$$\frac{\hbar^2}{M}\frac{d^2u}{dr^2}+E\,u+\left(g_1^2+\frac{?}{3}g_2^2\right)\frac{e^{-\kappa r}}{r}u+\sqrt{2}\,g_2^2\left(\frac{1}{3}+\frac{1}{\kappa r}+\frac{1}{\kappa^2r^2}\right)\frac{e^{-\kappa r}}{r}w$$

$=0$

$$\left\{\frac{\hbar^2}{M}\left(\frac{d^2v}{dr^2}-\frac{6}{r^2}v\right)+E\,v+\left\{g_1^2+g_2^2\left(\frac{4}{3}+\frac{2}{\kappa r}+\frac{2}{\kappa^2r^2}\right)\right\}\frac{e^{-\kappa r}}{r}w\right\}$$

$$\frac{1}{4}-4\sqrt{2\pi}\,g_2^2\left(\frac{1}{3}+\frac{1}{\kappa r}+\frac{1}{\kappa^2r^2}\right)\frac{e^{-\kappa r}}{r}u=0$$

$$g_1^2+g_2^2\left(\frac{4}{3}+\frac{2}{\kappa r}+\frac{2}{\kappa^2r^2}\right)$$

$v=r^3w.\quad \frac{dv}{dr}=r^3\frac{dw}{dr}+3r^2w\quad \frac{d^2v}{dr^2}=r^3\frac{d^2w}{dr^2}$

$+6r^2\frac{dw}{dr}+6rw$ $\qquad e^{-\kappa r}w$

$$\left\{\frac{\hbar^2}{M}\left(r^3\frac{d^2w}{dr^2}+6r^2\frac{dw}{dr}\right)+E\,r^3w+\left[\left(g_1^2+\frac{4}{3}g_2^2\right)r^2+g_2^2\frac{2r}{\kappa}+g_2^2\frac{2r^2}{\kappa^2}\right]\right.$$

$+4\sqrt{2\pi}$

$r^k \qquad k(k-1)=6$

$k=3$

$k=-2.$

$v=\frac{1}{r^2}(\cdots\cdots)$

$$1 - \frac{3(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2} = -3\left(\frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2} - \frac{1}{3}\right)$$

$$\left(\frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2} - \frac{1}{3}\right) = \begin{pmatrix} \cos^2\theta & -\sqrt{2}\cos\theta\sin\theta\, e^{i\varphi} & \sin^2\theta\, e^{2i\varphi} \\ -\sqrt{2}\cos\theta\sin\theta\, e^{-i\varphi} & -\cos^2\theta + \sin^2\theta - \frac{1}{3} & \sqrt{2}\cos\theta\sin\theta\, e^{i\varphi} \\ \sin^2\theta\, e^{-2i\varphi} & \sqrt{2}\cos\theta\sin\theta\, e^{-i\varphi} & \cos^2\theta \end{pmatrix}$$

$$\boxed{\frac{\hbar^2}{M}\frac{d^2u}{dr^2} + Eu + \left[g_1^2 + \frac{2}{3}g_2^2\right]\frac{e^{-\kappa r}}{r}u = 0}$$

$$\frac{\hbar^2}{M}\frac{d^2v}{dr^2} - \frac{6}{r^2}v + Ev + [\quad]\frac{e^{-\kappa r}}{r}v$$

$$+ \frac{\kappa}{3}\left(\frac{1}{3} + \frac{1}{\kappa r} + \frac{1}{\kappa^2 r^2}\right)v$$

$$- \frac{4}{3}\sqrt{2\pi}$$

$\boxed{{}^3S - {}^3D_1}$

$$\left\{ \frac{\hbar^2}{M}\Delta + E + \left(g_1^2 + \frac{2}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r} \right\}\psi$$

$$-3g_2^2\left(\frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2} - \frac{1}{3}\right)\left(\frac{1}{3} + \frac{1}{\varkappa r} + \frac{1}{\varkappa^2 r^2}\right)\frac{e^{-\varkappa r}}{r}\psi = 0$$

$$\frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2} - \frac{1}{3} = \begin{pmatrix} \cos^2\theta - \frac{1}{3}, & -\sqrt{2}\cos\theta\sin\theta e^{i\varphi} & \sin^2\theta e^{2i\varphi} \\ -\sqrt{2}\cos\theta\sin\theta e^{-i\varphi} & \boxed{-\cos^2\theta + \sin^2\theta - \frac{1}{3}} \\ -2\left(\cos^2\theta - \frac{1}{3}\right) & \sqrt{2}\cos\theta\sin\theta e^{i\varphi} \\ \sin^2\theta e^{-2i\varphi} & \sqrt{2}\cos\theta\sin\theta e^{-i\varphi} & \cos^2\theta - \frac{1}{3} \end{pmatrix}$$

$$\psi = \frac{u(r)}{r}\chi_{-1} + \frac{v(r)}{r}\left(\sqrt{\frac{1}{10}}Y_0\chi_{-1} - \sqrt{\frac{3}{10}}Y_{-1}\chi_0 + \sqrt{\frac{6}{10}}Y_{-2}\chi_1\right)$$

$$\sqrt{\frac{1}{10}}Y_0 = \sqrt{\frac{1}{10}\cdot\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right) = \sqrt{\frac{1}{8\pi}}\cdot\frac{3}{2}\left(\cos^2\theta - \frac{1}{3}\right)$$

$$\sqrt{\frac{3}{10}}Y_{-1} = \sqrt{\frac{3}{10}\cdot\frac{15}{8\pi}}\sin\theta\cos\theta e^{-i\varphi} = \sqrt{\frac{1}{8\pi}}\cdot\frac{3}{2}\left(\sqrt{2}\sin\theta\cos\theta e^{-i\varphi}\right)$$

