On the Interaction of Elementary Particles. IV.

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§1. Introduction and Summary.

In the previous papers[1] it was shown that the introduction of the neutral Bose particles, which were named N-particles by Bhabha[2], in addition to the charged Bose particles, i.e. U-particles, was necessary in order to account for approximate equality of unlike particle and like particle forces.[2]† In this paper, we want to deal with this problem by assuming the wave equations for the N-particles, which have the same ~~form~~ as those for the U-particles. ~~The~~ We can assume, further, that the interaction ~~between the form~~ of the heavy particle with the former ~~has~~ is the same ~~form~~ as that with the latter, except that the factor ~~$a+b\tau_3$~~ of the form $a+b\tau_3$ appears in place of $\tau_1+i\tau_2$ or $\tau_1-i\tau_2$, where a, b or $\tilde{a}+\tilde{b}\tau_3$ are arbitrary complex numbers. (§2) Thus, the forces between like particles as well as the ordinary forces between unlike particles ~~are~~ can be obtained in ~~a manner~~ the ~~similar to~~ usual way. ~~(§3) The arbitrary constants a, b can be determined by comparing the forces thus obtained with those employed in~~ By choosing the constants a, b suitably, ~~we arrive at the forces thus obtained~~ they can take exactly the same form as ~~obtain~~ ~~required form~~ the forces assumed in the current theory., ~~Very recently, Kemmer considered this problem in detail~~ as ~~investiga~~ shown by Kemmer recently.[3] (§2)

As already ~~shown in~~ mentioned in detail in III, §7, the forces between them obtained are not strictly ~~a~~ central, so that ~~we have to consider~~

1) Yukawa, Proc. Phys.-Math. Soc. 17, 48, 1935; Yukawa and Sakata, ibid. 19, 1084, 1937; Yukawa, Sakata and Taketani, ibid. 20, 319, 1938. They will be referred to as I, II and III.

3) ~~Kemmer~~ The authors wish to express their hearty thanks to Dr. Kemmer for sending the manuscript of his paper to be published in Proc. Camb. Phil. Soc.

~~In this connection, It should be noticed, however,~~ that, there are ~~many reasons~~ moreover, ~~it~~ is certainly true

It

~~The fo theory At smal~~ ; ~~There~~ are many reasons

It is cer In this connection, the validity of the interaction

The ~~intera~~ potential, ~~which~~ which ~~is obtained by second~~ as second order
interaction between the neutron and the proton

effect, is not valid for ~~small~~ distance small compared with $\frac{1}{\kappa}$,

~~is~~ that, ~~the complete de this problem, can be solved~~.

~~the~~ detailed calculations ~~may not a subject~~ of

~~deep~~ significance

the result of ~~little~~ in

can ~~have~~ be performed with ~~out~~ deep

may not have a not significance.

can not be ~~thought~~ as serious

~~may not~~ be taken too seriously.

can not be well ~~be have little~~

at or distances small

~~it~~ differs essentially from that obtained ~~in the U-field theory~~

as second order effect in our theory

~~the coupling between~~ the stationary states and ~~and~~ the ~~energy body levels~~ energy level scheme of ~~to of~~ the ~~syst~~ nuclear system become rather complicated. Already, ~~the solution of~~ the deuteron problem can not easily be solved owing to the coupling of, for example, of ${}^3_1S_1$ state with ~~a~~ ${}^3D_1$. (§4) ~~Moreover the st.~~ singula on the one hand and, ~~to the strong~~ term singularity ~~of the force of the~~ proportional to $\frac{1}{r^3}$ ~~small~~ the appearance of $\frac{1}{r^3}$ in the expression of at small distance on the other, $\frac{1}{r^2}$ and $\frac{1}{r}$ in the potential. (§3)

