16

数物報告

On the Interaction of Elementary Particles. IV.

By Hideki Yukawa, Shoichi Sakata, Minoru Kobayasi and Mitsuo Taketani

(Read May 28, 1938)

§1. Introduction and Summary.

The theory of the heavy quantum with charge, mass and spin satisfying linear tensor equations of Dirac-Proca type was developed recently by several authors,[1), 2), 3), 4), 5)] and was successful in explaining the exchange forces between the neutron and the proton. It was shown further that the introduction of the neutral quantum was necessary in order to account for approximate equality of the like particle and unlike particle forces. In §2 of the present paper, we deal with this problem by assuming the same equations for the neutral quanta as those for the charged quanta. It is assumed, moreover, that the interaction of the heavy particle with the former is the same as that with the latter, except that the factor of the form $a + b\tau_3$ (or $\tilde{a} + \tilde{b}\tau_3$) appears in place of $\tau_1 + i\tau_2$ (or $\tau_1 - i\tau_2$), where a and b are

1) Kemmer, Nature 141, 116 (1938); Proc. Roy. Soc. A 166, 127 (1938); Fröhlich, Heitler and Kemmer, ibid. 166, 154 (1938).
2) Bhabha, Nature 141, 117 (1938); Proc. Roy. Soc. A 166, 501 (1938).
3) Stueckelberg, Helv. Phys. 11, 299 (1938).
4) Yukawa, Sakata and Taketani, Proc. Phys.-Math. Soc. Japan 20, 319 (1938). This paper will be referred to as III.
5) Discussions of the scalar theory were made by Yukawa and Sakata, Proc. Phys.-Math. Soc. Japan 19, 1084 (1937), which will be referred to as II; Beck, Nature 141, 609, 832 (1938); Lamb and Schiff, Phys. Rev. 53, ~~651~~, (1938); Stueckelberg, Helv. Phys. 11, 225 (1938); Wentzel, Naturwiss. 26, ~~273~~, (1938).

complex numbers. Thus, the forces between like particles as well as the ordinary forces between unlike particles can be deduced as second order effects. By a suitable choice of the constants, these forces can take the same form as those assumed in the current theory.

As already mentioned in §7, III, the forces thus obtained are not strictly central, so that the stationary states and the energy level scheme of the nuclear system become rather complicated. Even the deuteron problem cannot easily be solved owing to the coupling, for example, of ^3S states with $^3D_1$ states on the one hand and the appearance, in the potential, of terms proportional to $1/r^2$ and $1/r^3$ at small distance on the other. It may well be true, however, that the interaction between the neutron and the proton at a distance small compared with $1/\kappa$ differs essentially from that obtained as second order effect in our theory, so that large significance cannot be attached to the details of such calculations. (§3.)

In the first paper,[6] the β-disintegration was considered as the result of exchange of a heavy quantum between the heavy and the light particles and thus we obtained a theory, which was equivalent to the original theory of Fermi. In the vectorial theory for the U-field, it is possible to construct a theory of the β-disintegration, which leads to the distribution of the β-ray equivalent to a combination of those of Fermi and Konopinski-Uhlenbeck. However, the quantization of the scheme including the interaction of K.-U. type is possible only in the first approximation. (§4, §5.) Moreover, the

---

6) Yukawa, Proc. Phys.-Math. Soc. Japan 17, 48, (1935). This paper will be referred to as I.

probability of the spontaneous annihilation of a heavy quantum with the positive charge, for example, by emitting a positive electron and a neutrino in this case is so large that the mean life time and the mean free path of the heavy quantum are much too short to be identified with the hard particle in the cosmic ray. Thus, only the simplest expressions for the interaction between the heavy quantum and the light particle as given by (25) and (26) in §4 of this paper seem to be consistent with the experiment on the cosmic ray, but, in this case, it is difficult to account for the asymmetry in the distribution of the β-ray. (§6).

Next, the problem of the creation of the heavy quantum by the following process is discussed in detail. A light quantum of energy larger than $m_u c^2$ is absorbed by the neutron, for example, and a heavy quantum with the negative charge is emitted subsequently, the neutron being transformed thereby into the proton. The cross section of this process turns out to be far larger than that of the creation of a pair of heavy quanta by a light quantum of energy larger than $2m_u c^2$, so that it is possible that the hard component of the cosmic ray observed on the sea level consists mainly of heavy quanta produced in the atmosphere from the soft component by the above process. The probability of the absorption of heavy quanta by the inverse process is also calculated. (§7, §8.)

Finally, the problem of the spin and the magnetic moment of the heavy quantum is discussed. (§9.)

獨逸文

數物報告

§2. Linear Equations for the Neutral Heavy Quanta.

We introduce a six vector $\vec{\mathfrak{F}}, \vec{\mathfrak{G}}$ and a four vector $\vec{\mathfrak{U}}, \mathfrak{U}_0$ for the field, which is accompanied by neutral heavy quanta, each of the German letters denoting a quantity equivalent to one for the U-field denoted by the corresponding Latin letter. Now, we assume a system of equations of the form

$$\frac{1}{c}\frac{\partial \vec{\mathfrak{F}}}{\partial t} - \mathrm{curl}\, \vec{\mathfrak{G}} - \varkappa \vec{\mathfrak{U}} = -4\pi g_1 \vec{\mathfrak{M}}, \quad \mathrm{div}\, \vec{\mathfrak{F}} + \varkappa \mathfrak{U}_0 = 4\pi g_1 \mathfrak{M}_0 \qquad (1)$$

$$\frac{1}{c}\frac{\partial \vec{\mathfrak{U}}}{\partial t} + \mathrm{grad}\, \mathfrak{U}_0 + \varkappa \vec{\mathfrak{F}} = 4\pi g_2 \vec{\mathfrak{T}}, \quad \mathrm{curl}\, \vec{\mathfrak{U}} - \varkappa \vec{\mathfrak{G}} = -4\pi g_2 \vec{\mathfrak{S}} \qquad (2)$$

in the presence of the heavy particle. This corresponds to the system of equations (36), (37) in §6 III for the U-field. The constants $\varkappa$, g_1 and g_2 in (1), (2) are considered for simplicity to be the same as in the previous case, although more general assumptions are not excluded. The four vector $\vec{\mathfrak{M}}, \mathfrak{M}_0$ and the six vector $\vec{\mathfrak{T}}, \vec{\mathfrak{S}}$ on the right hand sides of (1), (2) involve the operators, which cause the transition of the heavy particle between two neutron states as well as that between two proton states. The simplest possible expressions for them are

$$\vec{\mathfrak{M}} = \tilde{\Psi}\vec{\alpha}(a+b\tau_3)\Psi \qquad \mathfrak{M}_0 = \tilde{\Psi}(a+b\tau_3)\Psi \qquad (3)$$

$$\vec{\mathfrak{T}} = -\tilde{\Psi}\rho_2\vec{\sigma}(a+b\tau_3)\Psi, \quad \vec{\mathfrak{S}} = \tilde{\Psi}\rho_3\vec{\sigma}(a+b\tau_3)\Psi \qquad (4)$$

獨逸文字

corresponding to (38) and (39) in III, where a and b are complex numbers of the order of 1.

The quantization of this field can be performed in a manner similar to that of the U-field and we find that the field is accompanied by neutral quanta with the mass $m_{\bar{u}} = \varkappa\hbar/c$ and the spin 1, obeying Bose statistics.

The potential of forces between two heavy particles due to virtual emission and absorption of neutral quanta can be determined as the second order effect and turns out to be

$$\mathcal{H}_{12} = \{2(\tilde{a}a + \tilde{b}b\tau_3^{(1)}\tau_3^{(2)}) + (\tilde{a}b + a\tilde{b})(\tau_3^{(1)} + \tau_3^{(2)})\} \times \left\{g_1^2 + g_2^2(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) - g_2^2\frac{(\vec{\sigma}^{(1)}\mathrm{grad})(\vec{\sigma}^{(2)}\mathrm{grad})}{\varkappa^2}\right\}\frac{e^{-\varkappa r}}{r}, \qquad (5)$$

獨逸文字

which involves the forces between two neutrons and between two protons as well as those of Wigner and Bartlett types between the neutron and the proton. In order that these forces are symmetric with respect to the neutron and the proton, the condition

$$\tilde{a}b + a\tilde{b} = 0$$

shoud be fulfilled, which implies either $a = 0$ or $b = 0$ or $b/a = ci$, where c is a real number. According to recent result of Kemmer[7)], $a = 0$ seems to be a good choice. If we take $a = 0$ and $b = 1/2$, we obtain

$$\mathcal{H}_{12} = \frac{\tau_3^{(1)}\tau_3^{(2)}}{2}\left\{g_1^2 + g_2^2(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) - g_2^2\frac{(\vec{\sigma}^{(1)}\mathrm{grad})(\vec{\sigma}^{(2)}\mathrm{grad})}{\varkappa^2}\right\}\frac{e^{-\varkappa r}}{r}, \qquad (6)$$

By adding this to H_{12} as given by (59) in §7, III, we obtain the resultant potential

$$H_{12} + \mathcal{H}_{12} = \sum_{i=1,2,3}\frac{\tau_i^{(1)}\tau_i^{(2)}}{2}\left\{g_1^2 + g_2^2(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) - g_2^2\frac{(\vec{\sigma}^{(1)}\mathrm{grad})(\vec{\sigma}^{(2)}\mathrm{grad})}{\varkappa^2}\right\}\frac{e^{-\varkappa r}}{r}. \qquad (7)$$

Especially in S state, (7) reduces to

$$H_{12} + \mathcal{H}_{12} = \sum_i \frac{\tau_i^{(1)}\tau_i^{(2)}}{2}\left\{g_1^2 + \frac{2g_2^2}{3}(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})\right\}\frac{e^{-\varkappa r}}{r} \qquad (8)$$

---

7) The present authors wish to express their thanks to Dr. Kemmer for sending the manuscript of his paper to be published in Proc. Camb. Phil. Soc.

in the first approximation. If we take

$$g_1^2 = g^2/8, \qquad g_2^2 = 5g^2/16, \tag{9}$$

(8) becomes

$$H_{12} + \mathcal{H}_{12} = \sum_i \frac{\tau_i^{(1)}\tau_i^{(2)}}{2}\left\{\frac{1}{8} + \frac{5}{24}(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})\right\} g^2 \frac{e^{-\kappa r}}{r}, \tag{10}$$

which has the same form as the expression obtained by Kemmer[8] phenomenologically. It is surprising that the results reached from two widely different starting points agree with each other so well, but we cannot take it too seriously, if we consider various approximations in both calculations.