$$\sqrt{\frac{6}{10}}Y_{-2} = \frac{1}{4}\sqrt{\frac{6}{10}\cdot\frac{15}{2\pi}}\sin^2\theta e^{-2i\varphi} = \sqrt{\frac{1}{8\pi}}\cdot\frac{3}{2}\sin^2\theta e^{-2i\varphi}$$

$$\left(\frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2} - \frac{1}{3}\right)\chi_{-1} = \begin{pmatrix} \cos^2\theta - \frac{1}{3} \\ -\sqrt{2}\sin\theta\cos\theta e^{-i\varphi} \\ \sin^2\theta e^{-2i\varphi} \end{pmatrix} \qquad \chi_{-1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\cos^4\theta - \frac{2}{3}\cos^2\theta + \frac{1}{9}$$

$$\frac{8}{9} - \frac{2}{3}\left(\cos^2\theta - \frac{1}{3}\right)$$

$$\left(\frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2} - \frac{1}{3}\right)\begin{pmatrix} \cos^2\theta - \frac{1}{3} \\ -\sqrt{2}\sin\theta\cos\theta e^{-i\varphi} \\ \sin^2\theta e^{-2i\varphi} \end{pmatrix} = \begin{pmatrix} \left(\cos^2\theta - \frac{1}{3}\right)^2 + 2\sin^2\theta\cos^2\theta + \sin^4\theta \\ \frac{8}{9} - \frac{2}{3}\left(\cos^2\theta - \frac{1}{3}\right) \\ \cos\theta\sin\theta\left\{\left(-\sqrt{2} + 2\sqrt{2}\right)\left(\cos^2\theta - \frac{1}{3}\right) + \sqrt{2}\sin^2\theta\right\}e^{-i\varphi} \\ \frac{2}{3}\sqrt{2}\cos\theta\sin\theta e^{-i\varphi} \\ \sin^2\theta\left\{\cos^2\theta - \frac{1}{3} - 2\cos^2\theta + \cos^2\theta - \frac{1}{3}\right\}e^{-2i\varphi} \\ -\frac{2}{3}\sin^2\theta e^{-2i\varphi} \end{pmatrix}$$

$$= \frac{8}{9}\chi_{-1} - \frac{2}{3}\begin{pmatrix} \cos^2\theta - \frac{1}{3} \\ -\sqrt{2}\sin\theta\cos\theta e^{-i\varphi} \\ \sin^2\theta e^{-2i\varphi} \end{pmatrix}$$

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$$\left(\frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2}-\frac{1}{3}\right)\psi = \frac{u(r)}{r}\begin{pmatrix} \\ \\ \end{pmatrix}$$

$$+\frac{v(r)}{r}\sqrt{\frac{1}{8\pi}}\cdot\frac{4}{3}\begin{pmatrix}1\\0\\0\end{pmatrix}-\frac{v(r)}{r}\sqrt{\frac{1}{8\pi}}\begin{pmatrix}\cos^2\theta-\frac{1}{3}\\0\\0\end{pmatrix}$$

$$\left\{\frac{\hbar^2}{M}\frac{d^2u}{dr^2}+Eu+\left(g_1^2+\frac{2}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r}u\right.$$

$$-\frac{v(r)}{r}\sqrt{\frac{1}{8\pi}}\cdot 4g_2^2\left(\frac{1}{3}+\frac{1}{\varkappa r}+\frac{1}{\varkappa^2r^2}\right)\frac{e^{-\varkappa r}}{r}v=0$$

$$\frac{1}{\sqrt{8\pi}}\cancel{\frac{4}{3}}\left\{\frac{\hbar^2}{M}\left(\frac{d^2v}{dr^2}-\frac{6v}{r^2}\right)+Ev+\left(g_1^2+\frac{2}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r}v+2g_2^2\left(\frac{1}{3}+\frac{1}{\varkappa r}\pm\frac{1}{\varkappa^2r^2}\right)\frac{e^{-\varkappa r}}{r}v\right\}$$

$$-\frac{\cancel{3}}{2}g_2^2\left(\frac{1}{3}+\frac{1}{\varkappa r}\pm\frac{1}{\varkappa^2r^2}\right)\frac{e^{-\varkappa r}}{r}u=0$$

$$u'\sqrt{\frac{1}{4\pi}}$$

$$u=u'\sqrt{\frac{1}{4\pi}}$$

$$\left\{\begin{aligned}&\frac{\hbar^2}{M}\frac{d^2u'}{dr^2}+Eu'+\left(g_1^2+\frac{2}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r}u'\\&-2\sqrt{2}g_2^2\left(\frac{1}{3}+\frac{1}{\varkappa r}+\frac{1}{\varkappa^2r^2}\right)\frac{e^{-\varkappa r}}{r}v=0\\&\frac{\hbar^2}{M}\left(\frac{d^2v}{dr^2}-\frac{6v}{r^2}\right)+Ev+\left(g_1^2+\frac{2}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r}v+2g_2^2\left(\frac{1}{3}+\frac{1}{\varkappa r}-\frac{1}{\varkappa^2r^2}\right)\\&-2\sqrt{2}g_2^2\left(\frac{1}{3}+\frac{1}{\varkappa r}+\frac{1}{\varkappa^2r^2}\right)\frac{e^{-\varkappa r}}{r}u'=0.\end{aligned}\right.$$

$$\frac{e^{-\varkappa r}}{r}v$$

$$\varkappa^2r^2=\frac{-3\pm\sqrt{9+12}}{2}=\frac{-3\pm\sqrt{21}}{2}$$

$$\frac{\varkappa^2r^2+3\varkappa r-3}{3\varkappa^2r^2}$$

$$|\varkappa r|<1$$

$$\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$$

$$\frac{d}{dx}(x-x^2)=1-2x=0 \qquad x=\frac{1}{2}$$

$$\varkappa r=2. \qquad \frac{1}{2}-\frac{1}{4}=\frac{1}{4}$$

$\boxed{{}^1S}$ $\boxed{{}^1P}$ etc.