~~The theory of the β-disintegration~~ ~~the phenomenon~~ ~~process~~

~~As~~ As already proposed in I, §4, the β-disintegration can be explained ~~by~~ the help of the U-particle, is the second order process, by which heavy quantum is emitted virtually from the heavy particle and absorbed by a light particle. In this paper, ~~it is shown that~~ that ~~our the~~ ~~the~~ ~~we can arrived at the~~ consider the modification of the theory leads to the ~~result~~ scheme, which is essentially the same as, ~~that~~ of Konopinski ~~and~~ Uhlenbeck[4] ~~instead of~~ a combination of ~~Fermi~~ those of Fermi[3] and ~~But~~ In this case, ~~the assumption for~~ the interaction between the light particle and the U-particle in this case takes rather complicated form, ~~so that it is desired~~ We ~~hope that they~~ ~~more elegant simple~~ more satisfactory solution of this problem ~~will be obtained~~ It is a pity, however, that we is not sure whether this is the final solution of the problem. (§5)

Next, the ~~creation~~ problem of the creation of ~~the~~ U-particle ~~is important in conne. of~~ by the following process is discussed in detail. A γ-ray of energy larger than $\mu_U c^2$ ~~colliding with~~ is absorbed by ~~a~~ neutron (or a proton) in the nucleus ~~can change~~ and ~~emitting at the same time~~ ~~into~~ a U-particle with the negative charge (or with the positive charge) ~~by change and~~ is emitted at the same time, ~~leaving changing~~ a neutron (or a proton) in the nucleus being transformed into a proton (or a neutron). The cross section of this process is ~~a~~ far larger than that ~~due to the~~ of the ~~pair~~ creation of pair of U-particles by γ-rays, so that it is possible that the hard component of the

3) Fermi

cosmic ray observed on the [illegible] consists mainly of the U-particles produced in the atmosphere ~~for~~ by the ~~soft primary~~ above process from the soft primary. The absorption of the ~~U~~ U-particle by the inverse process, ~~i.e. by emitting~~ probability of is also discussed. (§6, §7)

~~Finally, the formal pro~~ In addendix, the equations for the U-field is written in spinor form in order to make clear that they are ~~the~~s a special case ~~of~~ of ~~the~~ generalised wave equations ~~of~~ ~~Dirac~~ Dirac's

§2. Linear Equations for the N-Field.

We introduce the six vector $\vec{\mathcal{F}}$, $\vec{\mathcal{G}}$ and the four vector $\vec{\mathcal{U}}$, $\mathcal{U}_0$ for the N-field corresponding to $\vec{F}$, $\vec{G}$ and $\vec{U}$, U_0 for the U-field. The equations for the N-field in the presence of the heavy particle We assume to have the form

$$\frac{1}{c}\frac{\partial \vec{\mathcal{F}}}{\partial t} - \mathrm{curl}\, \vec{\mathcal{G}} - \kappa \vec{\mathcal{U}} = -4\pi g_1 \vec{\mathcal{M}} \qquad \mathrm{div}\, \vec{\mathcal{F}} + \kappa \mathcal{U}_0 = 4\pi g_1 \mathcal{M}_0 \quad (1)$$

$$\frac{1}{c}\frac{\partial \vec{\mathcal{U}}}{\partial t} + \mathrm{grad}\, \mathcal{U}_0 + \kappa \vec{\mathcal{F}} = 4\pi g_2 \vec{\mathcal{I}} \qquad \mathrm{curl}\, \vec{\mathcal{U}} - \kappa \vec{\mathcal{G}} = -4\pi g_2 \vec{\mathcal{J}} \quad (2)$$

corresponding to the equations (36), (37) in III for the U-field. The constants κ, g_1, g_2 are ~~not necessary assume to~~ considered to be same ~~as~~ as in the ~~case~~ previous case for simplicity, although ~~it is not more general assumptions are not excluded, they differ each they~~ more general assumptions ~~are can~~ are not be excluded. Each of the four vector $\vec{\mathcal{M}}$, $\mathcal{M}_0$ and the six vector $\vec{\mathcal{I}}$, $\vec{\mathcal{J}}$ on the right hand sides of (1) and (2) involve each the operator which causes the transition of the heavy particle between two neutron states or between two proton states, the charge being conserved throughout. ~~the pr~~

The simplest possible ~~forms~~ expressions for them are

$$\vec{\mathcal{M}} = \tilde{\Psi}\vec{\alpha}(a + b\tau_3)\Psi, \qquad \mathcal{M}_0 = \tilde{\Psi}(a + b\tau_3)\Psi \quad (3)$$

and

$$\vec{\mathcal{I}} = -\tilde{\Psi}\rho_2\vec{\sigma}(a + b\tau_3)\Psi, \qquad \vec{\mathcal{J}} = \tilde{\Psi}\rho_3\vec{\sigma}(a + b\tau_3)\Psi, \quad (4)$$

corresponding to (38) and (39) in III, where a, b are complex constants of the order of 1.