§3. The Deuteron Problem.

If we consider a system consisting of a neutron and a proton, the Schrödinger equation for the relative motion takes the form

$$\left\{\frac{\hbar^2}{M}\Delta + E - V\right\}\psi(x, y, z, s_1, s_2) = 0 \tag{11}$$

with

$$V = \varepsilon_\pm\left\{g_1^2 + \frac{2}{3}g_2^2(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}) + g_2^2\left(\frac{\vec{\sigma}^{(1)}\vec{\sigma}^{(2)}}{3} - \frac{(\vec{\sigma}^{(1)}\vec{r})(\vec{\sigma}^{(2)}\vec{r})}{r^2}\right)\left(1 + \frac{3}{\kappa r} + \frac{3}{\kappa^2 r^2}\right)\right\}\frac{e^{-\kappa r}}{r}, \tag{12}$$

where x, y, z are relative coordinates and s_1, s_2 are z-components of the spins of two particles in units of $\hbar/2$ respectively. If we use the expression (7) for the potential V, we obtain

---

8) Kemmer, Nature 140, 192, (1937). See further Heisenberg, Naturwiss. 25, 749, (1937); Flügge, Zeits. f. Phys. 108, 545, (1938).

$$\varepsilon_+ = -\frac{1}{2} \quad \text{or} \quad \varepsilon_- = \frac{3}{2}$$

according as ψ is symmetric (^3S, ^1P, ^3D, ...) or antisymmetric (^1S, ^3P, ^1D, ...) with respect to coordinates and spins of two particles.

Now, the potential V are not commutative with the orbital angular momentum $\vec{r} \times \vec{p}$, but are commutative with the total angular momentum

$$(\vec{r} \times \vec{p}) + \frac{\hbar}{2}(\vec{\sigma}^{(1)} + \vec{\sigma}^{(2)}) \tag{13}$$

Thus, the coupling between two states with different azymuthal quantum numbers is possible, while that with different inner quantum numbers is not. Further, V is invariant with respect to the exchange of $\vec{\sigma}^{(1)}$ and $\vec{\sigma}^{(2)}$, so that singlet and triplet states do not combine with each other. Moreover, there is no coupling between the symmetric and antisymmetric states.

In this way, we find that each of the singlet states ^1S, ^1P, ^1D, ... is a stationary state by itself, for which the potential is given by

$$V = \varepsilon_\pm (g_1^2 - 2g_2^2) e^{-\chi r}/r \tag{14}$$

by the help of the relations

$$\vec{\sigma}^{(1)} \cdot \vec{\sigma}^{(2)} = -3, \quad (\vec{\sigma}^{(1)}\vec{r})(\vec{\sigma}^{(2)}\vec{r})/r^2 = -1.$$

The situation is not so simple in the case of the triplet states. For example, $^3S_1$ combines with $^3D_1$, while $^3P_0$ with none, etc. In the former case, the inner quantum number j is 1 for both sets of states and the z-component j_z takes the values -1, 0 and +1 respectively in units of $\hbar/2$ for both states, only those with the same j_z combining with each other. The eigenfunctions for ^3S and $^3D_1$ with $j_z = -1$, for instance, can be written in the form

$$\left.\begin{array}{l} \frac{u(r)}{r}\chi_{-1}(s_1, s_2) \\ \text{and} \quad \frac{v(r)}{r}\Psi_{-1}(\theta, \varphi, s_1, s_2), \end{array}\right\} \tag{15}$$

where

$$\chi_{-1} = \alpha(s_1)\alpha(s_2),\quad \chi_0 = \frac{1}{\sqrt{2}}\{\alpha(s_1)\beta(s_2)+\beta(s_1)\alpha(s_2)\},\quad \chi_1 = \beta(s_1)\beta(s_2), \tag{16}$$

α and β denoting the eigenfunctions for spins in z and -z directions respectively,[9] and

$$\left.\begin{aligned} \Psi_{-1} &= \sqrt{\tfrac{1}{10}}\left(Y_0\chi_{-1} - \sqrt{3}\,Y_{-1}\chi_0 + \sqrt{6}\,Y_{-2}\chi_1\right) \\ Y_0 &= \sqrt{5}\left(\tfrac{3}{2}\cos^2\theta - \tfrac{1}{2}\right) \\ Y_{-1} &= \sqrt{\tfrac{15}{2}}\sin\theta\cos\theta\, e^{-i\varphi} \\ Y_{-2} &= \tfrac{1}{2}\sqrt{\tfrac{15}{2}}\sin^2\theta\, e^{-2i\varphi} \end{aligned}\right\} \tag{17}$$

Now, we have to find the correct combination

$$\Psi = \frac{u(r)}{r}\chi_{-1} + \frac{v(r)}{r}\Psi_{-1}, \tag{18}$$

which satisfies the equation (11). By using the relations

$$\left.\begin{aligned} &(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})\chi_{-1} = \chi_{-1} \qquad (\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})\Psi_{-1} = \Psi_{-1}, \\ &\left\{\frac{(\vec{\sigma}^{(1)}\vec{r})(\vec{\sigma}^{(2)}\vec{r})}{r^2} - \frac{(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})}{3}\right\}\chi_{-1} = \frac{2\sqrt{2}}{3}\Psi_{-1}, \\ &\{\qquad\quad ''\qquad\quad\}\Psi_{-1} = -\frac{2}{3}\Psi_{-1} + \frac{2\sqrt{2}}{3}\chi_{-1}, \end{aligned}\right\} \tag{19}$$

which can be obtained by elementary calculations, we find that the radial functions u(r) and v(r) should satisfy the simultaneous equations

$$\left.\begin{aligned} &\frac{\hbar^2}{M}\frac{d^2u}{dr^2} + \left\{E - \varepsilon_+\left(g_1^2 + \frac{2}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r}\right\}u + \frac{2\sqrt{2}}{3}g_2^2\left(1 + \frac{3}{\varkappa r} + \frac{3}{\varkappa^2 r^2}\right)\frac{e^{-\varkappa r}}{r}v = 0, \\ &\frac{\hbar^2}{M}\left(\frac{d^2v}{dr^2} - \frac{6}{r^2}v\right) + \left\{E - \varepsilon_+\left(g_1^2 + \frac{2}{3}g_2^2\right)\frac{e^{-\varkappa r}}{r} - \frac{2}{3}\varepsilon_+ g_2^2\left(1 + \frac{3}{\varkappa r} + \frac{3}{\varkappa^2 r^2}\right)\frac{e^{-\varkappa r}}{r}\right\}v \\ &\qquad + \frac{2\sqrt{2}}{3}\varepsilon_+ g_2^2\left(1 + \frac{3}{\varkappa r} + \frac{3}{\varkappa^2 r^2}\right)\frac{e^{-\varkappa r}}{r}u = 0. \end{aligned}\right\} \tag{20}$$

It can easily be shown that these equations have no solutions, which are regular at r = 0, because of terms involving the factor $1/r^3$.

---

9) Costruction of eigenfunctions for any given values of ℓ, s and j was worked out, for example, by Wigner, Gruppentheorie, Braunschweig, (1931), Kap. XVII.

This, however, is not a fundamental difficulty, since the interaction potential derived as fourth order effect is larger than the above potential V for r small compared with $1/\kappa$, as already shown by Fröhlich, Heitler and Kemmer,[10] so that we can say little of the form of the interaction potential near r = 0. Moreover, there are many reasons to believe that the whole scheme of the theory in the present form is valid only for those regions with linear dimensions larger than $1/\kappa$, as discussed by Heisenberg in reference to the scalar theory.[11] In any case, it is true that the divergence difficulty in our theory is more pronounced than in the quantum theory of the electromagnetic field as developed by Heisenberg and Pauli.[12]

§4. Interaction of the U-Field with the Light Particle.

In our theory, the process of β-disintegration can be considered as the result of the following double transition. Namely, the heavy quantum with the negative charge, which is emitted virtually when a heavy particle jumps from a neutron to a proton state, can be absorbed by a light particle, which in turn rises from a neutrino state of negative energy to an electron state of positive energy. Thus an electron and an antineutrino are emitted simultaneously. This idea was developed in §4, I in the scheme of the scalar theory and lead to the energy distribution of the β-ray, which was essentially the same as that in the original theory of Fermi.[13]

Now, in the theory of the vectorial field for the heavy quantum, the

10) Fröhlich, Heitler and Kemmer, loc. cit.
11) Heisenberg, Ann. d. Phys. 32, 20, (1938).
12) Heisenberg and Pauli, Zeits. f. Phys. 56, 1, (1929); 59, 168, (1930).
13) Fermi, Zeits. f. Phys. 88, 161, (1934).