$$\begin{pmatrix} z & x-iy \\ x+iy & -z\end{pmatrix}_1 \begin{pmatrix} z & x-iy \\ x+iy & -z\end{pmatrix}_2 \frac{1}{\sqrt2}\left\{ \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right\}$$

$$= \frac{1}{\sqrt2}\left\{ \begin{pmatrix}z\\x+iy\end{pmatrix}_1 \begin{pmatrix}x-iy\\-z\end{pmatrix}_2 - \begin{pmatrix}x-iy\\-z\end{pmatrix}_1\begin{pmatrix}z\\x+iy\end{pmatrix}_2\right\}$$

$$= -\frac{r^2}{\sqrt2}\left\{\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right\}$$

$$\frac{(\sigma_1 r)(\sigma_2 r)}{r^2} = -1 \quad \text{for singlet state}$$

$$(\sigma_1\sigma_2) = \left\{ \begin{pmatrix}0&1\\1&0\end{pmatrix}_1\begin{pmatrix}0&1\\1&0\end{pmatrix}_2 + \begin{pmatrix}0&-i\\i&0\end{pmatrix}_1\begin{pmatrix}0&-i\\i&0\end{pmatrix}_2 + \begin{pmatrix}1&0\\0&-1\end{pmatrix}_1\begin{pmatrix}1&0\\0&-1\end{pmatrix}_2\right\}$$

$$\frac{1}{\sqrt2}\left\{\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right\}$$

$$= \frac{1}{\sqrt2}\left\{\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 + \begin{pmatrix}0\\i\end{pmatrix}_1\begin{pmatrix}-i\\0\end{pmatrix}_2 - \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 \right.$$

$$\left. - \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 - \begin{pmatrix}-i\\0\end{pmatrix}_1\begin{pmatrix}0\\i\end{pmatrix}_2 + \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right\}$$

$$= +\frac{3}{\sqrt2}\left\{\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 - \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2\right\} = -3 \qquad (\sigma_1\sigma_2) = -3,$$

$$V = +\left[g_1^2 + g_2^2\left\{(\sigma^{(1)}\sigma^{(2)}) - \frac{(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2}\right\} + g_2^2\left\{(\sigma^{(1)}\sigma^{(2)}) - \frac{3(\sigma^{(1)}r)(\sigma^{(2)}r)}{r^2}\right\} \times\left(\frac{1}{\kappa r} - \frac{1}{\kappa r^2}\right)\right]\frac{e^{-\kappa r}}{r}$$

$$\frac{g_1^2 + \frac{2}{3}g_2^2}{} \quad = \frac{5}{3}\frac{g^2}{g^2}$$

$$= +\left[g_1^2 - 2g_2^2\right]\frac{e^{-\kappa r}}{r} = -(2g_2^2 - g_1^2)\frac{e^{-\kappa r}}{r}$$

$$\iiint \frac{\partial f}{\partial z}(\vec{r})\,\frac{e^{-\kappa r}}{r}\,g(\vec{r})\,dv$$

$$= \iiint f(\vec{r})\,\frac{\partial}{\partial z}\left[\frac{e^{-\kappa r}}{r}\,g(\vec{r})\right] dx\,dy\,dz + \iint \left[f(\vec{r})\frac{e^{-\kappa r}}{r}\right] dx\,dy$$

$dx\,dy = df \cdot \cos\theta.$

$$\iint f(\vec{r})\frac{e^{-\kappa r}}{r}\cos\theta\,df$$

$$= \iint f(\vec{r})\,\frac{e^{-\kappa r}}{r^2}\,r^2\,dr\,\sin\theta\cos\theta\,d\theta\,d\varphi$$

$$= f(0)\cdot\frac{e^{-\kappa r}}{r} \quad dx\,dy\,dz$$

$$\iint \frac{\partial f(r,\theta,\varphi)}{\partial r}\,\frac{e^{-\kappa r}}{r}\,r^2\,dr\,\sin\theta\cos\theta\,d\theta\,d\varphi$$

$$\frac{\partial f(r)}{\partial z}\cdot\frac{e^{-\kappa r}}{r} = \iiint \frac{\partial f}{\partial r}\cdot\frac{\partial}{\partial r}(e^{-\kappa r}\cdot r) \qquad \boxed{\frac{1}{r}\frac{\partial}{\partial\theta}}$$

$$\iiint \frac{\partial^2 f}{\partial r^2}\,\frac{e}{r}\,dr\,r$$

$$f\ \frac{e^{-\kappa r}}{r} = \iiint f\,\frac{\partial^2}{\partial r^2}(e^{-\kappa r}\cdot r)$$

curl curl f

$$\iiint \frac{\partial f}{\partial z}\cdot\frac{\partial}{\partial z}\left(\frac{e}{r}\right) - \iiint \frac{\partial f}{\partial z}\frac{\partial}{\partial z}\left(\frac{e}{r}\right)$$

$$\frac{\partial}{\partial r}\left(f\frac{e}{r^2}\right) \qquad \iiint \qquad - \iiint \frac{\partial f}{\partial z}\frac{\partial}{\partial z}\left(\frac{e}{r}\right)$$

$$= \frac{\partial f}{\partial r}\frac{e}{r^2} + f\,\frac{\partial}{\partial r}\left(\frac{e}{r^2}\right) \qquad + \iiint f\,\frac{\partial^2}{\partial z^2}\left(\frac{e}{r}\right).$$

$$-\Delta U + \kappa^2 U = -4\pi g_2 \,\mathrm{curl}\, S$$

$$-\Delta \tilde{U}^\dagger + \kappa^2 \tilde{U}^\dagger = \frac{g_1}{\kappa c}\,\mathrm{grad}\, M_0$$

$$\mathrm{curl}_1 U(r_1) = g_2 \frac{e^{-\kappa r}}{r}\,\mathrm{curl}_2 \mathrm{curl}_2 S(r_2)$$

$$-\frac{g_2^2}{\kappa^2}\,\sigma_1 \frac{e^{-\kappa r}}{r}\,\sigma_1\,\mathrm{curl}\;\mathrm{curl}(\tilde{\Psi}\sigma_2\Psi)$$

