Neutron-Proton Problem ①

$$\Psi = \frac{u(r)}{r}\frac{\chi_{-1}}{\sqrt{4\pi}} + \frac{v(r)}{r}\Psi_{-1}$$

$$\frac{\hbar^2}{M}\frac{d^2u}{dr^2} + \left\{E + \left(g_1^2 + \frac{2}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r}\right\}u$$

$\frac{2}{3}(4-3$

$$-\frac{2\sqrt{2}}{3}g_2^2\left(1 + \frac{3}{\varkappa r} + \frac{3}{\varkappa^2 r^2}\right)\frac{e^{-\varkappa r}}{r}v = 0$$

$$\frac{\hbar^2}{M}\frac{d^2v}{dr^2} - \frac{6}{r^2}v + \left\{E + \left[g_1^2 + \frac{2}{3}g_2^2 - \frac{2}{3}g_2^2(\ldots)\right]\frac{e^{-\varkappa r}}{r}\right\}v$$

$$-\frac{2\sqrt{2}}{3}g_2^2\left(1 + \frac{3}{\varkappa r} + \frac{3}{\varkappa^2 r^2}\right)\frac{e^{-\varkappa r}}{r}u = 0$$

$$\left(g_1^2 + \frac{2}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r} = U$$

$$\frac{2}{3}g_2^2\left(1 + \frac{3}{\varkappa r} + \frac{3}{\varkappa^2 r^2}\right)\frac{e^{-\varkappa r}}{r} = V.$$

$$\left.\begin{aligned}&\frac{\hbar^2}{M}\frac{d^2u}{dr^2} + \{E + U\}u - \sqrt{2}Vv = 0\\ &\frac{\hbar^2}{M}\left(\frac{d^2v}{dr^2} - \frac{6}{r^2}v\right) + \{E + U - V\}v - \sqrt{2}Vu = 0\end{aligned}\right\}$$

$E<0$

$$\sqrt{\frac{-ME}{\hbar^2}} = \varepsilon \qquad \sqrt{\frac{MU}{\hbar^2}} = \alpha \qquad \sqrt{\frac{MV}{\hbar^2}} = \beta,$$

$$\varkappa = \frac{m_U c}{\hbar} = \sqrt{\frac{M_U m_U^2 c^2}{\hbar^2 M_U}}$$

$$\left.\begin{aligned}&\frac{d^2u}{dr^2} + (\alpha - \varepsilon)u - \sqrt{2}\beta^2 v = 0\\ &\frac{d^2v}{dr^2} - \frac{6}{r^2}v + (\alpha^2 - \varepsilon^2 - \beta^2)v - \sqrt{2}\beta u = 0\end{aligned}\right\}$$

$r > r_0$:
$$\frac{d^2u}{dr^2} + \varepsilon u = 0 \qquad \underline{u = C_1 e^{-\sqrt{\varepsilon}\cdot r}}$$

$$\frac{d^2v}{dr^2} - \frac{6}{r^2}v - \varepsilon v = 0.$$

$$v = v' e^{-\sqrt{\varepsilon}r} \qquad \frac{dv}{dr} = \frac{dv'}{dr}e^{-\sqrt{\varepsilon}r} - \sqrt{\varepsilon}v' e^{-\sqrt{\varepsilon}r}$$

$$\frac{d^2v}{dr^2} = \frac{d^2v'}{dr^2}e^{-\sqrt{\varepsilon}r} - 2\sqrt{\varepsilon}\frac{dv'}{dr}e^{-\sqrt{\varepsilon}r} + \varepsilon e^{-\sqrt{\varepsilon}r} =$$

$$\frac{d^2v}{dr^2} - 2\sqrt{\varepsilon}\frac{dv}{dr} - \frac{6}{r^2}v = 0$$

$$v = \sum_{n=n_0, n_0+1, \dots} c_n r^n$$

$$\sum_n \{n(n-1) - 6\} c_n r^{n-2} = 2\sqrt{\varepsilon}\sum_n (n-1) c_{n-1} r^{n-2}$$

$$c_n = \frac{2\sqrt{\varepsilon}(n-1)c_{n-1}}{n(n-1)-6}$$

$n_0(n_0-1) = 6$. $\quad n_0 = 3, -2$

$n_0 = -2$:

$$c_{-3} = c_{-4} = \cdots = 0$$

$$c_{-1} = \frac{2\sqrt{\varepsilon}\cdot 2\, c_{-2}}{4} = \sqrt{\varepsilon}\, c_{-2}$$

$$c_0 = \frac{2\sqrt{\varepsilon}\, c_{-1}}{3} \qquad c_{-1} = \frac{3}{\sqrt{\varepsilon}}c_0$$

$$c_1 = 0 \qquad c_{-2} = \frac{3}{(\sqrt{\varepsilon})^2}c_0$$

$n_0 = 3$ のものでは $e^{2\sqrt{\varepsilon}r}$ の order で 増大する。

$$v = c_2\left(1 + \frac{3}{\sqrt{\varepsilon}\cdot r} + \frac{3}{(\sqrt{\varepsilon})^2 r^2}\right)e^{-\sqrt{\varepsilon}r}$$