數物報告

system of equations (36), (37) in §6, III, which are valid when the interaction of the heavy quantum with the heavy particle alone is considered, should be modified by adding terms involving the wave functions for the light particle on the right hand sides, when the interaction with the latter is also taken into account. Thus, it takes the general form

$$\left.\begin{aligned} \frac{1}{c}\frac{\partial \vec{F}}{\partial t} - \mathrm{curl}\,\vec{G} - \varkappa\vec{U} &= -4\pi g_1\vec{M} - 4\pi g'\vec{M}' \\ \mathrm{div}\,\vec{F} + \varkappa U_0 &= 4\pi g_1 M_0 + 4\pi g' M_0' \end{aligned}\right\} \quad (21)$$

$$\left.\begin{aligned} \frac{1}{c}\frac{\partial \vec{U}}{\partial t} + \mathrm{grad}\,U_0 + \varkappa\vec{F} &= 4\pi g_2\vec{T} + 4\pi g'\vec{T}' \\ \mathrm{curl}\,\vec{U} - \varkappa\vec{G} &= -4\pi g_2\vec{S} - 4\pi g'\vec{S}', \end{aligned}\right\} \quad (22)$$

where g_1 and g_2 have the same meanings as in III and g' is the constant with the same dimension as g_1, g_2, but is far smaller in magnitude. $\vec{M}$, M_0 and $\vec{S}$, $\vec{T}$ are the four vector and the six vector, which have been given by (38) and (39) respectively in §6, III. If we denote ~~the first~~ the first four (neutron) components of wave functions for the heavy particle by Φ and the remaining four (proton) components by Ψ, we can write alternatively

$$M_0 = \tilde{\Phi}\Psi \qquad \vec{M} = \tilde{\Phi}\rho_1\vec{\sigma}\Psi \quad (23)$$

$$\vec{S} = \tilde{\Phi}\rho_3\vec{\sigma}\Psi \qquad \vec{T} = -\tilde{\Phi}\rho_2\vec{\sigma}\Psi \quad (24)$$

Similarly, ~~the simplest expressions for the~~ the corresponding quantities ~~in the case of~~ for the light particle may well be given by the simplest expressions

$$M_0' = \tilde{\psi}\phi \qquad \vec{M}' = \tilde{\psi}\rho_1\vec{\sigma}\phi \quad (25)$$

$$\vec{S}' = \tilde{\psi}\rho_3\vec{\sigma}\phi \qquad \vec{T}' = -\tilde{\psi}\rho_2\vec{\sigma}\phi, \quad (26)$$

where ψ denotes four electron components of the wave functions for the light particle and ϕ four neutrino components. But, as will be clear later, this assumption leads only to the energy distribution of the β-ray of Fermi type, as in §4, I, so that other expressions including the first derivatives of the neutrino wave functions should also be taken into account, in order to obtain asymmetric distribution of Konopinski-Uhlenbeck type.[14] Hence, we start from a more general assumption including the first derivatives of the wave functions for the neutrino

$$\left.\begin{aligned} M_0' &= \tilde{\varphi}\{\lambda_1 - i\lambda_2 \frac{\hbar}{mc}\rho_2 \vec{\sigma}\,\mathrm{grad} + i\lambda_3 \frac{\hbar}{mc^2}\rho_3 \frac{\partial}{\partial t}\}\phi \\ \vec{M}' &= \tilde{\varphi}\{\lambda_1 \rho_1 \vec{\sigma} + i\lambda_2 \frac{\hbar}{mc}(\rho_2 \vec{\sigma}\frac{\partial}{\partial t} - \rho_3 \vec{\sigma}\times\mathrm{grad}) - i\lambda_3 \frac{\hbar}{mc}\rho_3\,\mathrm{grad}\}\phi \end{aligned}\right\} \quad (27)$$

$$\left.\begin{aligned} \vec{S}' &= \tilde{\varphi}\{\mu_1 \rho_3 \vec{\sigma} - i\mu_2 \frac{\hbar}{mc}\rho_1 \vec{\sigma}\times\mathrm{grad} - i\mu_3 \frac{\hbar}{mc}(\vec{\sigma}\frac{1}{c}\frac{\partial}{\partial t} + \rho_1\,\mathrm{grad})\}\phi \\ \vec{T}' &= \tilde{\varphi}\{-\mu_1 \rho_2 \vec{\sigma} - i\mu_2 \frac{\hbar}{mc}(\rho_1 \vec{\sigma}\frac{1}{c}\frac{\partial}{\partial t} + \mathrm{grad}) + i\mu_3 \frac{\hbar}{mc}\vec{\sigma}\times\mathrm{grad}\}\phi \end{aligned}\right\} \quad (28)$$

where λ's and μ's are complex numbers of the order of magnitude 1.

The field equations (21) and ~~other~~ their complex conjugate can be derived from the Lagrangian

$$\bar{L}_U = \iiint L_U\, dv \quad (29)$$

with

$$L_U = \frac{1}{4\pi}(\tilde{F}F - \tilde{G}G + \tilde{U}_0 U_0 - \tilde{U}U) + \frac{g}{\kappa}(\tilde{U}M - \tilde{U}_0 M_0 + U\tilde{M} - U_0\tilde{M}_0)$$
$$+ \frac{g'}{\kappa}(\tilde{U}M' - \tilde{U}_0 M_0' + U\tilde{M}' - U_0 \tilde{M}_0') \quad (30)$$

by considering $\vec{F}$, $\vec{G}$ as functions of U_0, $\vec{U}$ defined by (22). The Lagrangian for for the total system consisting of the U-field, the heavy and light particles becomes thus

$$\bar{L} = \iiint L\, dv \quad (31)$$

with

$$\left.\begin{aligned} L &= L_U + L_s + L_l \\ L_s &= \tilde{\Psi}(i\hbar\frac{\partial}{\partial t} + i\hbar c\vec{\alpha}\,\mathrm{grad} - \beta M_P c^2)\Psi \\ &\quad + \tilde{\Phi}(i\hbar\frac{\partial}{\partial t} + i\hbar c\vec{\alpha}\,\mathrm{grad} - \beta M_N c^2)\Phi \\ L_l &= \tilde{\varphi}(i\hbar\frac{\partial}{\partial t} + i\hbar c\vec{\alpha}\,\mathrm{grad} - \beta mc^2)\psi \\ &\quad + \tilde{\phi}(i\hbar\frac{\partial}{\partial t} + i\hbar c\vec{\alpha}\,\mathrm{grad} - \beta\mu c^2)\phi \end{aligned}\right\} \quad (32)$$

where μ is the mass of the neutrino.

14) Konopinski and Uhlenbeck, Phys. Rev. 48, 7, (1935).
7"4

27

數物報告

Now, if we go over into the quantum theory by introducing the canonical variables, we ~~are met~~ meet with serious difficulties of the same sort as those which were pointed out by Fierz [15]) with reference to the ordinary K.-U. theory of the β-decay. Thus, the quantization can be carried out consistently only as far as we consider g' very small and neglect all terms higher than the first order with respect to g'. In this approximation, the terms involving time derivatives in (27) and (28) can be replaced by those involving space derivatives alone by the help of the wave equations for the light particle, so that the expressions containing λ_3, μ_3 become equivalent to those containing λ_2, μ_2. Thus, the most general expressions reduce to

$$\left.\begin{aligned} M_0' &= \tilde{\psi}\left\{\lambda_1 - i\lambda_2 \frac{\hbar}{mc}\rho_2 \vec{\sigma}\,\mathrm{grad}\right\}\phi \\ \vec{M}' &= \tilde{\psi}\left\{(\lambda_1 + i\lambda_2\zeta)\rho_1\vec{\sigma} - \lambda_2\frac{\hbar}{mc}\rho_3\,\mathrm{grad}\right\}\phi \end{aligned}\right\} \qquad (33)$$

$$\left.\begin{aligned} \vec{S}' &= \tilde{\psi}\left\{\mu_1\rho_3\vec{\sigma} - i\mu_2\frac{\hbar}{mc}\rho_1\vec{\sigma}\times\mathrm{grad}\right\}\phi \\ \vec{T}' &= \tilde{\psi}\left\{(-\mu_1 - i\mu_2\zeta)\rho_2\vec{\sigma} + \mu_2\frac{\hbar}{mc}\vec{\sigma}\times\mathrm{grad}\right\}\phi \end{aligned}\right\} \qquad (34)$$

where $\zeta = \hbar/m.$

The variables canonically conjugate to U_0, $\vec{U}$ etc., Ψ etc., ψ etc. are defined by

$$\left.\begin{aligned} &U_0^+ = \tilde{U}_0^+ = 0, \quad U_x^+ = -\frac{1}{4\pi\varkappa c}\tilde{F}_x, \ \text{etc.} \quad \tilde{U}_x^+ = -\frac{1}{4\pi\varkappa c}F_x, \ \text{etc.}, \\ &\Psi^+ = i\hbar\tilde{\Psi}, \quad \Phi^+ = i\hbar\Phi, \quad \tilde{\Psi}^+ = \tilde{\Phi}^+ = 0, \\ &\psi^+ = i\hbar\tilde{\psi}, \quad \phi^+ = i\hbar\tilde{\phi}, \quad \tilde{\psi}^+ = \tilde{\phi}^+ = 0 \end{aligned}\right\} \qquad (35)$$

Hence, the Hamiltonian for the total system takes the form

$$\bar{H} = \iiint H\,dv \qquad (36)$$

15) Fierz, Helv. Phys. 10, 123, 284, (193~~8~~7).