~~From these the equations (1) and (2)~~ The quantization of this neutral field can be performed in a manner similar to that of the U-field. ~~Also the further interaction~~ Accordingly, the force of two heavy particles 1 and 2 due to virtual emission of the N-particles (and absorption) can be ~~deduced~~ determined ~~in corresp.~~ ~~accordingly~~. and takes the ~~following~~ form,

$$\mathcal{H}_{12} = \left[2\{|a|^2 + |b|^2\tau_3^{(1)}\tau_3^{(2)}\} + (\tilde{a}b + a\tilde{b})(\tau_3^{(1)} + \tau_3^{(2)})\right] \times \left[g_1^2 + g_2^2\left\{(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) - \frac{(\vec{\sigma}^{(1)}\mathrm{grad})(\vec{\sigma}^{(2)}\mathrm{grad})}{\kappa^2}\right\}\right]\frac{e^{-\kappa r}}{r}. \quad (5)$$

~~In order that the forces~~ $\mathcal{H}_{12}$ involves the forces between two neutrons and two protons as well as those ~~forces~~ of Wigner and Bartlett types between the neutron and the proton. In order that the forces are symmetric with respect to the neutron and the proton, the condition

$$\tilde{a}b + a\tilde{b} = 0$$

should be fulfilled, which ~~imply leads to~~ implies ~~so that we can write~~ either of the conditions

$$a = 0 \quad \text{or} \quad b = 0$$

or

$$\frac{b}{a} = -\frac{\tilde{a}}{\tilde{b}} = \text{pure} = ci$$

where c is a real constant. According to recent result of ~~In a paper to be published in Proc., ...,~~ Kemmer, ~~suggested that the case~~ ~~$a = 0$~~ ~~adopted~~ the case $a = 0$, ~~and~~, so that ~~the~~ we ~~he obtained~~ ~~the force~~ the resultant

$$\mathcal{H}_{12} = \cancel{2}\left(\tau_3^{(1)}\tau_3^{(2)}\right)\left[g_1^2 + g_2^2\left\{\sigma^{(1)}\sigma^{(2)} - \frac{(\sigma^{(1)}\text{grad})(\sigma^{(2)}\text{grad})}{\varkappa^2}\right\}\right]\frac{e^{-\varkappa r}}{r}$$

~~$ab = 1$~~ seems to be the correct one and

we can adopt the case $a = 0$ $b = \frac{1}{2}$, so that we obtain

$$\mathcal{H}_{12} = \frac{\tau_3^{(1)}\tau_3^{(2)}}{2}\left[\qquad\right]\frac{e^{-\varkappa r}}{r}. \qquad (6)$$

Thus, the resultant force between two heavy particles due to the virtual presence of U-particles ~~and~~ that of the N-particles becomes

$$\cancel{V_{12} + \mathcal{H}_{12} =}$$

$$H_{12} + \mathcal{H}_{12} = \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)} + \tau_3^{(1)}\tau_3^{(2)}}{2}\left[g_1^2 + g_2^2\left\{\sigma^{(1)}\sigma^{(2)} - \frac{(\sigma^{(1)}\text{grad})(\sigma^{(2)}\text{grad})}{\varkappa^2}\right\}\right]\frac{e^{-\varkappa r}}{r} \qquad (7)$$

In the S-state, ~~the~~ (7) reduce to ~~takes the approximate~~

$$H_{12} + \mathcal{H}_{12} = \frac{\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)} + \tau_3^{(1)}\tau_3^{(2)}}{2}\left\{g_1^2 + \frac{2g_2^2}{3}(\sigma^{(1)}\sigma^{(2)})\right\}\frac{e^{-\varkappa r}}{r} \qquad (8)$$

in the first approx. ~~reduces (8) takes the form~~

~~The these, this~~ ~~result~~ is in fair agreement with that obtained by Kemmer phenomenologically.[7)]

to be ...