=

^3S, ^3D の Coupling に於ける $\left(\frac{1}{\kappa r} - \frac{1}{\kappa^2 r^2}\right)\frac{e^{-\kappa r}}{r}$ の形の singularity の為めに、0 迄で finite な solution を得られない。然し之は interaction の式で partial integration をやられたものと考へられる。

実際 interaction energy は

$$\overline{H}' = \iiint H(r_1)\,dv_1$$

$$H(r_1) = \tilde{\Psi}(r_1)\cdots\Big[\;\cdot\; + \frac{g_2}{\kappa^2}\big(\mathrm{curl}\,\tilde{U}\cdot\vec{\sigma}\tilde{\Omega} + \mathrm{curl}\,U\cdot\vec{\sigma}\Omega\big) + \frac{4\pi g_2}{\kappa^2}\{(\tilde{S}\sigma)\tilde{\Omega} + (S\sigma)\Omega\}\Big]\Psi(r_1)$$

$$= -\frac{g_2^2}{\kappa^2}\Big\{S(r_1)\frac{e^{-\kappa r}}{r}\,\mathrm{curl}\,\mathrm{curl}\,\tilde{S}(r_2) + \tilde{S}(r_1)\frac{e^{-\kappa r}}{r}\,\mathrm{curl}\,\mathrm{curl}\,S(r_2)\Big\}$$

$$+ \frac{4\pi g_2^2}{\kappa^2}\{\tilde{S}(r_2)S(r_1) + S(r_2)\tilde{S}(r_1)\}$$

$$= -\frac{g_1^2}{\kappa^2}\,\tilde{u}_m(r_1)\vec{\sigma}\,v_n(r_1)\frac{e^{-\kappa r}}{r}\Big\{\cancel{\mathrm{curl}_2\,\mathrm{curl}_2}\;\nabla_2\times\{\nabla_2\times(\tilde{v}_p(r_2)\vec{\sigma}\,u_q(r_2)\}\Big\} + \frac{4\pi g_2^2}{\kappa^2}\{u_m(r_1)\vec{\sigma}\,v_n(r_1)\,v_p(\vec{r}_2)\vec{\sigma}\,u_q(r_2)\} +$$

$$= \tilde{u}_m(r_1)\vec{\sigma}_1 \qquad (\nabla_2\times\vec{\sigma}_2)\,u_q(r_2)\;\; v_n(r_1)$$

$$\vec{\sigma}^{(1)}\{\nabla_2\times[\nabla_2\times\tilde{v}_p\vec{\sigma}^{(2)}\ u_q(r_2)]$$

$$=-\vec{\sigma}^{(1)}\{\nabla_2\times\{\tilde{v}_p(\sigma^{(2)}\times\nabla_2)u_q(r_2)\pm\tilde{v}_p(\sigma^{(2)}\times\nabla_2)\cdot u_q(r_2)\}$$

$$=-(\vec{\sigma}^{(1)}\times\nabla_2)\{\tilde{v}_p(\sigma^{(2)}\times\nabla_2)u_q(r_2)\pm\cancel{\sigma^{(1)}\times\nabla}\tilde{v}_p(\sigma^{(2)}\times\nabla_2)\cdot u_q(r_2)\}$$

$$=-\{\vec{\sigma}^{(1)}\tilde{v}_p(r_2)\cdot(\vec{\sigma}^{(1)}\times\nabla_2)(\vec{\sigma}^{(2)}\times\nabla_2)u_q(r_2)$$

$$-\ \tilde{v}_p(r_2)(\vec{\sigma}^{(1)}\times\nabla_2)\cdot(\vec{\sigma}^{(2)}\times\nabla_2)u_q(r_2)$$

$$-\ \tilde{v}_p(r_2)(\vec{\sigma}^{(2)}\times\nabla_2)\cdot(\vec{\sigma}^{(1)}\times\nabla_2)u_q(r_2)$$

$$+\ \tilde{v}_p(r_2)(\sigma^{(2)}\times\nabla_2)(\sigma^{(1)}\times\nabla_2)\cdot u_q(r_2)$$

$$\frac{g_1^2}{\kappa^2}\tilde{u}_m(r_1)v_p(r_2)\Big\{\frac{e^{-\kappa r}}{r}(\vec{\sigma}^{(1)}\times\nabla_2)(\vec{\sigma}^{(2)}\times\nabla_2)$$

$$-(\sigma^{(1)}\times\nabla_2)\frac{e^{-\kappa r}}{r}(\sigma^{(2)}\times\nabla_2)-(\sigma^{(2)}\times\nabla_2)\frac{e^{-\kappa r}}{r}(\sigma^{(1)}\times\nabla_2)$$

$$+(\sigma^{(2)}\times\nabla_2)(\sigma^{(1)}\times\nabla_2)\frac{e^{-\kappa r}}{r}\Big\}v_n(r_1)u_q(r_2)$$