$$H_{gg'} = \frac{4\pi g_1 g_1'}{\varkappa^2}\tilde{M}_0 M_0' + \frac{4\pi g_2 g_2'}{\varkappa^2}\tilde{S}S' \qquad (44)$$

with
$$H = \vec{U}^{\dagger}\frac{\partial \vec{U}}{\partial t} + U_0^{\dagger}\frac{\partial U_0}{\partial t} + \Psi^{\dagger}\frac{\partial \Psi}{\partial t} + \Phi^{\dagger}\frac{\partial \Phi}{\partial t} + \psi^{\dagger}\frac{\partial \psi}{\partial t} + \phi^{\dagger}\frac{\partial \phi}{\partial t} + \text{comp. conj.} - L$$
$$= H_S + H_L + H_U + H_g + H_{g'} + H_{g^2} + H_{gg'} \qquad (37)$$

$$H_S = \tilde{\Psi}(-i\hbar c\rho_1\vec{\sigma}\,\mathrm{grad} + \rho_3 M_P c^2)\Psi + \tilde{\Phi}(-i\hbar c\rho_1\vec{\sigma}\,\mathrm{grad} + \rho_3 M_N c^2)\Phi. \qquad (38)$$

$$H_L = \tilde{\psi}(-i\hbar c\rho_1\vec{\sigma}\,\mathrm{grad} + \rho_3 m c^2)\psi + \tilde{\phi}(-i\hbar c\rho_1\vec{\sigma}\,\mathrm{grad} + \rho_3 \mu c^2)\phi \qquad (39)$$

$$H_U = 4\pi\varkappa^2 c^2 \tilde{U}^{\dagger}U^{\dagger} + 4\pi c^2\,\mathrm{div}\tilde{U}^{\dagger}\,\mathrm{div}U^{\dagger} + \frac{1}{4\pi}\tilde{U}U + \frac{1}{4\pi\varkappa^2}\mathrm{curl}\tilde{U}\,\mathrm{curl}U \qquad (40)$$

$$H_g = \frac{4\pi g_1 c}{\varkappa}\mathrm{div}U^{\dagger}\cdot M_0 - \frac{g_1}{\varkappa}\tilde{U}M + \frac{g_2}{\varkappa}\mathrm{curl}U\cdot S + 4\pi g_2 c\,U^{\dagger}T + \text{comp. conj.} \qquad (41)$$

$$H_{g'} = \frac{4\pi g_1' c}{\varkappa}\mathrm{div}U^{\dagger}\cdot M_0' - \frac{g_1'}{\varkappa}\tilde{U}M' + \frac{g_2'}{\varkappa}\mathrm{curl}U\cdot S' + 4\pi g_2' c\,U^{\dagger}T' + \text{comp. conj.} \qquad (42)$$

$$H_{g^2} = \frac{4\pi g_1^2}{\varkappa^2}\tilde{M}_0 M_0 + \frac{4\pi g_2^2}{\varkappa^2}\tilde{S}S \qquad (43)$$

~~(44)~~ ~~$H_{gg'} = 4\pi g_1 g_1'$~~

In the quantum theory, the canonical variables U_x, $U_x^{\dagger}$ etc., Ψ, $\Psi^{\dagger}$ etc., ψ, $\psi^{\dagger}$, etc. should satisfy the commutation relations

$$\left.\begin{aligned} &U_x(\vec{r},t)U_x^{\dagger}(\vec{r}',t) - U_x^{\dagger}(\vec{r}',t)U_x(\vec{r},t) = i\hbar\,\delta(\vec{r},\vec{r}') \\ &U_x(\vec{r},t)U_y^{\dagger}(\vec{r}',t) - U_y^{\dagger}(\vec{r}',t)U_x(\vec{r},t) = 0 \\ &\text{etc.} \end{aligned}\right\} \qquad (45)$$

$$\left.\begin{aligned} &\Psi^{(i)}(\vec{r},t)\tilde{\Psi}^{(j)}(\vec{r}',t) + \tilde{\Psi}^{(j)}(\vec{r}',t)\Psi^{(i)}(\vec{r},t) = \delta_{ij}\,\delta(\vec{r},\vec{r}') \\ &\Psi^{(i)}(\vec{r},t)\Phi^{(j)}(\vec{r}',t) + \Phi^{(j)}(\vec{r}',t)\Psi^{(i)}(\vec{r},t) = 0 \\ &\Psi^{(i)}(\vec{r},t)\Psi^{(j)}(\vec{r}',t) + \Psi^{(j)}(\vec{r}',t)\Psi^{(i)}(\vec{r},t) = 0 \\ &\text{etc.} \end{aligned}\right\} \qquad (46)$$

$$\left.\begin{aligned}&\psi^{(i)}(\vec{r},t)\,\psi^{(j)}(\vec{r}',t)+\tilde{\psi}^{(j)}(\vec{r}',t)\,\psi^{(i)}(\vec{r},t)=\delta_{ij}\,\delta(\vec{r},\vec{r}')\\&\psi^{(i)}(\vec{r},t)\,\phi^{(j)}(\vec{r}',t)+\phi^{(j)}(\vec{r}',t)\,\psi^{(i)}(\vec{r},t)=0\\&\psi^{(i)}(\vec{r},t)\,\psi^{(j)}(\vec{r}',t)+\psi^{(j)}(\vec{r}',t)\,\psi^{(i)}(\vec{r},t)=0\end{aligned}\right\}\qquad(47)$$

etc.,

any other pair of variables commuting with each other. From these commutation relations and the Hamiltonian (37), we can derive all fundamental equations for the heavy and light particles as well as for the U-field.

§5. Theory of β-Disintegration.

According to the formulation made in the preceding section, the β-disintegration takes place in two ways. Firstly, a neutron in the nucleus can change into a proton with the simultaneous transition of the light particle from a neutrino state of negative energy to an electron state of positive energy, owing to the term $H_{gg'}$ in the Hamiltonian as given by (44), which expresses the direct interaction between the heavy and light particles as in the ordinary theory of β-disintegration. If we write the eigenfunctions (each with four components) for the neutron, proton, neutrino and electron by $u_n(\vec{r})$, $v_m(\vec{r})$, $\phi_\sigma(\vec{r})$ and $\psi_s(\vec{r})$ respectively and the corresponding energies (each including the proper energy) by W_n, W_m, $-E_\sigma$ and E_s respectively, the energy condition becomes

$$W_n - E_\sigma = W_m + E_s \qquad (48)$$

and the matrix element for the corresponding ~~expression~~ transition takes the form

$$H_\rho^{(1)} = \frac{4\pi g_1 g'}{\varkappa^2}\iiint \tilde{M}_0(\vec{r})M_0'(\vec{r})\,d\vec{r} + \frac{4\pi g_2 g'}{\varkappa^2}\iiint \tilde{S}(\vec{r})S'(\vec{r})\,d\vec{r} \qquad (49)$$

~~where~~ with

$$\left.\begin{aligned} &\tilde{M}_0(\vec{r}) = \tilde{v}_m(\vec{r})\, u_n(\vec{r}) \qquad \tilde{S}(\vec{r}) = \tilde{v}_m(\vec{r})\,\vec{\sigma}\, u_n(\vec{r}) \\ &M_0'(\vec{r}) = \tilde{\psi}_s(\vec{r})\left\{\lambda_1 - i\lambda_2 \frac{\hbar}{mc}\rho_2\vec{\sigma}\,\mathrm{grad}\right\}\phi_\alpha(\vec{r}) \\ &S'(\vec{r}) = \tilde{\psi}_s(\vec{r})\left\{\mu_1\rho_3\vec{\sigma} - i\mu_2 \frac{\hbar}{mc}\rho_1\vec{\sigma}\times\mathrm{grad}\right\}\phi_\alpha(\vec{r}), \end{aligned}\right\} \tag{50}$$

where nonrelativistic approximation is made for the heavy particle, so that each of the wave functions u_n, v_m has ~~only~~ only two components.

Secondly, a neutron can change into a ~~pro~~ proton, due to the term H_g as ~~as~~ given by (41), with the virtual emission of a heavy quantum of negative charge, which is absorbed subsequently by a neutrino of negative energy, due to the term $H_{g'}$ as given by (42), the latter changing thereby into an electron of positive energy. ~~The matrix element for these second order por process~~ The process can ~~be~~ take place in reverse order by emitting and absorbing a heavy quantum of positive energy. The matrix element for these second order processes can be calculated according to the usual perturbation theory. It is more convenient, however, to adopt an alternative method used in I and III. Namely, we consider the U-field of the form

$$\left.\begin{aligned} &\tilde{U}(\vec{r},t) = \tilde{U}(\vec{r})\, e^{-\frac{i\Delta W}{\hbar}t} \\ &U^{\dagger}(\vec{r},t) = U^{\dagger}(\vec{r})\, e^{-\frac{i\Delta W}{\hbar}t} \end{aligned}\right. \tag{51}$$

with $\Delta W = W_n - W_m$

corresponding to the transition of the heavy particle from the neutron state $u_n(\vec{r})$ with the energy W_n to the proton state $v_m(\vec{r})$ with the energy W_m. By a procedure similar to that in §7, III, we can show that ~~U and~~ $\tilde{U}(\vec{r})$ and $U^{\dagger}(\vec{r})$ should satisfy the second order equations

$$\left.\begin{aligned} &(\Delta - \varkappa^2)\,\tilde{U}(\vec{r}) = 4\pi g_2\,\mathrm{curl}\,\tilde{S}(\vec{r}) \\ &(\Delta - \varkappa^2)\,U^{\dagger}(\vec{r}) = -\frac{g_1}{\varkappa c}\,\mathrm{grad}\,\tilde{M}_0(\vec{r}), \end{aligned}\right\} \tag{52}$$

where $\omega^2 = \varkappa^2 - \left(\frac{\Delta W}{\hbar c}\right)^2$ and S and M_0 are given by (50). These equations can be integrated at once with the result

$$\left.\begin{aligned} \tilde{U}(\vec{r}_1) &= -g_2 \,\mathrm{curl}_1 \iiint \frac{e^{-\omega r}}{r} \tilde{S}(\vec{r}_2)\, d\vec{r}_2 \\ U^{\dagger}(\vec{r}_1) &= \frac{g_1}{4\pi\varkappa c} \,\mathrm{grad}_1 \iiint \frac{e^{-\omega r}}{r} \tilde{M}_0(\vec{r}_2)\, d\vec{r}_2 \end{aligned}\right\} \qquad (53)$$

where $r = |\vec{r}_1 - \vec{r}_2|$.

