in the theory of interaction of matter and radiation.
~~At present, however, this method has serious difficulties~~
The thorough ~~the~~ development of this method, however, is hindered by the serious difficulties as shown in the following sections, so that at present ~~we~~ can not say much about ~~its~~ its applicability ~~of the method~~ to various problems ~~at present~~.

§ 2. Wave Equations for Electrons, Protons and Neutrons.

Following the above consideration, wave equations for electrons should ~~be~~ ~~have~~ consist ~~a term~~ not only of the terms which are linear in the wave functions ψ for electrons, but also of the terms which do not contain ψ at all and depend only on the wave functions ~~for~~ χ for neutrons and protons. ~~We~~ Thus we write down the wave equations ~~for~~ in the forms

$$\cancel{\frac{p_0}{c} + \frac{e}{c}\varphi_0}$$

$$\left\{ \frac{W}{c} + \frac{e}{c}A_0 + \rho_1\left(\sigma, p + \frac{e}{c}A\right) + \rho_3 mc \right\}\psi = \chi^\dagger \gamma \chi \qquad (1)$$

or $L_1 \psi = \cancel{e}\chi^\dagger\gamma\chi$ for shortness

where ~~$W = i\hbar\frac{\partial}{\partial t}$, $p = -i\hbar\,\mathrm{grad}$~~

~~and the left hand side is~~

$$W = i\hbar\frac{\partial}{\partial t}, \quad p = -i\hbar\,\mathrm{grad}$$

~~and~~, ~~other~~ the other symbols in the left hand side ~~are~~ being identitical with those used by ~~a~~ Dirac. If we put the left hand side equal to 0, it's ~~not~~ is no other than the ordinary wave equations of Dirac. In the right hand side, $\chi^\dagger$ denote wave functions conjugate to χ ~~and γ~~, each with $2\times4=8$ components and γ, ~~a symbol~~ denote matrices with four components corresponding to those of ~~wave~~ ψ. Each matrix of γ ~~having~~ has ~~of~~ ~~twenty~~ eight rows and columns, operating on ~~the~~ χ and $\chi^\dagger$.

We assume

Next ~~The~~ the wave ~~fun~~ equations for ~~electrons~~ neutrons and protons ~~should have the terms~~ containing ψ and $\psi^\dagger$, the conjugate ~~complex to ψ as the outer~~ in the following manner the following form

$$\left\{\frac{W}{c} - \frac{e}{c}\frac{1+\tau_3}{2}A_0 + \rho'_1\left(\sigma', \mathfrak{p} - \frac{e}{c}\frac{1+\tau_3}{2}A\right) + \rho'_3\left(\frac{1+\tau_3}{2}Mc + \frac{1-\tau_3}{2}M'c\right) - (\gamma\psi^\dagger + \gamma^\dagger\psi)\right\}\chi = 0 \qquad (2)$$

or $L_2\chi = c(\gamma\psi^\dagger + \gamma^\dagger\psi)\chi$ for shortness

where ~~ρ'_1~~ ρ' and σ' ~~denote~~ the ~~sp~~ ~~operators similar~~ ~~to ρ and σ~~ the matrices, differ from ρ and σ ~~only~~ by the fact only that the former operate on χ and the latter on ψ. τ_3 denotes a matrix, ~~so where~~ which commutes with ~~other matrices~~ ρ' and σ' operating on χ, and has ~~the ±1~~ +1 and −1 as eigenvalues, ~~+1 which~~ corresponding to proton and neutron respectively. M, M' are the masses of proton and neutron respectively. $\gamma^\dagger$ denotes the ~~operator~~ Hermitian conjugate ~~complex conjugate~~ of γ.

The equations (1) and (2) can be derived from the Lagrangian function

$$\bar{L} = \iiint (\psi^\dagger L_1 \psi + \chi^\dagger L_2 \chi - c\chi^\dagger(\gamma\psi^\dagger + \gamma^\dagger\psi)\chi] \qquad (3)$$

by the variation principles.

Canonical conjugate to ψ and χ are

$$\psi^* = \frac{\partial \bar{L}}{\partial \frac{\partial\psi}{\partial t}} = i\hbar\psi^* \quad \text{and} \quad \chi^* = \frac{\partial \bar{L}}{\partial \frac{\partial\chi}{\partial t}} = i\hbar\chi^\dagger$$

respectively. Hamiltonian for the system is thus

$$\bar{H} = \int\left(i\hbar\psi^\dagger\frac{\partial\psi}{\partial t} + i\hbar\chi^\dagger\frac{\partial\chi}{\partial t}\right)dv - \bar{L}$$

$$\left.\begin{aligned}\text{or }\ \bar{H} &= c\int \psi^{\dagger}\left\{-\frac{e}{c}A_0-\rho_1\left(\sigma,\ p+\frac{e}{c}A\right)-\rho_3 mc\right\}\psi\, dv\\ &+c\int \chi^{\dagger}\left(\gamma\psi^{\dagger}+\gamma^{\dagger}\psi\right)\chi\, dv\\ &+c\int \chi^{\dagger}\left\{\frac{e}{c}\frac{1+\tau_3}{2}A_0-\rho_1'\left(\sigma',\ p-\frac{e}{c}\frac{1+\tau_3}{2}A\right)-\rho_3'\left(\frac{1+\tau_3}{2}Mc\right.\right.\\ &\qquad\left.\left.+\frac{1-\tau_3}{2}M'c\right)\right\}\chi\, dv\end{aligned}\right\} \quad (4)$$