$$=\frac{g_1^2}{\kappa^2}\tilde{u}_n(r_1)\tilde{v}_p(r_2)\Big\{\cancel{(\vec{\sigma}^{(1)}\times\nabla_2)\frac{e^{-\kappa r}}{r}-\frac{e^{-\kappa r}}{r}\vec{\sigma}^{(1)}}$$

$$(\sigma^{(2)}\times\nabla_2)\Big((\sigma^{(1)}\times\nabla_2)\Big(\frac{e^{-\kappa r}}{r}\Big)-\Big(\frac{e^{-\kappa r}}{r}\Big)(\sigma^{(1)}\times\nabla_2)\Big)$$

$$-\Big((\sigma^{(1)}\times\nabla_2)\Big(\frac{e^{-\kappa r}}{r}\Big)-\frac{e^{-\kappa r}}{r}(\sigma^{(1)}\times\nabla_2)\Big)(\sigma^{(2)}\times\nabla_2)\Big\}v_n(r_1)u_q(r_2)$$

注

$$(\sigma^{(2)}\times\nabla_2)(\sigma^{(1)}\times\nabla_2)\frac{e^{-\kappa r}}{r}$$

$$\sigma^{(2)}\qquad (\vec{A}\times\vec{B})(\vec{C}\times\vec{D})=\{(A\times B)\times C\}\cdot D$$

$$=\cancel{\{(A\times B)\cdot C\cdot D\}C}$$

$$(\sigma^{(2)}\times\nabla_2)(\sigma^{(1)}\times\nabla_2)=\{(\sigma^{(2)}\times\nabla_2)\times\sigma^{(1)}\}\cdot\nabla_2=\cancel{\{(\sigma^{(2)}\times\nabla_2)\times\nabla_2\}\sigma^{(1)}}$$

$$=-(\sigma^{(1)}\nabla)(\sigma^{(2)}\nabla)+\sigma^{(1)}\sigma^{(2)}\Delta_2+\cancel{\sigma^{(2)}\sigma^{(1)}\Delta_2-(\sigma^{(1)}\nabla_2)(\sigma^{(2)}\nabla_2)}$$

$$=$$

$$H' = \frac{4\pi g_1 c}{\varkappa}\left(\mathrm{div}\,\tilde{U}^{+}\cdot\tilde{Q} + \mathrm{div}\,U\cdot Q\right) + \frac{g_2}{\varkappa^2}\left(\mathrm{curl}\,\tilde{U}\cdot\vec{\sigma}\tilde{Q} + \mathrm{curl}\,U\cdot\vec{\sigma}Q\right) + \frac{4\pi g_1^2}{\varkappa^2}\left(\tilde{M}_0\tilde{Q} + M_0 Q\right) + \frac{4\pi g_2^2}{\varkappa^2}\left\{(\tilde{S}\sigma)\tilde{Q} + (S\sigma)Q\right\} \quad (54),\ \mathrm{II}.$$

$$\left.\begin{aligned} -\Delta U + \varkappa^2 U &= -4\pi g_2\,\mathrm{curl}\,S \\ -\Delta\tilde{U}^{+} + \varkappa^2\tilde{U}^{+} &= \frac{g_1}{\varkappa c}\,\mathrm{grad}\,M_0 \\ M_0 = \tilde{\Psi}\tilde{Q}\Psi \quad &, \quad \vec{S} = \tilde{\Psi}\vec{\sigma}\tilde{Q}\Psi \end{aligned}\right\} \quad (55),\ \mathrm{II}.$$

$\cancel{\Psi =}$

Interaction Energy $\quad \Psi = \sum a_i u_i$

$$\bar{H}' = \frac{4\pi g_1 c}{\varkappa^2}\iiint\tilde{\Psi}\left(\mathrm{div}\,U^{+}\cdot\tilde{Q} + \mathrm{div}\,\tilde{U}^{+}\cdot Q\right)\Psi\,dv + \frac{4\pi g_1^2}{\varkappa^2}\iiint\tilde{\Psi}\left(\tilde{M}_0\tilde{Q} + M_0 Q\right)\Psi\,dv$$
$$+ \frac{g_2}{\varkappa^2}\iiint\tilde{\Psi}\left(\mathrm{curl}\,\tilde{U}\cdot\vec{\sigma}\tilde{Q} + \mathrm{curl}\,U\cdot\vec{\sigma}Q\right)\Psi\,dv + \frac{4\pi g_2^2}{\varkappa^2}\iiint\tilde{\Psi}\left\{(\tilde{S}\sigma)\tilde{Q} + (S\sigma)Q\right\}\Psi\,dv$$

$$= \sum a_i^{+} a_i$$

$$\boxed{\frac{\partial}{\partial z}\frac{e^{-\varkappa r}}{r} = \frac{z}{r}\frac{\partial}{\partial r}\left(\frac{e^{-\varkappa r}}{r}\right)}$$

$$(\sigma^{1}\times D_1)(\sigma^{2}\times D_2) = \begin{pmatrix} \frac{\partial}{\partial z} & \frac{\partial}{\partial x}-i\frac{\partial}{\partial y} \\ \frac{\partial}{\partial x}+i\frac{\partial}{\partial y} & -\frac{\partial}{\partial z} \end{pmatrix}_1 \begin{pmatrix} \frac{\partial}{\partial z} & \frac{\partial}{\partial x}-i\frac{\partial}{\partial y} \\ \frac{\partial}{\partial x}+i\frac{\partial}{\partial y} & -\frac{\partial}{\partial z} \end{pmatrix}_2$$

