

Constant mag. field = su transition

E 04110P14

electromagnetic field

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constant mag field H

①

$$A_x = \frac{1}{2}yH \quad A_y = -\frac{1}{2}xH \quad A_z = 0 \quad A_0 = 0$$

1存在 2u H. U + Electromagnetic field H interaction.

$$H_U' = \frac{4\pi e c}{\hbar} (U^+ A \operatorname{div} \tilde{U} + \tilde{U}^+ A \operatorname{div} U^+)$$

$$+ \frac{1}{4\pi \hbar^2} \frac{ie}{\hbar c} ([A \tilde{U}] \operatorname{curl} U - [AU] \operatorname{curl} \tilde{U})$$

$$H_U'' = -\frac{4\pi e^2}{\hbar^2} (AU^+)(A\tilde{U}^+) + \frac{e^2}{4\pi \hbar^2 c^2 \hbar^2} [AU][A\tilde{U}]$$

$$\operatorname{div} U^+ = \sum_{\mathbf{k}} (-ik) p_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{r}}$$

$$H_U' = \sum_{\mathbf{k}, \mathbf{j}} \left[ \frac{4\pi e c}{\hbar} \left\{ \begin{aligned} & (p_{\mathbf{k}l} + p_{\mathbf{k}}^{(1)l_1} + p_{\mathbf{k}}^{(2)l_2}) (+ij) p_{\mathbf{j}}^* \cdot \frac{1}{2} H \int y e^{-i(\mathbf{k}-\mathbf{j})\cdot\mathbf{r}} dV \\ & - (p_{\mathbf{k}m} + p_{\mathbf{k}}^{(1)m_1} + p_{\mathbf{k}}^{(2)m_2}) (ij) p_{\mathbf{j}}^* \cdot \frac{1}{2} H \int x e^{-i(\mathbf{k}-\mathbf{j})\cdot\mathbf{r}} dV \\ & - (p_{\mathbf{k}l}^* + p_{\mathbf{k}}^{*(1)l_1} + p_{\mathbf{k}}^{*(2)l_2}) (-ij) p_{\mathbf{j}} \cdot \frac{1}{2} H \int y e^{-i(\mathbf{j}-\mathbf{k})\cdot\mathbf{r}} dV \\ & + (p_{\mathbf{k}m}^* + p_{\mathbf{k}}^{*(1)m_1} + p_{\mathbf{k}}^{*(2)m_2}) (-ij) p_{\mathbf{j}} \cdot \frac{1}{2} H \int x e^{-i(\mathbf{j}-\mathbf{k})\cdot\mathbf{r}} dV \end{aligned} \right\} \right]$$

$$[A\tilde{U}]_x = A_y \tilde{U}_z - A_z \tilde{U}_y = -\frac{1}{2}xH \tilde{U}_z$$

$$[A\tilde{U}]_y = A_z \tilde{U}_x - A_x \tilde{U}_z = -\frac{1}{2}yH \tilde{U}_z$$

$$[A\tilde{U}]_z = A_x \tilde{U}_y - A_y \tilde{U}_x = \frac{1}{2}yH \tilde{U}_y + \frac{1}{2}xH \tilde{U}_x$$

$$\operatorname{curl} U = \sum_{\mathbf{k}} (ik) \left( -q_{\mathbf{k}}^{(2)l_1} + q_{\mathbf{k}}^{(1)l_2} \right) e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$+ \frac{1}{4\pi \hbar^2} \frac{ie}{\hbar c} \left\{ - (q_{\mathbf{k}n}^* + q_{\mathbf{k}}^{*(1)n_1} + q_{\mathbf{k}}^{*(2)n_2}) (ij) \left( -q_{\mathbf{j}}^{(2)l_1} + q_{\mathbf{j}}^{(1)l_2} \right) \cdot \frac{1}{2} H \int x e^{-i(\mathbf{k}-\mathbf{j})\cdot\mathbf{r}} dV \right.$$