$r < r_0$:

$u = \sum_{n=1}^{\infty} u_n r^n \qquad r^n: (n+2)(n+1)u_{n+2} + (\alpha-\varepsilon)u_n - \sqrt{2}\beta v_n = 0$

$v = \sum_{n=1}^{\infty} v_n r^n \qquad r^n: (n+2)(n+1)v_{n+2} - 6v_{n+2} + (\alpha-\varepsilon-\beta)v_n - \sqrt{2}\beta u_n = 0$

$$n^2 + 3n - 4 = (n+4)(n-1)$$

$$(n+4)(n-1)v_{n+2} + (\alpha-\varepsilon-\beta)v_n - \sqrt{2}\beta u_n = 0$$

$$(n+2)(n+1)\{a u_{n+2} + b v_{n+2}\} + (\alpha-\varepsilon)\{a u_n + b v_n\} - \sqrt{2}\beta\{b u_n + a v_n\}$$

$$- 6v_{n+2} - \beta v_n = 0.$$

$$(n+2)(n+1)\{u_{n+2} \pm v_{n+2}\} + (\alpha - \varepsilon \mp \sqrt{2}\beta)\{u_n \pm v_n\}$$

$$-6v_{n+2} - \beta v_n = 0$$

$$u_{n+2} \pm v_{n+2} = -\frac{(\alpha - \varepsilon \mp \sqrt{2}\beta)}{(n+2)(n+1)}(u_n \pm v_n)$$

$$v_{n+2} = -\frac{\beta}{6}v_n$$

③

$\cancel{f = u \pm v}$

$\frac{d^2 f}{dr^2} + (\alpha - \varepsilon) f - \sqrt{2}\beta v$

$$u_{n+2} = \frac{-(\alpha-\varepsilon)u_n}{(n+2)(n+1)} + \frac{\sqrt{2}\beta v_n}{(n+2)(n+1)}$$

$$v_{n+2} = \frac{-(\alpha-\varepsilon+\beta)v_n}{(n+4)(n-1)} + \frac{\sqrt{2}\beta u_n}{(n+4)(n-1)}$$

$n=1$: $(\alpha-\varepsilon-\beta)v_1 = 2\sqrt{2}\beta u_1$　　v_3: arbitrary

$$u_3 = \frac{-(\alpha-\varepsilon)u_1 + \sqrt{2}\beta v_1}{3 \cdot 2} =$$

$u = \sum_{n=0}^{\infty} u_{2n+1} r^{2n+1}$, $v = \sum_{n=0}^{\infty} v_{2n+1} r^{2n+1}$

$n=2$: $u_4 = \frac{-(\alpha-\varepsilon)u_2 + \sqrt{2}\beta v_2}{3 \cdot 2}$

$v_4 = \frac{-(\alpha-\varepsilon+\beta)v_2 + \sqrt{2}\beta u_2}{3 \cdot 6}$

etc.

$u = \sum_{n=1}^{\infty} u_{2n} r^{2n}$, $v = \sum_{n=1}^{\infty} v_{2n} r^{2n}$

$r = r_0$

外側: $u = c_1 e^{-\sqrt{\varepsilon} r_0}$

$$v = c_2\left(1 + \frac{3}{\sqrt{\varepsilon} r_0} + \frac{3}{(\sqrt{\varepsilon})^2 r_0^2}\right) e^{-\sqrt{\varepsilon} r_0}$$

内側: $u = \sum_{n=0}^{\infty} u_{2n+1} r_0^{2n+1}$, $v = \sum v_{2n+1} r_0^{2n+1}$　　$u = \sum_{n=1}^{\infty} u_{2n} r_0^{2n}$, $v = \sum_{n=1} v_{2n} r_0^{2n}$