On the other hand, the wave equations for the light particle, which are derived from the Hamiltonian (37) and the commutation relations (47) are

$$\left.\begin{aligned} i\hbar\frac{\partial\psi}{\partial t} &= \{-i\hbar c\vec{\alpha}\,\mathrm{grad} + \beta mc^2\}\psi + H'\phi \\ i\hbar\frac{\partial\phi}{\partial t} &= \{-i\hbar c\vec{\alpha}\,\mathrm{grad} + \beta \mu c^2\}\phi + H''\psi \end{aligned}\right\} \qquad (54)$$

where

$$\begin{aligned} H' = {} & \frac{4\pi g'}{\varkappa}\left(c\,\mathrm{div}\,U^{\dagger} + \frac{g_1}{\varkappa}\tilde{M}_0\right)\left(\lambda_1 - i\lambda_2\frac{\hbar}{mc}\rho_2\vec{\sigma}\,\mathrm{grad}\right) \\ & - \frac{g'}{\varkappa}\tilde{U}\{(\lambda_1 + i\lambda_2\zeta)\rho_1\vec{\sigma} - \lambda_2\frac{\hbar}{mc}\rho_3\,\mathrm{grad}\} \\ & + \frac{g'}{\varkappa^2}(\mathrm{curl}\,\tilde{U} + 4\pi g_2\tilde{S})\left(\mu_1\rho_3\vec{\sigma} - i\mu_2\frac{\hbar}{mc}\rho_1\vec{\sigma}\times\mathrm{grad}\right) \\ & + 4\pi g' c\, U^{\dagger}\{-(\mu_1 - i\mu_2\zeta)\rho_2\vec{\sigma} + \mu_2\frac{\hbar}{mc}\vec{\sigma}\times\mathrm{grad}\} \end{aligned} \qquad (55)$$

is the term responsible for the emission of the negative electron, while H'' with the similar form is responsible for the emission of the positive electron. The required matrix element for the simultaneous transitions of the heavy and light particles from the states $u_n(\vec{r})$ and $\phi_\sigma(\vec{r})$ to $v_m(\vec{r})$ and $\psi_s(\vec{r})$ respectively due to the virtual emission and absorption of heavy quanta can be obtained by inserting (53) in (55). The result is

$$\begin{aligned} H_\beta^{(2)} = {} & \frac{g_1 g'}{\varkappa}\int\cdots\int \mathrm{div}_1\,\mathrm{grad}_1\left(\frac{e^{-\omega r}}{r}\right)\tilde{M}_0(\vec{r}_2)\, M_0'(\vec{r}_1)\, d\vec{r}_1\, d\vec{r}_2 \\ & + \frac{g_2 g'}{\varkappa}\int\cdots\int \mathrm{curl}_1\left\{\frac{e^{-\omega r}}{r}\tilde{S}(\vec{r}_2)\right\} M'(\vec{r})\, d\vec{r}_1\, d\vec{r}_2 \\ & + \frac{g_1 g'}{\varkappa}\int\cdots\int \mathrm{grad}_1\left(\frac{e^{-\omega r}}{r}\right)\tilde{M}_0(\vec{r}_2)\, T'(\vec{r}_1)\, d\vec{r}_1\, d\vec{r}_2 \\ & - \frac{g_2 g'}{\varkappa}\int\cdots\int \mathrm{curl}_1\,\mathrm{curl}_1\left\{\frac{e^{-\omega r}}{r}\tilde{S}(\vec{r}_2)\right\} S'(\vec{r}_1)\, d\vec{r}_1\, d\vec{r}_2 \end{aligned} \qquad (56)$$

where

$$\left.\begin{aligned} M'(\vec{r}_1) &= \tilde{\psi}_s\{(\lambda_1 + i\lambda_2\zeta)\rho_1\vec{\sigma} - \lambda_2\frac{\hbar}{mc}\rho_3\,\mathrm{grad}\}\phi_\sigma \\ T'(\vec{r}) &= \tilde{\psi}_s\{-(\mu_1 - i\mu_2\zeta)\rho_2\vec{\sigma} + \mu_2\frac{\hbar}{mc}\vec{\sigma}\times\mathrm{grad}\}\phi_\sigma \end{aligned}\right\} \qquad (57)$$

If we perform partial integrations by using the relation

$$\Delta\left(\frac{e^{-\omega r}}{r}\right) = \omega^2 \frac{e^{-\omega r}}{r} - 4\pi\delta(\vec{r}) \tag{58}$$

and add the result to (49), we obtain the resultant matrix element

$$H_\beta = H_\beta^{(1)} + H_\beta^{(2)} = g_1 g' \int\cdots\int \frac{e^{-\omega r}}{r} \tilde{M}_0(\vec{r}_2)\left\{\frac{\omega^2}{\kappa^2} M_0'(\vec{r}_1) - \frac{1}{\kappa}\,\mathrm{div}\,T'(\vec{r}_1)\right\} d\vec{r}_1\, d\vec{r}_2$$
$$+ g_2 g' \int\cdots\int \frac{e^{-\omega r}}{r} \tilde{S}(\vec{r}_2)\left\{\frac{\omega^2}{\kappa^2} S'(\vec{r}_1) + \frac{1}{\kappa}\,\mathrm{curl}\,M'(\vec{r}_1) - \frac{1}{\kappa^2}\,\mathrm{grad\,div}\,S'(\vec{r}_1)\right\} d\vec{r}_1\, d\vec{r}_2 \tag{59}$$

Now, the energy of disintegration ΔW is always small compared with $m_\nu c^2$, so that ω is approximately equal to κ. Hence, if we neglect the terms involving $\frac{1}{\kappa}\frac{\partial}{\partial x}$ etc., which are smaller by a factor $\Delta W/m_\nu c^2$ than the remaining terms, (59) reduces to

$$H_\beta = g_1 g' \int\cdots\int \frac{e^{-\kappa r}}{r} \tilde{M}_0(\vec{r}_2) M_0'(\vec{r}_1)\, d\vec{r}_1\, d\vec{r}_2$$
$$+ g_2 g' \int\cdots\int \frac{e^{-\kappa r}}{r} \tilde{S}(\vec{r}_2) S'(\vec{r}_1)\, d\vec{r}_1\, d\vec{r}_2, \tag{60}$$

where $\tilde{M}_0$, M_0', $\tilde{S}$, S' are given by (50). Further, the wave numbers of the emitted electron and antineutrino are small compared with κ, so that the integration in (60) with respect to the light particle can be carried out by considering $\frac{e^{-\kappa r}}{r}$ as $\frac{4\pi}{\kappa^2}\delta(\vec{r})$.[16] In this approximation, (60) becomes

$$H_\beta = f_1 \iiint (\tilde{\Psi}_s \phi_\sigma)(\tilde{v}_m u_n)\, d\vec{r} + f_2 \iiint \tilde{\Psi}_s \rho_2 (\vec{\sigma}\vec{\eta}) \phi_\sigma \cdot (\tilde{v}_m u_n)\, d\vec{r}$$
$$+ f_1' \iiint (\tilde{\Psi}_s \rho_3 \vec{\sigma} \phi_\sigma)(\tilde{v}_m \vec{\sigma} u_n)\, d\vec{r} + f_2' \iiint (\tilde{\Psi}_s \rho_1 [\vec{\sigma} \times \vec{\eta}] \phi_\sigma)(\tilde{v}_m \vec{\sigma} u_n)\, d\vec{r} \tag{61}$$

where

$$\left.\begin{aligned} \vec{\eta} &= -\frac{i\hbar}{mc}\,\mathrm{grad} \\ f_1 &= \frac{4\pi g_1 g' \lambda_1}{\kappa^2} \qquad & f_2 &= \frac{4\pi g_1 g' \lambda_2}{\kappa^2} \\ f_1' &= \frac{4\pi g_2 g' \mu_1}{\kappa^2} \qquad & f_2' &= \frac{4\pi g_2 g' \mu_2}{\kappa^2} \end{aligned}\right\} \tag{62}$$

16) Compare I, §4.

The constants f's have the same dimension as g in the original theory of Fermi.[17] The terms in (63) involving f_1 and f_1' are the matrix elements of Fermi type, whereas those involving f_2 and f_2' are of K.-U. type.[18] The latter, however, differ from the matrix elements, which are in use in the current theory, for example, by Hoyle,[19] in that only the space derivatives appear in our case, while the time derivatives are also included in the current theory. However, the disappearance of the time derivatives in our theory is an inevitable consequence of the quantization of the wave field for the light particle and probably the same result will be obtained, if the quantization will be carried out in the scheme of the current theory. Besides this, our formulation is more restricted than that of Hoyle in that the quantities such as pseudo-scalar and pseudo-vector are excluded from the beginning.

Hereafter, the calculation can be made on the same line as in the current theory and the energy distribution function and the decay constant can be determined without any difficulty. Especially, if we neglect the Coulomb force between the nucleus and the electron, the probability per unit time that the electron with the energy between E and E+dE is emitted is given by

$$\begin{aligned} P_\beta dE = {} & \frac{64\pi^4}{h^6 c^6} E\sqrt{E^2-m^2c^4}\,(\Delta W-E)\sqrt{(\Delta W-E)^2-\mu^2c^4}\,dE \times \\ & \Big[\Big\{1-\frac{m\mu c^4}{E(\Delta W-E)}\Big\}\Big\{f_1^2\Big|\int\tilde{v}_m u_n d\vec{r}\Big|^2+f_1'^2\Big|\int\tilde{v}_m\vec{\sigma}u_n d\vec{r}\Big|^2\Big\} \\ & +\frac{\{(\Delta W-E)^2-\mu^2c^4\}}{m^2c^4}\Big\{1+\frac{m\mu c^4}{E(\Delta W-E)}\Big\}\Big\{f_2^2\Big|\int\tilde{v}_m u_n d\vec{r}\Big|^2+\frac{2}{3}f_2'^2\Big|\int\tilde{v}_m\vec{\sigma}u_n d\vec{r}\Big|^2\Big\} \\ & +\frac{\{(\Delta W-E)^2-\mu^2c^4\}}{E(\Delta W-E)}\Big\{i(\tilde{f}_1 f_2-f_1\tilde{f}_2)\Big|\int\tilde{v}_m u_n d\vec{r}\Big|^2+\frac{2i(\tilde{f}_2 f_1'-f_2'\tilde{f}_1')}{3}\Big|\int\tilde{v}_m\vec{\sigma}u_n d\vec{r}\Big|^2\Big\}\Big] \end{aligned} \quad (63)$$

17) Fermi, loc. cit.
18) The appearance of the terms involving the spin of the heavy particle is in accord with the generalization, in the scheme of the current theory, made by Gamow and Teller, Phys. Rev. 49, 895, (1936).
19) Hoyle, Proc. Camb. Phil. Soc. 33, 277, (1937); Proc. Roy. Soc. A 166, (1938), 249.