$$- (q_{\mathbf{k}n}^* + q_{\mathbf{k}}^{*(1)n_1} + q_{\mathbf{k}}^{*(2)n_2}) (ij) \left( -q_{\mathbf{j}}^{(2)m_1} + q_{\mathbf{j}}^{(1)m_2} \right) \cdot \frac{1}{2} H \int y e^{-i(\mathbf{k}-\mathbf{j})\cdot\mathbf{r}} dV$$

$$+ (q_{\mathbf{k}m}^* + q_{\mathbf{k}}^{*(1)m_1} + q_{\mathbf{k}}^{*(2)m_2}) (ij) \left( -q_{\mathbf{j}}^{(2)n_1} + q_{\mathbf{j}}^{(1)n_2} \right) \cdot \frac{1}{2} H \int y e^{-i(\mathbf{k}-\mathbf{j})\cdot\mathbf{r}} dV$$

$$+ (q_{\mathbf{k}l}^* + q_{\mathbf{k}}^{*(1)l_1} + q_{\mathbf{k}}^{*(2)l_2}) (ij) \left( -q_{\mathbf{j}}^{(2)n_1} + q_{\mathbf{j}}^{(1)n_2} \right) \cdot \frac{1}{2} H \int x e^{-i(\mathbf{k}-\mathbf{j})\cdot\mathbf{r}} dV$$

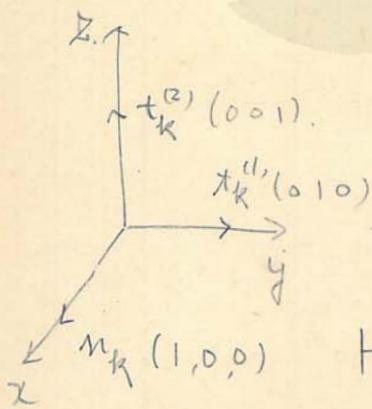
- conj compl. }

$$H'_U = \sum_k \sum_j \left[ \frac{4\pi e c}{\hbar} \frac{1}{2} j H \right] \left\{ \begin{aligned} & - (p_k^l + p_k^{(1)l} + p_k^{(2)l}) p_j^* \cdot y_{kj} \\ & + (p_k^m + p_k^{(1)m} + p_k^{(2)m}) p_j^* \cdot x_{kj} \\ & - (p_k^* l + p_k^{(1)*l} + p_k^{(2)*l}) p_j \cdot y_{jk} \\ & + (p_k^{*m} + p_k^{(1)*m} + p_k^{(2)*m}) p_j \cdot x_{jk} \end{aligned} \right\}$$

$$x_{kj} = \int x e^{-i(\vec{k}-\vec{j})\cdot\vec{r}} dV$$

$$y_{kj} = \int y e^{-i(\vec{k}-\vec{j})\cdot\vec{r}} dV$$

$$+ \left[ \frac{1}{4\pi \hbar^2} \frac{e}{\hbar c} \frac{1}{2} j H \right] \left\{ \begin{aligned} & (q_k^* n + q_k^{*(1)n} + q_k^{*(2)n}) (-q_j^{(3)l'} + q_j^{(1)l'}) \cdot x_{kj} \\ & + (q_k^* n + q_k^{*(1)n} + q_k^{*(2)n}) (-q_j^{(2)m'} + q_j^{(1)m'}) \cdot y_{kj} \\ & - (q_k^* m + q_k^{*(1)m} + q_k^{*(2)m}) (-q_j^{(2)n'} + q_j^{(1)n'}) \cdot y_{kj} \\ & - (q_k^* l + q_k^{*(1)l} + q_k^{*(2)l}) (-q_j^{(2)n'} + q_j^{(1)n'}) \cdot x_{kj} \\ & + (q_k n + q_k^{(1)n} + q_k^{(2)n}) (-q_j^{*(2)l'} + q_j^{*(1)l'}) \cdot x_{jk} \\ & + (q_k n + q_k^{(1)n} + q_k^{(2)n}) (-q_j^{*(2)m'} + q_j^{*(1)m'}) \cdot y_{jk} \\ & - (q_k m + q_k^{(1)m} + q_k^{(2)m}) (-q_j^{*(2)n'} + q_j^{*(1)n'}) \cdot y_{jk} \\ & - (q_k l + q_k^{(1)l} + q_k^{(2)l}) (-q_j^{*(2)n'} + q_j^{*(1)n'}) \cdot x_{jk} \end{aligned} \right\}$$