6) The authors wish to express their hearty thanks to Dr. Kemmer for sending the manuscript of this paper before publication.

If we take

$$\left.\begin{aligned} g_1^2 &= \cancel{\tfrac{5}{24}g^2}\ \frac{g^2}{8} \\ g_2^2 &= \cancel{\tfrac{3}{16}g^2}\ \frac{5}{16}g^2 \end{aligned}\right\} \quad (6)$$

(8) becomes

$$H_{12}+\mathcal{H}_{12} = \frac{\tau_1^{(1)}\tau_1^{(2)}+\tau_2^{(1)}\tau_2^{(2)}+\tau_3^{(1)}\tau_3^{(2)}}{2}\left\{\frac{1}{8}+\frac{5}{24}(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})\right\} g^2\frac{e^{-\varkappa r}}{r}, \quad (7)$$

which has the same form as the ~~expr~~ ~~for~~ expression obtained by Kemmer phenomenologically[7]. ~~This seems very satisfor~~ ~~It should be noticed however, that the expression~~ Thus, ~~whether such an agreement between~~ it is surprising that the results ~~obtained~~ reached ~~by two different~~ from two wide different starting points ~~seems to be very satisfactory~~ are obtained agree ~~very~~ well, but, ~~owing~~ we are not sure ~~whether~~ owing to various approximations ~~made~~ in both calculations, ~~to~~ we ~~the~~ can ~~say that~~ not take it too seriously.

In the former case, ~~the state with z-component of the inner quantum number j=1.~~ ~~If we~~ ~~corresponding to~~ in which both sets of states have the inner quantum number $j=1$ for both sets of states, each pair of states with the same j_z combining with each other ~~which~~ and ~~the z-component~~ magnetic quantum number j_z ~~of the total~~. The ~~z~~-component j_z of the total angular momentum (13) can take ~~the~~ one of the values −1, 0 and +1 in the units of $\frac{h}{2\pi}$. The eigenfunctions for $j_z=-1$ ~~to the~~ ^3S states with $j_z=-1$ and $^3D_1$ can, for example, be written in the forms

$$\frac{u(r)}{r}\chi_{-1}(s_1, s_2),$$

~~where $\chi_{-1}(1,-1)=1$ $\chi_{-1}=1$~~ while that for $^3D_1$ ~~state with $j_z=-1$~~ and

7) Kemmer, Nature 140, 192, 1937. See further Heisenberg, Naturwiss. 25, 749, 1937; Flügge, Zeits. f. Phys. 108, 545, 1938.

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which lead to ~~energy~~ distribution of Fermi type for β-rays as shown in the previous section.

§6. Annihilation of Heavy Quanta ~~in Free Space~~. Spontaneous

In §8, III, the probability of annihilation of a heavy quantum ~~in vacuum~~ with the positive (or negative) charge by emitting a positive (or negative) electron and a neutrino (or an antineutrino) was calculated on the assumption that the interaction between the heavy quantum and the light particle could be given by the simplest expressions such as (25) and (26) in §4 of this paper. Thus, we found that the mean life time of the heavy quantum at rest was about 0.5×10^{-6} sec, which ~~agrees~~ is in qualitative agreement with the recent result ~~of Euler~~[18] ~~determined from the~~ (2×10^{-6} sec) obtained by Euler as the result of detailed analysis of the cosmic ray[18]. It is a pity, however, that there was an error of a factor 2 in our calculation and the correct value was to be ~~2×10^{-6} sec~~ 0.25×10^{-6} sec instead of 0.5×10^{-6} sec[19]. This makes the agreement between theory and experiment a little worse, ~~although In the previous section, it was shown that the~~

~~Now we want to~~ ~~It is important, however, to calculate~~ Sakata and ~~Tanikawa~~
~~We can easily~~ extend the above calculation ~~to more general~~

18) Euler, Naturwiss.