$$(\quad \chi \quad)\frac{e^{-\varkappa r}}{r} =$$

$$(\sigma^{1}\times D_2)(\sigma^{2}\times D_2) = \left(\sigma^{(1)}_y\frac{\partial}{\partial z} - \sigma^{(1)}_z\frac{\partial}{\partial y}\right)\left(\sigma^{(2)}_y\frac{\partial}{\partial z} - \sigma^{(2)}_z\frac{\partial}{\partial y}\right)$$
$$+\left(\sigma^{(1)}_z\frac{\partial}{\partial x} - \sigma^{(1)}_x\frac{\partial}{\partial z}\right)\left(\quad\right)$$
$$= \left(\quad\right) + \left(\sigma^{(1)}_x\frac{\partial}{\partial y} - \sigma^{(1)}_y\frac{\partial}{\partial x}\right)\left(\quad\right)$$

$$= \begin{pmatrix} \frac{\partial}{\partial y} & -i\frac{\partial}{\partial z} \\ i\frac{\partial}{\partial z} & -\frac{\partial}{\partial y} \end{pmatrix}_1 \begin{pmatrix} & \\ & \end{pmatrix}_2$$
$$+ \begin{pmatrix} \frac{\partial}{\partial x} & -\frac{\partial}{\partial z} \\ -\frac{\partial}{\partial z} & -\frac{\partial}{\partial x} \end{pmatrix}_1 \begin{pmatrix} & \\ & \end{pmatrix}_2$$
$$+ \begin{pmatrix} 0 & \frac{\partial}{\partial y}+i\frac{\partial}{\partial x} \\ \frac{\partial}{\partial y}-i\frac{\partial}{\partial x} & 0 \end{pmatrix} \begin{pmatrix} & \\ & \end{pmatrix}$$
$$=$$

$= (\sigma_x^{(1)}\sigma^{(2)})\Delta$

$\cancel{-(\sigma_y^{(1)}\sigma_z^{(2)} + \sigma_z^{(1)}\sigma_y^{(2)})\frac{\partial}{\partial y}\frac{\partial}{\partial z}}$

$- (\sigma_y^{(1)}\sigma_z^{(2)})\frac{\partial}{\partial y}\frac{\partial}{\partial z} - \cdots$

$= (\sigma^{(1)}\sigma^{(2)})\Delta - (\sigma_x^{(1)}\frac{\partial}{\partial x} + \cdots)(\quad)$

$(\sigma^{(1)}\mathrm{grad})(\sigma^{(2)}\mathrm{grad})\cdot\frac{e^{-\varkappa r}}{r}$

$= (\sigma^{(1)}\mathrm{grad})\{\sigma_x^{(2)}\frac{x}{r}\cdot\frac{e^{-\varkappa r}}{r}(-\varkappa - \frac{1}{r}) + \sigma_y^{(2)}\frac{y}{r}\cdot\frac{e^{-\varkappa r}}{r}(-\varkappa - \frac{1}{r}) + \cdots\}$

$=$

$\iiint \frac{e^{-\varkappa r}}{r}\cdot(\sigma^{(1)}\mathrm{grad}_2)(\sigma^{(2)}\mathrm{grad}_2) f(\vec{r}_2)\, dv$

$=$

wave equation (54) 中 Ψ に関する Hamiltonian

中に(53)を代入すれば

$\mathrm{curl}\, U = -g_2 \iiint \mathrm{curl}\,\mathrm{curl}\, S\cdot\frac{e^{-\varkappa r}}{r}\, dv$

$\mathrm{div}\,\tilde{U}^{+} = \frac{g_1}{4\pi\varkappa c}\iiint\{\Delta M_0\cdot\frac{e^{-\varkappa r}}{r}\}dv$

を入れれば

$c\vec{\alpha}\vec{p} + \rho\{\cancel{\tfrac{1+\tau_3}{2}} M_N c^2 M c^2 +$

$- \frac{g_1^2}{\varkappa^2}$

$\vec{\sigma}''' \vec{r}$

$$\iiint \frac{e^{-\kappa r}}{r^2} \operatorname{div} \vec{f}(\vec{r}_2)\, dv$$

$$= \iiint \operatorname{div} \left\{ \frac{e^{-\kappa r}}{r^2} \vec{f}(\vec{r}_2) \right\} dv$$

$$- \iiint \operatorname{grad} \left(\frac{e^{-\kappa r}}{r^2} \right) \cdot \vec{f}(\vec{r}_2)\, dv$$

$$= \iiint \operatorname{div} \{ \qquad \}$$

$$\iiint \frac{e^{-\kappa r}}{r}(\sigma^{(1)}\mathrm{grad}_2)(\sigma\, \cancel{\mathrm{grad}_2 f(\vec r_2)}\, dv_2$$

$$(\mathrm{grad}_2\ \mathrm{div}_2\ \vec f(\vec r_2)\, dv_2$$

$$= \iiint \mathrm{div}\left\{\sigma^{(1)}\frac{e^{-\kappa r}}{r}\right\} \mathrm{div}_2 \vec f(\vec r_2)\, dv,$$

$$= \iiint \left(\frac{\sigma^{(1)}\vec r}{r}\left(-\kappa - \frac{1}{r}\right)\frac{e^{-\kappa r}}{r}\right) \mathrm{div}_2 \vec f(\vec r_2)\, dv$$

$$= \iiint -(\sigma^{(1)}\vec r)\left(\frac{\kappa}{r^2} + \frac{1}{r^3}\right)e^{-\kappa r}\ \mathrm{div}_2 \vec f(\vec r_2)\, dv.$$

$$= -\iint (\sigma^{(1)}\vec r)\left(\frac{\kappa}{r^2}+\frac{1}{r^3}\right)e^{-\kappa r}\ \vec f \cdot d\vec S$$

$$+ \iiint (\vec f(\vec r_2)\,\mathrm{grad})(\sigma^{(1)}\vec r)\left(\frac{\kappa}{r^2}+\frac{1}{r^3}\right)e^{-\kappa r}\, dv$$