- $l=1 \quad m=0 \quad n=0$
- $l_1=0 \quad m_1=1 \quad n_1=0$
- $l_2=0 \quad m_2=0 \quad n_2=1$

$$H'_U = \sum_k \sum_j \left[ \frac{4\pi e c}{\hbar} \frac{1}{2} j H \right] \left\{ \begin{aligned} & - p_k p_j^* y_{kj} + p_k^{(1)} p_j^* x_{kj} \\ & - p_k^* p_j y_{jk} + p_k^{*(1)} p_j x_{jk} \end{aligned} \right\} \\ + \left[ \frac{1}{4\pi \hbar^2} \frac{e}{\hbar c} \frac{1}{2} j H \right] \left\{ \begin{aligned} & q_k^{*(2)} (-q_j^{(3)l'} + q_j^{(1)l'}) x_{kj} + q_k^{*(2)} (-q_j^{(2)m'} + q_j^{(1)m'}) y_{kj} \\ & - q_k^{*(1)} (-q_j^{(2)n'} + q_j^{(1)n'}) y_{kj} - q_k^* (-q_j^{(2)n'} + q_j^{(1)n'}) x_{kj} \\ & + \text{comp conj} \end{aligned} \right\}$$

$$\begin{aligned}
 H_4' = & \sum_k \sum_j \left[ \frac{4\pi e^2}{k} \frac{1}{2} H \right. \\
 & - \frac{\hbar}{8\pi c} \frac{j}{\sqrt{k_0} \sqrt{j_0}} (a_k^* + b_k^*) (a_j + b_j^*) y_{kj} \\
 & - \frac{\hbar}{8\pi c} \frac{j}{\sqrt{k_0} \sqrt{j_0}} (a_k + b_k^*) (a_j^* + b_j) y_{jk} \\
 & + \frac{\hbar}{8\pi c} \frac{\sqrt{k_0} j}{\sqrt{j_0}} (a_k^{(1)*} + b_k^{(1)}) (a_j + b_j^*) x_{kj} \\
 & \left. + \frac{\hbar}{8\pi c} \frac{\sqrt{k_0} j}{\sqrt{j_0}} (a_k^{(1)} + b_k^{(1)*}) (a_j^* + b_j) x_{jk} \right] \\
 & + \frac{1}{4\pi k^2} \frac{e}{\hbar c} \frac{1}{2} H \left\{ + 2\pi \hbar c k^2 \frac{j}{\sqrt{k_0} \sqrt{j_0}} (a_k^{*(2)} - b_k^{(2)}) (-a_j^{(2)} + b_j^{*(2)}) (l_1' x_{kj} + m_1' y_{kj}) \right. \\
 & + 2\pi \hbar c k^2 \frac{j}{\sqrt{k_0} \sqrt{j_0}} (-a_k^{(2)} + b_k^{*(2)}) (a_j^{*(2)} - b_j^{(2)}) (l_1' x_{jk} + m_1' y_{jk}) \\
 & - 2\pi \hbar c k^2 \frac{j}{\sqrt{k_0} \sqrt{j_0}} (a_k^{*(2)} - b_k^{(2)}) (-a_j^{(1)} + b_j^{*(1)}) (l_2' x_{kj} + m_2' y_{kj}) \\
 & - 2\pi \hbar c k^2 \frac{j}{\sqrt{k_0} \sqrt{j_0}} (-a_k^{(2)} + b_k^{*(2)}) (a_j^{*(1)} - b_j^{(1)}) (l_2' x_{jk} + m_2' y_{jk}) \\
 & - 2\pi \hbar c k^2 \frac{j}{\sqrt{k_0} \sqrt{j_0}} (a_k^{*(1)} - b_k^{(1)}) (-a_j^{(1)} + b_j^{*(1)}) n_1' y_{kj} \\
 & - 2\pi \hbar c k^2 \frac{j}{\sqrt{k_0} \sqrt{j_0}} (-a_k^{(1)} + b_k^{*(1)}) (a_j^{*(2)} - b_j^{(2)}) n_1' y_{jk} \\
 & + 2\pi \hbar c k^2 \frac{j}{\sqrt{k_0} \sqrt{j_0}} (a_k^{*(1)} - b_k^{(1)}) (-a_j^{(1)} + b_j^{*(1)}) n_2' y_{kj} \\
 & + 2\pi \hbar c k^2 \frac{j}{\sqrt{k_0} \sqrt{j_0}} (-a_k^{(1)} + b_k^{*(1)}) (a_j^{*(1)} - b_j^{(1)}) n_2' y_{jk} \\
 & - 2\pi \hbar c \frac{\sqrt{k_0} j}{\sqrt{j_0}} (a_k^* - b_k) (-a_j^{(2)} + b_j^{*(2)}) n_1' x_{kj} \\
 & - 2\pi \hbar c \frac{\sqrt{k_0} j}{\sqrt{j_0}} (-a_k + b_k^*) (a_j^{*(2)} - b_j^{(2)}) n_1' x_{jk} \\
 & + 2\pi \hbar c \frac{\sqrt{k_0} j}{\sqrt{j_0}} (a_k^* - b_k) (-a_j^{(1)} + b_j^{*(1)}) n_2' x_{kj} \\
 & \left. + 2\pi \hbar c \frac{\sqrt{k_0} j}{\sqrt{j_0}} (-a_k + b_k^*) (a_j^{*(1)} - b_j^{(1)}) n_2' x_{jk} \right\} ]
 \end{aligned}$$