~~19) Prof. Heisenberg called informed us of the result of Euler.~~ ~~The present~~ The present authors wish to express their thanks to Prof. W. Heisenberg, who was kind informed them of the result of Euler.

19) The energy E in (62), §8, III, should be $m_0c^2/2$ instead of m_0c^2, ~~$\hbar$ in III should be multiplied by 2,~~ ~~E in III should be divided by 2,~~ so that (63), §8, III becomes $-dW_0 = \frac{m_0 c}{16\pi^2\hbar^4}\sum|V|^2 d\Omega$. Thus W_0 and W^0

by assuming more general ~~ex~~ form ~~for~~ (as given in §4 of this paper,) the interaction between the ~~heavy~~ quantum and the light particle ~~as made in §4 of this paper~~. By taking $\lambda_3 = \mu_3 = 0$, they obtained the probability for the annihilation of the heavy quantum with the ~~$W = W_0 \sqrt{1-\beta^2}$ velocity~~ energy E

$$W = \frac{g'^2}{\hbar c}\frac{m_U c^2}{\hbar}\frac{m_U c^2}{E}\left\{\frac{2}{3}\left|\lambda_1 + i\frac{1-\mu}{2}\lambda_2\right|^2 + \frac{1}{3}\left|\mu_1 + i\frac{1-\mu}{2}\mu_2\right|^2\right\} \tag{71}$$

~~the calcula~~ ~~g' is the same order of m~~

~~this value~~ Since $\lambda_1, \lambda_2, \mu_1, \mu_2$ are quantities of the order of 1 and g' is the same order of magnitude of with g_1', g_2' in §8, III, . the m ( ) is ~~about equal to~~ ( ) the probability ( ) does not differ essentially from ~~that~~ the previous one. ~~Thus,~~ ~~the qualitative agreement~~ The mean life time $\tau = \frac{1}{W}$ ~~for this case and~~ thus obtained is still in qualitatively agreement with the experiment. ~~Thus~~

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~~2)~~ Similar ~~opinion~~ conclusions were reached independently by Kemmer,[2)] Bhabha,[3)] ~~Heider~~ Fröhlich, Heitler ~~and~~ Kemmer[4)] and Stueckelberg[5)].

2) Kemmer, Nature **141**, 116, 1938; Proc. Roy. Soc. ~~in press~~. 166, 127, 1938
3) Bhabha, Nature **141**, 117, 1938; in press
4) Fröhlich, Heitler and Kemmer, Proc. Roy. Soc. ~~in press~~. 166, 154, 1938
5) Stueckelberg, Helv. Phys. Acta **11**, 225, 1938.

In the previous paper[1)], the theory of ~~the charge particle with~~ the ~~charged~~ Bose ~~quanta~~ particle with spin 1 obeying Bose statistics was developed by assuming ~~field~~ linear equations of Dirac-Proca type. ~~Detai~~ ~~Full account of similar~~ Kemmer, Fröhlich and Heitler[2)] Detailed account of the theory

The theory of the ~~charged~~ Bose particle with spin ~~1 obeying Bose statistics~~ ~~was developed~~ charged and ~~based on~~ satisfying linear equations of Dirac-Proca type was developed, ~~independently~~ by several authors[1) 2) 3)]† ~~In these papers~~ ~~and~~

The theory was successful in explaining the ~~inter~~ exchange forces ~~Yukawa~~ between the neutron and the proton, ~~but it was not sufficient~~ it was shown, however, tha

~~1)~~ Kemmer, Nature 141, 116, 1938; Proc. Roy. Soc. 166, 127, 1938;
T. H. and Kemmer,
~~2)~~ Bhabha, ; Proc. Roy. Soc. in press
Serber, Lamb and Schiff.
3) Yukawa, Sakata and Taketani, — — and Beck, . . .

†† Discussions of the Scalar theory ~~was given~~ were made by Stueckelberg.