$\vec f \to$

$$= \iint \sigma^{(1)}_r(\kappa r+1)e^{-\kappa r} f_r(\vec r)\sin\theta\, d\theta\, d\varphi + \iiint$$

$$= \underbrace{-4\pi\sigma^{(1)}_r f_r(\vec r_1)}. + \iiint \cdots$$

$\sigma^{(1)}_x f_x \cos^2\theta$

$$\| $$

$$-\frac{4\pi}{3}(\sigma^{(1)}\vec f(\vec r_1))$$

$(\sigma^{(1)}\mathrm{grad})$

$M_0 = \tilde{\Psi}\beta\frac{1}{c}\frac{\partial}{\partial t}\Psi$

$\vec{M} = \tilde{\Psi}\beta\,\mathrm{grad}\,\Psi$

curl

$\vec{M} = \tilde{\Psi}\beta(\frac{\partial}{c\partial t} + \frac{\partial}{\partial x})\Psi$

$\vec{S} = \tilde{\Psi}\beta(\vec{\alpha}\times\mathrm{grad})\Psi$

Deuteron の理論(續)

$$\left\{\frac{\hbar^2}{M}\Delta + E - V\right\}\psi = 0$$

$$V = \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}\left\{g_1^2 + g_2^2(\sigma^{(1)}\sigma^{(2)}) - g_2^2\frac{(\sigma^{(1)}\mathrm{grad})(\sigma^{(2)}\mathrm{grad})}{\varkappa^2}\right\} \times \frac{e^{-\varkappa r}}{r}$$

a) $\left\{\begin{array}{l}\text{symmetric state}\\ \text{anti-symmetric state}\end{array}\right\}$ $\dfrac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}}{2}$ $\left\{\begin{array}{l}= -1 \quad ({}^3S, {}^1P, {}^3D, \ldots)\\ = 1 \quad ({}^1S, {}^3P, {}^1D, \ldots)\end{array}\right.$

b) $\left\{\begin{array}{l}\text{Triplet state}\\ \text{Singlet state}\end{array}\right\}$ $\sigma^{(1)}\sigma^{(2)}$ $\left\{\begin{array}{l}1\\ -3\end{array}\right.$

$$[(\sigma^{(1)}\sigma^{(2)}), (\sigma^{(1)}\vec{\mathrm{grad}})] = [(\sigma_x^{(1)}\sigma_x^{(2)} + \sigma_y^{(1)}\sigma_y^{(2)} + \sigma_z^{(1)}\sigma_z^{(2)}), (\sigma_x^{(1)}\frac{\partial}{\partial x} + \sigma_y^{(1)}\frac{\partial}{\partial y} + \sigma_z^{(1)}\frac{\partial}{\partial z})]$$

$$= 2i\left(\sigma_z^{(1)}\sigma_x^{(2)}\frac{\partial}{\partial y} - \sigma_z^{(1)}\sigma_y^{(2)}\frac{\partial}{\partial x} + \sigma_y^{(1)}\sigma_z^{(2)}\frac{\partial}{\partial x} - \sigma_y^{(1)}\sigma_x^{(2)}\frac{\partial}{\partial z} \cdots\right)$$

$$= 2i[\sigma_y$$

$$[(\sigma^{(1)}\sigma^{(2)}), (\sigma^{(1)}\mathrm{grad})(\sigma^{(2)}\mathrm{grad})] = [(\sigma^{(1)}\sigma^{(2)}), \sigma^{(1)}\mathrm{grad}](\sigma^{(2)}\mathrm{grad})$$
$$+ (\sigma^{(1)}\mathrm{grad})[(\sigma^{(1)}\sigma^{(2)}), (\sigma^{(2)}\mathrm{grad})]$$

$$= 2i[(\sigma_z^{(1)}\sigma_x^{(2)}\frac{\partial}{\partial y} \qquad)(\sigma_x^{(2)}\frac{\partial}{\partial x} + \sigma_y^{(2)}\frac{\partial}{\partial y} + \sigma_z^{(2)}\frac{\partial}{\partial z})]$$

$$+ 2i[(\sigma_x^{(1)}\frac{\partial}{\partial x} + \sigma_y^{(1)}\frac{\partial}{\partial y} + \sigma_z^{(1)}\frac{\partial}{\partial z})(\qquad - \sigma_x^{(1)}\sigma_y^{(2)}\frac{\partial}{\partial z})]$$

$$= 4i\left(\sigma_z^{(1)}\sigma_y^{(2)}\frac{\partial}{\partial y}\frac{\partial}{\partial z} - \cdots\right)$$

$$+ 4\left(-\sigma_z^{(1)}\sigma_y^{(2)}\frac{\partial}{\partial y}\frac{\partial}{\partial z} + \cdots\right) = 0,$$

故に、Triplet - Singlet の Intercomb. はない。

同様、Symm. - Anti. の Intercomb. もない。

Total Angular Momentum $(\vec{r}\times\vec{p}) + \frac{\hbar}{2}(\sigma^{(1)} + \sigma^{(2)})$ は V と commutative。