$$\frac{4\pi e^2 \hbar}{k \times 8\pi c} = \frac{e \hbar}{2m_0 c}$$

$$\frac{1}{4\pi k^2} \frac{e}{\hbar c} \cdot 2\pi \hbar c k^2 = \frac{e}{2k} = \frac{e \hbar}{2m_0 c}$$

longitudinal k の part

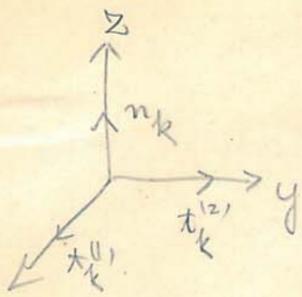
$$H'_L = \sum_j \frac{e\hbar}{2m_0c} \frac{1}{2} H \left[ - \frac{\kappa_j}{\sqrt{k_0} \sqrt{j_0}} a_k (a_j^* + b_j) y_{jk} \right. \\
 + \left( \frac{\sqrt{k_0} j}{\sqrt{j_0}} \right) a_k (a_j^{*(2)} - b_j^{(2)}) n_1' x_{jk} \\
 \left. - \left( \frac{\sqrt{k_0} j}{\sqrt{j_0}} \right) a_k (a_j^{*(1)} - b_j^{(1)}) n_2' x_{jk} \right]$$

transversal k の part  $a_k^{(1)}$ , term.

$$H'_L = \sum_j \frac{e\hbar}{2m_0c} \frac{1}{2} H \left[ \left( \frac{\sqrt{k_0} j}{\sqrt{j_0}} \right) a_k^{(1)} (a_j^* + b_j) x_{jk} \right. \\
 + \frac{\kappa_j}{\sqrt{k_0} \sqrt{j_0}} a_k^{(1)} (a_j^{*(2)} - b_j^{(2)}) n_1' y_{jk} \\
 \left. - \frac{\kappa_j}{\sqrt{k_0} \sqrt{j_0}} a_k^{(1)} (a_j^{*(1)} - b_j^{(1)}) n_2' y_{jk} \right]$$

trans  $a_k^{(2)}$ , term.