Especially, if we take $\lambda_2 = \mu_2 = 0$, i.e. $f_2 = f_2' = 0$, we obtain pure Fermi distribution, while, if we take $\lambda_1 = \mu_1 = 0$, i.e. $f_1 = f_1' = 0$, we obtain pure K.-U. distribution. In gerneral, the distribution function can be expressed in the form

$$\left\{A + B(\varepsilon_0-\varepsilon)^2 + C\frac{\varepsilon_0-\varepsilon}{\varepsilon} - (\zeta A + \zeta^3 B + \zeta^2 C)\frac{1}{\varepsilon(\varepsilon_0-\varepsilon)}\right\} \times \varepsilon\sqrt{\varepsilon^2-1}\,(\varepsilon_0-\varepsilon)\sqrt{(\varepsilon_0-\varepsilon)^2-\zeta^2} \qquad (64)$$

if we measure the energy in units of mc^2, i.e.

$$E = \varepsilon mc^2 \qquad \Delta W = \varepsilon_0 mc^2 \qquad \mu c^2 = \zeta mc^2. \qquad (65)$$

A, B, C are quantities independent of the energies of the electron and the neutrino and are defined by

$$\left.\begin{aligned} A &= a_1\left|\int \tilde{v}_m u_n d\vec{r}\right|^2 + a_2\left|\int \tilde{v}_m \vec{\sigma} u_n d\vec{r}\right|^2 \\ B &= b_1\left|\int \tilde{v}_m u_n d\vec{r}\right|^2 + b_2\left|\int \tilde{v}_m \vec{\sigma} u_n d\vec{r}\right|^2 \\ C &= c_1\left|\int \tilde{v}_m u_n d\vec{r}\right|^2 + c_2\left|\int \tilde{v}_m \vec{\sigma} u_n d\vec{r}\right|^2 \end{aligned}\right\} \qquad (66)$$

where $a_1, b_1, c_1, a_2, b_2, c_2$ are constants, which can be expressed by f_1, f_2, f_1', f_2', satisfying the conditions

$$\left.\begin{aligned} & a_1+\zeta^2 b_1,\quad a_2+\zeta^2 b_2,\quad b_1,\quad b_2 > 0 \\ & |c_1-\zeta b_1| \leqq \sqrt{b_1(a_1+\zeta^2 b_1)} \\ & |c_2-\zeta b_2| \leqq \tfrac{2}{3}\sqrt{b_2(a_2+\zeta^2 b_2)}. \end{aligned}\right\} \qquad (67)$$

Detailed discussions of the subject will be made by Sakata and Tanikawa in a subsequent paper.

§6. Spontaneous Annihilation of the Heavy Quanta.

In §8, III, the probability of annihilation of a heavy quantum with the positive (or negative) charge by emitting a positive (or negative) electron and a neutrino (or an antineutrino) was calculated on the assumption that the interaction between the heavy quantum and the light particle could be given by the simplest expressions such as (25) and (26) in §4 of this paper, which, however, was found to result in Fermi distribution for β-ray. In this case, the mean life time of the heavy quantum at rest was about 0.5×10^{-6} sec, assuming $m_u = 100\,m$. This is in qualitative agreemnet with the value 2×10^{-6} sec obtained by Euler as the result of detailed analysis of the cosmic ray.[20] It is a pity that there was an error by a factor 2 in our calculation and the correct value is to be 0.25×10^{-6} sec instead of 0.5×10^{-6} sec,[21] which is a little too small compared with the value of Euler.

Under these circumstances, it is important to perform the calculation by assuming more general form for the interaction as given in the previous sections. Thus, the probability per unit time of the annihilation of the heavy quantum with the energy E turns out to be

$$w = \frac{g'}{\hbar c}\frac{m_u c^2}{\hbar}\frac{m_u c^2}{E}\left\{\frac{2}{3}\left|\lambda_1 + i\frac{1-\zeta}{2}\lambda_2 - i\frac{m_u}{2m}\mu_2\right|^2 + \frac{1}{3}\left|\mu_1 + i\frac{1-\zeta}{2}\mu_2 - i\frac{m_u}{2m}\lambda_2\right|^2\right\}, \qquad (68)$$

which reduces to (67) in §8, III multiplied by 2, if we take $\lambda_2 = \mu_2 = 0$, $\lambda_1 = \mu_1 = 1$, $g_1' = g_2' = g'$, as it should be. The details of the calculation will be described in a paper by Sakata and Tanikawa.

---

20) Euler, Naturwiss. 26, 382, (1938).
21) The energy E in (62), §8, III should be $m_u c^2/2$ instead of $m_u c^2$, so that (63) becomes

$$dw_0 = \frac{m_u c}{16\pi^2\hbar^4}\sum |V|^2 d\Omega.$$

Thus, w_0 and w in III should be multiplied by 2, while the mean life time $\tau = 1/w$ should be divided by 2.

36 數物報告

Especially, in the case of pure K.-U. interaction, i,e. $\lambda_1=\mu_1=0,\ \lambda_2=\mu_2=1$, we obtain

$$w=\frac{g'}{4\hbar c}\frac{m_u c^2}{\hbar}\frac{m_u c^2}{E}\left(\frac{m_u}{m}\right)^2\left\{\frac{2}{3}|\mu_2|^2+\frac{1}{3}|\lambda_2|^2\right\}, \tag{69}$$

which shows that the mean life time $\tau = 1/w$ of the heavy quantum is shorter by a factor $(m/m_u)^2$ than that in the case of pure Fermi interaction, i.e. $\lambda_1=\mu_1=1,\ \lambda_2=\mu_2=0$. The numerical values of τ and the mean free path in these cases are summarized in Table 1 for several values of the kinetic energy $E - m_u c^2$, assuming $g' = 4\times10^{-17}$, $m_u = 200$ m[22] (instead of 100 m) and $\zeta = 0$.

Table 1.

Kinetic Energy		0	10^9	10^{10}	10^{11}	eV
Mean Life Time	Fermi	1.3×10^{-7}	1.3×10^{-6}	1.3×10^{-5}	1.3×10^{-4}	sec
	K.-U.	1.3×10^{-11}	1.3×10^{-10}	1.3×10^{-9}	1.3×10^{-8}	sec
Mean Free Path	Fermi	———	0.4	4	40	km
	K.-U.	———	4	40	400	cm

Thus, the mean free path is very much too short to account for the hard cosmic rays as heavy quanta, as long as the interaction of them with the light particles involves the terms of K.-U. type in appreciable amount.[23] In this way, if we want to adopt the scheme containing the interaction of K.-U. type in the theory of the U-field, we meet with the above substantial difficulty in addition to the formal difficulty connected with the quantization of the light particle wave as discussed in §4. On the contrary, in the case of pure Fermi interaction, there is no serious discrepancy between theory and experiment with regard to the cosmic ray phenomena, while it is impossible for the time being to account for the asymmetry of the energy distribution of the β-ray.

22) Recent experiments seem to indicate a value for m_u nearer to 200 m than to 100 m. Compare Ehrenfest, C. R. 206, ~~428~~, (1938); Williams and Pickup, Nature 141, 684, (1938); Corson and Brode, Phys. Rev. 53, 773, (1938).
23) The authors wish to express their thanks to Prof. W. Heisenberg, who called our attention to the importance of this point.

In this connection, a cloud chamber photograph obtained by Ehrenfest[24] is very interesting. According to him, a track of the heavy electron of positive charge with the mass about about 200 m ends at the wall and from there starts a positron track with $H\rho = 1.6 \times 10^5$ gauss.cm, i.e. with the energy about 100 mc^2. This is just the phenomena to happen, when the heavy quantum with the mass about 200 m slowed down in the wall annihilates into a positron and a neutrino each with the energy about 100 mc^2. Moreover, according to Blackett,[25] it is likely that most of the hard particles in the cosmic ray transform into the soft particles, when they are slowed down. This phenomenon finds its natural explanation in the above annihilation process. Further, it is possible that the neutrino or the antineutrino accompanying this process forms a constituent of the extremely penetrating component of the cosmic ray.

§7. Creation of the Heavy Quanta in Matter.

As shown in §8, III and in the previous section, the life time of the heavy quantum is very short, so that those quanta which are found in the cosmic ray on the sea level should be the secondary created in the atmosphere. This conclusion seems to be in accord with the recent experimental result,[26] at least, qualitatively. In order to settle the matter, it is important to discuss in detail the problem of the creation in matter of heavy quanta by the soft component of the cosmic ray. The probability of creation of a pair of heavy quanta in the Coulomb field of the nucleus by a light quantum of energy larger than $2m_u c^2$ is a little too small to account for the main part of the hard cosmic ray. There is, however, another possibility of the creation. Namely,

24) Ehrenfest, loc. cit.
25) Blackett, Proc. Roy. Soc. A 165, 11, (1938).
26) Bowen, Millikan and Neher, Phys. Rev. 52, 80, (1937); 53, 217, 855, (1938).