故に 3S と Combine するのは 3D 丈。

$^3S - {}^3D_1$ - Combination

$$V = -\left\{ g_1^2 + \frac{2}{3} g_2^2 + g_2^2 \left(\frac{1}{3} + \frac{1}{\kappa r} + \frac{1}{\kappa^2 r^2} \right) \left(1 - \frac{3(\sigma^{(1)} r)(\sigma^{(2)} r)}{r^2} \right) \right\} \frac{e^{-\kappa r}}{r}$$

$$\frac{\hbar^2}{M} \frac{d^2 u'}{dr^2} + E u' + \left(g_1^2 + \frac{2}{3} g_2^2\right) \frac{e^{-\kappa r}}{r} u' - 2\sqrt{2} g_2^2 \left(\frac{1}{3} + \frac{1}{\kappa r} + \frac{1}{\kappa^2 r^2} \right) \frac{e^{-\kappa r}}{r} v = 0$$

$$\frac{\hbar^2}{M} \left(\frac{d^2 v}{dr^2} - \frac{6v}{r^2} \right) + E v + \left(g_1^2 + \frac{2}{3} g_2^2\right) \frac{e^{-\kappa r}}{r} v + 2 g_2^2 \left(\frac{1}{3} + \frac{1}{\kappa r} + \frac{1}{\kappa^2 r^2} \right) \frac{e^{-\kappa r}}{r} v - 2\sqrt{2} g_2^2 \left(\frac{1}{3} + \frac{1}{\kappa r} + \frac{1}{\kappa^2 r^2} \right) \frac{e^{-\kappa r}}{r} u' = 0$$

$H^{(1)}_{\frac{5}{2}}(i\alpha r)$

$r \gg \frac{1}{\kappa}$. $\quad \frac{\hbar^2}{M} \frac{d^2 u'}{dr^2} + E u' = 0 \qquad \frac{d^2 u'}{dr^2} - \alpha^2 u' = 0$

$\frac{\hbar^2}{M} \left(\frac{d^2 v}{dr^2} - \frac{6v}{r^2} \right) + E v = 0 \qquad \frac{d^2 v}{dr^2} - \frac{6v}{r^2} - \alpha^2 v = 0$

$u' = e^{-\alpha r} \qquad \alpha = \frac{\sqrt{-ME}}{\hbar}, \qquad \frac{d^2 v}{d(\alpha r)^2} - \cancel{6} \left(1 + \frac{6}{(\alpha r)^2}\right) v = 0$

$\frac{4n^2 - 1}{4} = 6. \quad 4n^2 = 25 \quad n = \frac{5}{2}$

$v = \sqrt{\alpha r}\, H^{(1)}_{\frac{5}{2}}(i\alpha r)$

$v = v' e^{-\alpha r}: \quad \frac{dv}{dr} = \frac{dv'}{dr} e^{-\alpha r} - \alpha v' e^{-\alpha r}$

$\frac{d^2 v}{dr^2} = \frac{d^2 v'}{dr^2} e^{-\alpha r} - 2\alpha \frac{dv'}{dr} e^{-\alpha r} + \alpha^2 v' e^{-\alpha r}$

$\frac{d^2 v'}{dr^2} - 2\alpha \frac{dv'}{dr} - \frac{6v'}{r^2} = 0$

$v' = \sum_n c_n r^n \quad \sum_n \{n(n-1) - 6\} c_n r^{n-2} = 2\alpha \sum_n n c_n r^{n-1}$

$\sum_n \left[\{n(n-1) - 6\} c_n - 2\alpha (n-1) c_{n-1} \right] r^{n-2} = 0$

$n = 0.$

$n = 3, -2; \qquad v' = \Sigma$

i) $n = -2$:

-1 : $\quad -4c_{-1} + \cancel{4}\,6\alpha c_{-2} = 0 \qquad c_{-1} = \alpha c_{-2}$

0 : $\quad -6c_0 + 2\alpha c_{-1} = 0 \qquad c_0 = \frac{\alpha}{3} c_{-1}$

1 : $\quad c_1 = 0$

$$v' = c_{-2}\left\{ \frac{1}{r^2} + \frac{\alpha}{r} + \frac{\alpha^2}{3} \right\}$$

ii) $n = 3$: $\quad c_2 = 0$

$n = 3, \cdots$ $\quad C_n = \dfrac{2\alpha(n-1)}{n(n-1)-6} c_{n-1}$

$n = m+3 \qquad c_m = \dfrac{2\alpha(m+2)}{m(m+5)} c_{m-1} \qquad m = 0, 1, 2, \cdots$

$(m+2)(m+3)$

$m^2 + 5m \qquad = \dfrac{(2\alpha)^m (m+2)!\; 5!}{m!\,(m+5)!\; 2!} c_0 \qquad \sim e^{2\alpha r}$

$$\frac{d^2 v'}{d(\alpha r)^2} - 2\frac{dv'}{d(\alpha r)} - \frac{6v'}{(\alpha r)^2} = 0$$

Bessel.

$r < \frac{1}{\alpha}$: $\quad \dfrac{\hbar^2}{M}\dfrac{d^2 u'}{dr^2} + (E + V_0)u'$

$\quad - \sqrt{2} V_1 v = 0$

$V_1 > V_0$!! $\quad \dfrac{\hbar^2}{M}\left(\dfrac{d^2 v}{dr^2} - \dfrac{6v}{r^2}\right) + (E + V_0)v$

$\quad + 2V_1 v - \sqrt{2} V_1 u' = 0$

$u' = \sum_n u_n r^n$

$v = \sum_n v_n r^n$

$\frac{\hbar^2}{M}\sum_n u_{n+2} r^{n+2} + \sum_n \frac{M(E+V_0)}{\hbar^2} u_n r^n$

$\quad - \sqrt{2} V_1 \sum_n v_n r^n = 0$

$r \gg \frac{1}{\kappa}$

$$\frac{\hbar^2}{M}\frac{d^2u'}{dr^2}+Eu'=0 \qquad \frac{d^2u'}{dr^2}-\alpha^2u'=0$$

$$\frac{\hbar^2}{M}\left(\frac{d^2v}{dr^2}-\frac{6v}{r^2}\right)+Ev=0 \qquad \frac{d^2v}{dr^2}-\frac{6v}{r^2}-\alpha^2v=0$$