$$H'_L = \sum_j \frac{e\hbar}{2m_0c} \frac{1}{2} H \left[ - \frac{\kappa_j}{\sqrt{k_0} \sqrt{j_0}} a_k^{(2)} (a_j^{*(2)} - b_j^{(2)}) (l_1' x_{jk} + m_1' y_{jk}) \right. \\
 \left. + \frac{\kappa_j}{\sqrt{k_0} \sqrt{j_0}} a_k^{(2)} (a_j^{*(1)} - b_j^{(1)}) (l_2' x_{jk} + m_2' y_{jk}) \right]$$



$$l = m = 0, \quad n = 1.$$

$$l_1 = 1, \quad m_1 = n_1 = 0.$$

$$l_2 = n_2 = 0, \quad m_2 = 1.$$

$$\begin{aligned}
 H_u' &= \sum_k \sum_j \left[ \frac{4\pi e c}{h} \frac{1}{2} \dot{J} H \right] - p_k^{(1)} p_j^* y_{kj} + p_k^{(2)} p_j^* x_{kj} \\
 &\quad - p_k^{*(1)} p_j y_{jk} + p_k^{*(2)} p_j x_{jk} \} \\
 &+ \frac{1}{4\pi k^2} \frac{e}{h c} \frac{1}{2} \dot{J} H \left\{ g_k^* (-g_j^{(2)} l_1' + g_j^{(1)} l_2') x_{kj} + g_k^* (-g_j^{(2)} m_1' + g_j^{(1)} m_2') y_{kj} \right. \\
 &\quad - g_k^{*(2)} (-g_j^{(2)} n_1' + g_j^{(1)} n_2') y_{kj} + g_k^{*(1)} (-g_j^{(2)} n_1' + g_j^{(1)} n_2') x_{kj} \\
 &\quad \left. + \text{comple conj} \right\} \\
 &= \sum_k \sum_j \frac{e h}{2 m_0 c} \frac{1}{2} H \left[ -\frac{\sqrt{k_0} \dot{J}}{\sqrt{j_0}} \left\{ (a_k^{(1)*} + b_k^{(1)}) (a_j + b_j^*) y_{kj} + (a_k^{(1)} + b_k^{(1)*}) (a_j^* + b_j) y_{jk} \right\} \right. \\
 &\quad \left. + \frac{\sqrt{k_0} \dot{J}}{\sqrt{j_0}} \left\{ (a_k^{(2)*} + b_k^{(2)}) (a_j + b_j^*) x_{kj} + (a_k^{(2)} + b_k^{(2)*}) (a_j^* + b_j) x_{jk} \right\} \right. \\
 &\quad \left. + \frac{\sqrt{k_0} \dot{J}}{\sqrt{j_0}} \left\{ (a_k^* - b_k) (-a_j^{(2)} + b_j^{*(2)}) (l_1' x_{kj} + m_1' y_{kj}) \right. \right. \\
 &\quad \left. + (-a_k + b_k^*) (a_j^{*(2)} + b_j^{(2)}) (l_1' x_{jk} + m_1' y_{jk}) \right. \\
 &\quad \left. - (a_k^* - b_k) (-a_j^{(1)} + b_j^{*(1)}) (l_2' x_{kj} + m_2' y_{kj}) \right. \\
 &\quad \left. - (-a_k + b_k^*) (a_j^{*(1)} + b_j^{(1)}) (l_2' x_{jk} + m_2' y_{jk}) \right\} \\
 &\quad \left. + \frac{\kappa \dot{J}}{\sqrt{k_0} \sqrt{j_0}} \left\{ - (a_k^{*(2)} - b_k^{(2)}) (-a_j^{(2)} + b_j^{*(2)}) n_1' y_{kj} - (-a_k^{(2)} + b_k^{*(2)}) (a_j^{*(2)} - b_j^{(2)}) n_1' y_{jk} \right. \right. \\
 &\quad \left. + (a_k^{*(2)} - b_k^{(2)}) (-a_j^{(1)} + b_j^{*(1)}) n_2' y_{kj} + (-a_k^{(2)} + b_k^{*(2)}) (a_j^{*(1)} - b_j^{(1)}) n_2' y_{jk} \right. \\
 &\quad \left. + (a_k^{*(1)} - b_k^{(1)}) (-a_j^{(2)} + b_j^{*(2)}) n_1' x_{kj} + (-a_k^{(1)} + b_k^{*(1)}) (a_j^{*(2)} - b_j^{(2)}) n_1' x_{jk} \right. \\
 &\quad \left. - (a_k^{*(1)} - b_k^{(1)}) (-a_j^{(1)} + b_j^{*(1)}) n_2' x_{kj} - (-a_k^{(1)} + b_k^{*(1)}) (a_j^{*(1)} - b_j^{(1)}) n_2' x_{jk} \right\} ]
 \end{aligned}$$