$\left.\begin{matrix}\frac{u}{r} \\ \frac{v}{r}\end{matrix}\right\}$ even　　$\left.\begin{matrix}\frac{u}{r} \\ \frac{v}{r}\end{matrix}\right\}$ odd

$$r < \frac{1}{\kappa} \begin{cases} \dfrac{d^2u}{dr^2} + (\alpha - \text{[illegible]} \\ \dfrac{d^2v}{dr^2} - 6\kappa^2 v + (\alpha - \varepsilon - \beta) v - \sqrt{2}\beta u = 0 \end{cases}$$

$$\varepsilon = \frac{ME}{\hbar^2}, \quad \alpha = \frac{MU}{\hbar^2} \qquad \beta = \frac{MV}{\hbar^2}$$

$$U = \left(g_1^2 + \frac{2}{3} g_2^2\right) \kappa.$$

$$V = \frac{2}{3} g_2^2 \times 7\kappa.$$

$$r > \frac{1}{\kappa} \begin{cases} \dfrac{d^2u}{dr^2} - \varepsilon u = 0 \\ \dfrac{d^2v}{dr^2} - \dfrac{6}{r^2} v = 0 \end{cases} \qquad \begin{cases} u = C_1 e^{-\sqrt{\varepsilon} r} \\ v = C_2 \left(1 + \dfrac{3}{\sqrt{\varepsilon} r} + \dfrac{3}{(\sqrt{\varepsilon} r)^2}\right) e^{-\sqrt{\varepsilon} r} \end{cases}$$

$$A \quad \frac{d^2u}{dr^2} + au + bv = 0 \qquad a = \alpha - \varepsilon \qquad b = -\sqrt{2}\beta$$

$$B \quad \frac{d^2v}{dr^2} + cu + dv = 0 \qquad c = -\sqrt{2}\beta \qquad d = \alpha - \varepsilon - \beta - 6\kappa^2$$

$$\frac{d^2}{dr^2}(Au + Bv) + (Aa + Bc)u + (Ab + Bd)v = 0$$

$$\frac{A}{B} = \frac{Aa + Bc}{Ab + Bd} = \frac{\frac{A}{B} a + c}{\frac{A}{B} b + d}$$

$$b\left(\frac{A}{B}\right)^2 - \left(\frac{A}{B}\right)(a - d) - c = 0$$

$$\frac{A}{B} = \frac{(a-d) \pm \sqrt{(a-d)^2 + 4bc}}{2b}$$

$$= \frac{(\beta + 6\kappa^2) \pm \sqrt{(\beta + 6\kappa^2)^2 + 8\beta^2}}{-2\sqrt{2}\beta}$$

$$\frac{A}{B} b + d = \frac{(a+d) \pm \sqrt{(a-d)^2 + 4bc}}{2}$$

$$= \frac{2(\alpha - \varepsilon) - \beta - 6\kappa^2 \pm \sqrt{(\beta + 6\kappa^2)^2 + 8\beta^2}}{2}$$

±

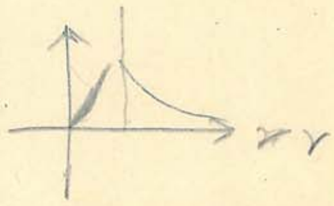
$$\varphi_\pm = [(\beta+6\varkappa^2) \pm \sqrt{(\beta+6\varkappa^2)^2+8\beta^2}]u - 2\sqrt{2}\beta v$$

$$\frac{d^2\varphi_\pm}{dr^2} + \frac{2(\alpha-\varepsilon)-\beta-6\varkappa^2\pm\sqrt{\quad ''}}{2}\varphi_\pm = 0$$

$$(\beta+6\varkappa^2)^2+8\beta^2 \gtrless \{2(\alpha-\varepsilon)-\beta-6\varkappa^2\}^2$$

$$8\beta^2 \gtrless 4(\alpha-\varepsilon)^2 - 4(\alpha-\varepsilon)(\beta+6\varkappa^2)$$

$e^{\cdots} - e$



$$(\alpha-\varepsilon)^2 - (\alpha-\varepsilon)(\beta+6\varkappa^2) - 2\beta^2 > 0$$

$$\frac{d}{dr}(e^{\gamma r} - e^{-\gamma x}) = \gamma(e^{\gamma r}+e^{-\gamma x})$$

$$\varepsilon = \frac{-ME}{\hbar^2}$$

$$\alpha = \frac{MU}{\hbar^2} \cong \frac{Mg^2}{\hbar^2}\left(\frac{5}{3}\right)\varkappa$$

$$\beta = \frac{MV}{\hbar^2} \cong \frac{Mg^2}{\hbar^2}\left(\frac{2\times7}{3}\right)\varkappa$$

$$\frac{\hbar^2}{M^2c^2}\cdot\frac{\hbar c^2}{g^2}\cdot\varkappa$$

$\varepsilon \cong 0$ とおいて,

$$\left(\frac{5}{3}\right)^2 - \frac{5}{3}\left(\frac{14}{3} + \frac{6\hbar^2\varkappa}{Mg^2}\right) - 2\left(\frac{14}{3}\right)^2 < 0$$

$$\frac{\hbar c}{g^2}\cdot\frac{\hbar}{Mc}\varkappa$$

$$\therefore\ 8\beta^2 > 4(\alpha$$