In this connection, a cloud chamber photograph obtained by Ehrenfest[24] is very interesting. According to him, a track of the heavy electron of positive charge with the mass about about 200 m ends at the wall and from there starts a positron track with $H\rho = 1.6 \times 10^5$ gauss.cm, i.e. with the energy about 100 mc^2. This is just the phenomena to happen, when the heavy quantum with the mass about 200 m slowed down in the wall annihilates into a positron and a neutrino each with the energy about 100 mc^2. Moreover, according to Blackett,[25] it is likely that most of the hard particles in the cosmic ray transform into the soft particles, when they are slowed down. This phenomenon finds its natural explanation in the above annihilation process. Further, it is possible that the neutrino or the antineutrino accompanying this process forms a constituent of the extremely penetrating component of the cosmic ray.

§7. Creation of the Heavy Quanta in Matter.

As shown in §3, III and in the previous section, the life time of the heavy quantum is very short, so that those quanta which are found in the cosmic ray on the sea level should be the secondary created in the atmosphere. This conclusion seems to be in accord with the recent experimental result,[26] at least, qualitatively. In order to settle the matter, it is important to discuss in detail the problem of the creation in matter of heavy quanta by the soft component of the cosmic ray. The probability of creation of a pair of heavy quanta in the Coulomb field of the nucleus by a light quantum of energy larger than $2m c^2$ is a little too small to account for the main part of the hard cosmic ray. There is, however, another possibility of the creation. Namely,

24) Ehrenfest, loc. cit.
25) Blackett, Proc. Roy. Soc. A 165, 11, 1938.
26) Bowen, Millikan and Neher, Phys. Rev. 52, 80, 1937; 53, 217, 855, 1938.

a light quantum with the energy larger than $m_u c^2$ is absorbed by the nucleus and a heavy quantum with the negative (or positive) charge is emitted, the nuclear charge being increased (or decreased) thereby by one. This can be decomposed into following two processes. Consider a light quantum and a neutron, for example, in the initial state A. The neutron can change into a proton by emitting a heavy quantum with the negative charge (intermediate state Z). Subsequently, the latter absorbs the light quantum, so that a proton and a heavy quantum remain in the final state B. Alternatively, the light quantum is annihilated by producing a pair of heavy quanta (intermediate state Z') and subsequently the neutron changes into the proton by absorbing the heavy quantum with the positive charge, so that a proton and a heavy quantum with the negative charge remain again in the final state.

The calculation of the cross section of this process according to the vectorial theory for the U-field is rather complicated and will be made by Kobayasi in other place.[27] In this paper, we want to content ourselves with the calculation based on the scalar U-field theory as developed in I and II.

Now, the matrix element for the above transition is given by

$$H_{AB} = \sum_Z \frac{(A|H_g|Z)(Z|H_e|B)}{E_A - E_Z} + \sum_{Z'} \frac{(A|H_e|Z')(Z'|H_g|B)}{E_A - E_{Z'}} \qquad (70)$$

where E_A, E_Z, $E_{Z'}$ denote the energies of the states A, Z, Z' respectively and the summations extend over all possible intermediate states Z and Z'

27) Very recently, a paper by Heitler (Proc. Roy. Soc. A 166, 529 (1938)) dealing with the same subject came to our notice.

respectively. H_g is the energy of interaction between the heavy quantum and the heavy particle and is given by (14) in II, i.e.

$$H_g = -i\hbar g c \sum_k \sqrt{\frac{2\pi}{E_k}} \{(a_k^* - b_k) e^{-i\vec{k}\vec{r}} Q^* - (a_k - b_k^*) e^{i\vec{k}\vec{r}} Q\} \beta \qquad (71)$$

where (a_k^*, b_k^*) and (a_k, b_k) are the operators increasing and decreasing respectively the number of the heavy quanta with the momentum $\hbar\vec{k}$ and the energy E_k. Q^* and Q are the operators changing the proton into the neutron and vice versa respectively and β is the usual Dirac matrix for the heavy particle. H_e is the energy of interaction between the heavy and light quanta and is given by (53) in the paper of Pauli and Weisskopf,[28] i.e.

$$H_e = \frac{e\hbar c^2}{2} \sum_k \sum_{k'} \sum_{k_0} \sqrt{\frac{2\pi}{E_k E_{k'}}} (\vec{e}_\lambda, \vec{k} + \vec{k'}) \times$$
$$\{ \sqrt{\frac{\hbar n_\lambda}{c k_0}} q_\lambda (a_k^* a_{k'} + b_k b_{k'}^* - a_k^* b_{k'}^* - b_k a_{k'}) e^{i(\vec{k}_0 - \vec{k} - \vec{k'})\vec{r}}$$
$$+ \sqrt{\frac{\hbar(n_\lambda + 1)}{c k_0}} q_\lambda^* (a_k a_{k'}^* + b_k^* b_{k'} - a_k b_{k'} - b_k^* a_{k'}^*) e^{-i(\vec{k}_0 - \vec{k} - \vec{k'})\vec{r}} \}, \qquad (72)$$

where n_λ is the number of the light quanta with the momentum $\hbar\vec{k}_0$, the energy $E_0 = \hbar c k_0$ and the polarization λ (= 1 or 2), $\vec{e}_\lambda$ being the unit vector in the direction of polarization. q_λ^* and q_λ are the operators increasing and decreasing n_λ by one respectively.

If we consider the system in a unit cube and perform the summations in (70) by inserting (71), (72), we obtain the matrix element for the transition from the state, in which there are a light quantum with the momentum $\hbar\vec{k}_0$ and a neutron at rest, to the state, in which there are a heavy quantum with the momentum $\hbar\vec{k}$ and a proton with the momentum $\hbar\vec{k}' = \hbar(\vec{k}_0 - \vec{k})$,

$$H_{AB} = \frac{2\pi i g e (\hbar c)^3}{E_{k'} \sqrt{E_k E_0}} \left(\frac{1}{Mc^2 - W - E_{k'}} + \frac{1}{E_0 - E_k - E_{k'}} \right) (v_q^* \beta u_0)(\vec{e}_\lambda, \vec{k} - \vec{k'}) \qquad (73)$$

28) Pauli and Weisskopf, Helv. Phys. 7, 709, (1934).

where M is the mass of the heavy particle and W the energy of the proton. u_0 and v are constant spinors for the neutron and the proton respectively. By taking the average of $|H_{AB}|^2$ with respect to the directions of polarization of the light quantum and of the spin of the neutron and further by performing the summation with respect to the directions of the spin of the proton, we obtain

$$\overline{|H_{AB}|^2} = \pi^2 (ge)^2 (\hbar c)^6 \frac{k^2 \sin^2\theta \left(1 + \frac{Mc^2}{W}\right)}{E_k E_0 \{ E_{k'}^2 - (E_0 - E_k)^2 \}^2} \qquad (74)$$

where θ is the angle between the vectors $\vec{k}_0$ and $\vec{k}$.

Now, the differential cross section that a light quantum with the energy E_0 is absorbed by a neutron at rest and a heavy quantum is emitted in a solid angle $d\Omega$ making an angle θ with the direction of incidence is given by

$$d\phi = \frac{2\pi}{\hbar c} \overline{|H_{AB}|^2} \frac{\partial k}{\partial (W + E_k)} \frac{k^2}{(2\pi)^3} d\Omega$$

$$= (ge)^2 (\hbar c)^3 \frac{k^4 (W + Mc^2) \sin^2\theta \, d\Omega}{E_0 \{ E_{k'}^2 - (E_0 - E_k)^2 \}^2 \{ E_k (k - k_0 \cos\theta) + W k \}}, \qquad (75)$$

where $E_{k'}$ is the energy of the heavy quantum with the momentum $\hbar \vec{k}'$ in the intermediate state, k, E_k, W and $E_{k'}$ being functions of θ.

The total cross section ϕ can be obtained by integrating (75) over all directions. Especially, when E_0 is the order of Mc^2, it is given approximately by

$$\phi \doteqdot 4\pi (ge)^2 \frac{1}{E_0^2} \log \frac{2E_0}{m_u c^2}, \qquad (76)$$

whereas it becomes approximately

$$\phi \doteqdot \frac{\pi}{2} \frac{(ge)^2}{Mc^2 \cdot E_0}, \qquad (77)$$

when E_0 is large compared with Mc^2. Thus, the probability of creation of the heavy quantum by the above process decreases with the energy E_0 of the light quantum, when E_0 is large compared with $m_u c^2$. On the contrary, it increases with E_0 in the vectorial theory for the U-field, as will be shown in another paper by Kobayasi. Moreover, in the latter theory, the cross sections for the higher order processes increase more rapidly with the energy, so that we can expect, as emphasized by Heisenberg, the explosion consisting of heavy quanta and heavy particles, although the vectorial theory in the present form can not be applied to the region, where the energies of the particles concerned is large compared with $m_u c^2$.[29)]

The cross section ϕ as given by (76) or (77) refers to a single heavy particle, so that the cross section of the nucleus with the mass number A is A times the above value (76) or (77). Thus, for $E_0 = Mc^2 \doteqdot 10^9$eV, ϕ is the order of 10^{-29}cm^2, so that $A\phi$ is about 2×10^{-27}cm^2 for Pb and 1.4×10^{-28}cm^2 for air (A = 14).

On the other hand, the cross section ϕ' for the creation of pair of heavy quanta by a light quantum with the energy E_0 ($> 2m_u c^2$) in the field of the nucleus with the atomic number Z can be written down at once by changing the mass m into m_u in the corresponding expression for the Bose electron calculated by Pauli and Weisskopf.[30)] Thus, we obtain

$$\phi' = Z^2 \frac{e^2}{\hbar c}\left(\frac{e^2}{m_u c^2}\right)^2 \left(\frac{16}{9}\log\frac{2E_0}{m_u c^2} - \frac{104}{27}\right), \qquad (78)$$

---

29) Recently, these problems were discussed by Heitler (loc. cit.).
30) Pauli and Weisskopf, loc. cit. p. 729.