longitudinal  $k \rightarrow$

①

$$H_u' = \sum_j \frac{e\hbar}{2m_j c} \frac{1}{2} H. \left[ - \frac{\sqrt{k_0 j}}{V_{j0}} a_k (a_j^{*(2)} + b_j^{(2)}) (l_1' x_{jk} + m_1' y_{jk}) \right. \\ \left. + \frac{\sqrt{k_0 j}}{V_{j0}} a_k (a_j^{*(1)} + b_j^{(1)}) (l_2' x_{jk} + m_2' y_{jk}) \right]$$

transversal  $k \rightarrow$  put  $\neq$   $a_k^{(1)}$ , term.

$$H_u' = \sum_j \frac{e\hbar}{2m_j c} \frac{1}{2} H. \left[ - \frac{\sqrt{k_0 j}}{V_{j0}} a_k^{(1)} (a_j^* + b_j) y_{jk} \right. \\ \left. - \frac{\kappa j}{\sqrt{k_0} V_{j0}} a_k^{(1)} (a_j^{*(2)} - b_j^{(2)}) n_1' x_{jk} + \frac{\kappa j}{\sqrt{k_0} V_{j0}} a_k^{(1)} (a_j^{*(1)} - b_j^{(1)}) n_2' x_{jk} \right]$$

$a_k^{(2)}$ , term.

$$H_u' = \sum_j \frac{e\hbar}{2m_j c} \frac{1}{2} H. \left[ + \frac{\sqrt{k_0 j}}{V_{j0}} a_k^{(2)} (a_j^* + b_j) x_{jk} \right. \\ \left. + \frac{\kappa j}{\sqrt{k_0} V_{j0}} a_k^{(2)} (a_j^{*(2)} - b_j^{(2)}) n_1' y_{jk} - \frac{\kappa j}{\sqrt{k_0} V_{j0}} a_k^{(2)} (a_j^{*(1)} - b_j^{(1)}) n_2' y_{jk} \right]$$

$$\begin{aligned} \xi &= (x+iy) & \xi + \bar{\xi} &= x \\ &= r \sin \theta e^{i\phi} & \xi - \bar{\xi} &= y \\ \bar{\xi} &= r \sin \theta e^{-i\phi} & \vec{k} - \vec{j} &= \vec{p} \end{aligned}$$

$$\int x e^{-i\vec{p}\vec{r}} r^2 \cos \theta dV = \int r \sin \theta (e^{i\phi} + e^{-i\phi}) e^{-i\vec{p}\vec{r}} r^2 \sin \theta dr d\theta d\phi \\ = \int r^3 \sin^2 \theta \left\{ e^{i(\phi - \vec{p}\vec{r})} + e^{-i(\phi + \vec{p}\vec{r})} \right\} dr d\theta d\phi$$

Constant field =  $su$

L 1 transition