For energy $E_0 = Mc^2 \doteqdot 10^9$eV, ϕ' is the order of $Z^2 \times 2 \times 10^{-32}$cm², which is about 2×10^{-28}cm² for Pb and 10^{-30}cm² for air (Z=7). Thus, the cross section ϕ^z for $E_0 = 10^9$eV is larger by a factor 10 for Pb than the corresponding cross section ϕ'. This factor is still larger for air. Hence, main portion of the hard cosmic rays observed on the sea level can be attributed to the secondary heavy quanta created by the high energy γ-rays by the former process. It is also possible that a part of them are the heavy quanta and heavy particles, which are created by the explosion above discussed.[31]

§8. Absorption of the Heavy Quanta in Matter.

In §5, II, we calculated the cross section for the scattering of the high energy heavy quantum by the heavy particle and that for the absorption by the nucleus with subsequent emission of a heavy particle ~~in~~ according to the scalar theory. It turned out that both of them were so small that they had little contribution to the absorption of the heavy quantum in matter, as long as the energy is large compared with $m_U c^2$. Recently, Bhabha[32] showed that the cross section for the former process increases rapidly with the energy large compared with Mc^2 according to the vectorial theory. However, ~~but~~ it is probable that the theory is not applicable to such a high energy region without serious modification, ~~it~~ so that it is difficult for the time being to draw definite conclusion about the absorption and energy loss of the high energy heavy quanta.

---

31) Recent experiments indicate ~~show that~~ the presence of the large shower consisting of penetrating particles. See, for example, Schmeizer, Naturwiss. **25**, 137, (1937); Schmeizer and Bothe, ibid. **25**, 669, 833, (1937); Maier-Leibnitz, ibid. **26**, 217, (1938); Auger, Maze and Grivet-Meyer, C. R. **206**, 1721, (1938).

32) Bhabha, Proc. Roy. Soc. loc. cit.

Besides these processes, the heavy quantum can be absorbed by the heavy particle with subsequent emission of a light quantum as the process inverse to that considered in §7. In the case of the scalar theory, the calculation of the cross section can be performed in like manner. Thus, the total cross section of a neutron, for example, for the heavy quantum with the positive charge and the energy E_k of the order of $Mc^2 \doteqdot 10^9$eV is about

$$\phi \cong 8\pi \frac{(ge)^2}{E_k^2} \log \frac{2E_k}{m_u c^2} \qquad (79)$$

For E_k large compared with Mc^2, ϕ decreases as $1/E_k$. This cross section is also very small and can not have appreciable contribution to the absorption of the high energy heavy quantum. For example, if we consider this process alone, the mean free path of the quantum with the negative charge and energy about 10^9eV in Pb is the order of 2×10^4cm. The corresponding calculation in the case of the vectorial theory will be made in other place by Kobayasi.[33]

§9. Spin and Magnetic Moment of the Heavy Quantum.

As shown in §3, III, it is reasonable to consider that the spin of the heavy quantum is 1 inasmuch as it has the extra degree of freedom characterized by the suffix j, which can take either of three values. On the other hand, the spin can be defined as a part of the moment of momentum, which does not contain space coordinates explicitly, according to Proca.[34] Durandin and Erschow, however, showed that this lead to the value 0 for the spin.[35] More unambiguous way of defining the spin is to construct an infinitesimal operator

---

33) See also Heitler, loc. cit.
34) Proca, Jour. d. Phys. 7, 347, (1936).
35) Durandin and Erschow, Phys. Zeits. d. Sowj. 12, 466, (1937).

然しながら Proca の definition に於ては Orbital angular momentum + spin は constant of motion にならない事がわかった。

~~for/the/rotation/of~~

for the rotation of the U-field in space. This can be done by a method similar to that of Heisenberg and Pauli for the electromagnetic field.[36]

Consider the infinitesimal rotation

$$x_i \to x_i + \varepsilon s_{ik} x_k, \qquad s_{ik} = -s_{ki}, \quad i,k = 1,2,3, \tag{80}$$

where x_i ~~(i = 1, 2, 3)~~ are space coordinates and ε is a small parameter. The symbol for the summation with respect to k ~~(= 1, 2, 3)~~ in (80) is omitted as usual. The vector U is transformed thereby

$$U_i \to U_i + \varepsilon s_{ik} U_k - \varepsilon \frac{\partial U_i}{\partial x_k} s_{kl} x_l \tag{81}$$

These expressions show that the total angular momentum for the heavy quantum $\vec{M}$, which is a constant of motion, consists of the orbital angular momentum

$$\vec{m} = -ih(\vec{r} \times \mathrm{grad})$$

and the spin angular momentum $\vec{s}$ with the components

$$s_x = \hbar\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad s_y = \hbar\begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad s_z = \hbar\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{82}$$

each of which has eigenvalues 1, 0, -1. In this sense, the heavy quantum can be said to have the spin 1.

Now, if we go over into the quantized field, the law of transformation is given by

$$U_i \to U_i + \frac{2\pi\varepsilon i}{\hbar}(\bar{\Lambda} U_i - U_i \bar{\Lambda}) \tag{83}$$

with

$$\left.\begin{aligned} \bar{\Lambda} &= \iiint \Lambda \, dv \\ \Lambda &= \left(s_{jk} U_k - \frac{\partial U_j}{\partial x_i} s_{ik} x_k\right) U_j^{+}, \end{aligned}\right\} \tag{84}$$

---

36) Heisenberg and Pauli, loc. cit.

とこの definition では $\vec{m}+\vec{s}$ が const of motion になる

which shows that the orbital angular momentum is given by

$$\vec{m} = -\iiint \{ U_i^{+} (\vec{r} \times \mathrm{grad}) U_i + \mathrm{comp.\ conj.} \} dv, \qquad (85)$$

while the spin angular momentum is given by

$$\vec{S} = -\iiint \{ (\vec{U}^{+} \times \vec{U}) + \mathrm{comp.\ conj.} \} dv. \qquad (86)$$

These expressions can be expressed by normal coordinates according to the general method of §3, III. When there are heavy quanta moving in the x direction with the wave number k, the z-component, for example, of the spin takes the form

$$S_z = \hbar \left(-a_k^{*(1)} b_k^{*(1)} + a_k^{*(2)} b_k^{*(2)} + b_k^{(1)} a_k^{(1)} - b_k^{(2)} a_k^{(2)} \right) \qquad (87)$$

where $a_k^{(1)}$ and $a_k^{(2)}$, for example, are the operators which decrease the numbers of quanta by one circularly polarized in xy-plane with the positive charge. Thus, (87) involves only those operators which increase or decrease the number of quanta by two, so that there is no term corresponding to the static spin. The results are quite similar for s_x and s_y. This rather queer consequence has its origin in the commutation relations for the U-field corresponding to Bose statistics instead of Fermi.

In §4, III, it was shown that the heavy quantum had the magnetic moment $\frac{e\hbar}{2m_U c}$ in non-relativistic approximation.[37] According to Proca, the magnetic moment can be defined in general by the expression

$$\vec{\mu} = \frac{ie}{4\pi\hbar c \kappa^2} \iiint (\vec{U}^{*} \times \vec{U}) dv. \qquad (88)$$

It is easy to express (88) by normal coordinates according to the method of

---

37) See also Proca, Jour. d. Phys. 9, 61, (1938).

§3, III. Namely, when there are ~~n~~ heavy quanta with the wave number k moving in x direction, the z-compone~~t~~nt of the resultant magnetic moment can ~~be~~ be written in the form

$$\mu_z = \frac{e\hbar}{2m_u c}\left(-a_k^{*(1)}a_k^{(1)} + a_k^{*(2)}a_k^{(2)} - b_k^{*(1)}b_k^{(1)} + b_k^{*(2)}b_k^{(2)} + a_k^{*(1)}b_k^{*(1)} - a_k^{*(2)}b_k^{*(2)} + a_k^{(1)}b_k^{(1)} - a_k^{(2)}b_k^{(2)}\right) \quad \text{(~~98~~) (89)}$$

On the other hand, when the heavy quanta are moving in the z direction, we obtain

$$\mu_z = \frac{e\hbar}{2m_u c}\frac{\kappa}{k_0}\left(-a_k^{*(1)}a_k^{(1)} + a_k^{*(2)}a_k^{(2)} - b_k^{*(1)}b_k^{(1)} + b_k^{*(2)}b_k^{(2)} + a_k^{*(1)}b_k^{*(1)} - a_k^{*(2)}b_k^{*(2)} + a_k^{(1)}b_k^{(1)} - a_k^{(2)}b_k^{(2)}\right) \quad \text{(~~99~~) (90)}$$

where $k_0 = \sqrt{k^2 + \kappa^2}$. Thus, the magnetic moment in the direction of motion is smaller by a factor $\frac{\kappa}{k_0}$ than that perpendicular to the direction of motion, which takes always the normal value $e\hbar/m_u c$.

κ/k_0

(Received August 2, 1938)

6 pt

Physical Institute, Faculty of Science,
Osaka Imperial University.

~~Note added in Proof:~~

In conclusion the authors desire to acknowledge the assistance rendered by ~~Mrs. S. Takata~~ Mr. ~~H.~~ S. Okayama, Mr. Z. Hai and Mr. Y. Tanikawa to the completion of this paper.

Note added in proof: Recent ~~cloud chamber~~ photograph taken by ~~Messrs.~~ Neddermeyer and Anderson (   ) indicates ~~a mass~~ 240 m for ~~the hard cosmic~~ the mass of a cosmic ray particle with positive charge and also the annihilation of it ~~into~~ by emitting an positron