Now we ~~determine~~ should determine the form of the interaction term between neutron, proton and electron, ~~Equation~~ namely the form of γ. For this purpose we ~~writing~~ write down the equation corresponding to the equation of continuity of charge in the case of (ordinary) Dirac's electron. ~~thus~~ From ~~(1)~~ the equations (1) and ~~its~~ (their) conjugate, we obtain

$$-e\frac{\partial}{\partial t}\left(\psi^{\dagger}\psi\right)+ec\,\mathrm{div}\left\{\psi^{\dagger}(\rho_1\sigma)\psi\right\}=-e\left(\frac{c}{i\hbar}\right)\chi^{\dagger}\left(\gamma\psi^{\dagger}-\gamma^{\dagger}\psi\right)\chi \quad (5)$$

which means the fact that the total charge of system of electrons does not conserve~~s~~ since there can happen creation or annihilation of electrons. Similarly the equation in the case of proton, which can be obtained from (2) and ~~its~~ their conjugate~~s~~, are

$$e\frac{\partial}{\partial t}\left(\chi^{\dagger}\frac{1+\tau_3}{2}\chi\right)-ec\,\mathrm{div}\left\{\chi^{\dagger}(\rho_1'\sigma')\frac{1+\tau_3}{2}\chi\right\}$$

$$=e\left(\frac{c}{i\hbar}\right)\chi^{\dagger}\left\{\left(\frac{1+\tau_3}{2}\gamma-\gamma\frac{1+\tau_3}{2}\right)\psi^{\dagger}+\left(\frac{1+\tau_3}{2}\gamma^{\dagger}-\gamma^{\dagger}\frac{1+\tau_3}{2}\right)\psi\right\}\chi \quad (6)$$

In any process the total charge of electrons and protons conserves, so that the equation of continuity

$$\frac{\partial}{\partial t}\left(-e\psi^{\dagger}\psi+e\chi^{\dagger}\frac{1+\tau_3}{2}\chi\right)+\mathrm{div}\left\{\psi^{\dagger}(ec\rho_1\sigma)\psi+\chi^{\dagger}\left(-ec\rho_1'\sigma'\frac{1+\tau_3}{2}\right)\chi\right\}=0 \quad (7)$$

should be satisfied for any ψ, $\psi^{\dagger}$ ~~and~~, $\chi^{\dagger}$ and χ.

Hence, comparing the right hand sides of (5) and (6), we have

$$\left.\begin{aligned} \gamma &= \frac{1+\tau_3}{2}\gamma - \gamma\frac{1+\tau_3}{2} \\ -\gamma^\dagger &= \frac{1+\tau_3}{2}\gamma^\dagger - \gamma^\dagger\frac{1+\tau_3}{2} \end{aligned}\right\} \quad (8)$$

Denoting ~~$\tau_1 =$~~ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \tau_1$ $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \tau_2$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \tau_3$,

the solution of (8) can be written in the form

$$\left.\begin{aligned} \gamma &= \tfrac{1}{2}(\tau_1 + i\tau_2)\lambda \\ \gamma^\dagger &= \tfrac{1}{2}(\tau_1 - i\tau_2)\lambda^\dagger \end{aligned}\right\} \quad (9)$$

where λ and $\lambda^\dagger$ commute with τ's and are Hermitian conjugate to each other. The form of λ and $\lambda^\dagger$ ~~itself~~ themselves can not be determined by such general considerations.

§ 3. Quantization of the Wave Equations

Thus far ~~we did not take into~~ no account is taken for the noncommutativility of ~~ψ~~ the wave functions ~~in the quantum electrodynamics~~ as the dynamical variables. Now we ~~should~~ have to construct the "Vertauschungsrelation" for ψ, $\psi^\dagger$, χ and $\chi^\dagger$.

~~Both protons and neutrons obey Fermi's statistics~~

~~Pauli's~~ Since the exclusion principle of Pauli holds both for ~~electrons~~ protons and neutrons, Vertauschungsrelation for χ and $\chi^\dagger$ ~~takes~~ may have the form ~~We assume the~~

$$\left.\begin{aligned} &\chi_i(r,t)\chi_j^\dagger(r',t) + \chi_j^\dagger(r',t)\chi_i(r,t) = \delta_{ij}\,\delta(r-r') \\ &\chi_i(r,t)\chi_j(r',t) + \chi_j(r',t)\chi_i(r,t) = 0 \\ &\chi_i^\dagger(r,t)\chi_j^\dagger(r',t) + \chi_j^\dagger(r',t)\chi_i^\dagger(r,t) = 0 \end{aligned}\right\} \quad (10)$$

~~But there is another alternative, namely~~

~~If~~ Also we assume For electrons ~~the Vertauschungsrelation has~~ the form as usual

$$\left.\begin{aligned} &\psi_\mu(r,t)\,\psi_\nu^\dagger(r',t) + \psi_\nu^\dagger(r',t)\,\psi_\mu(r,t) = \delta_{\mu\nu}\,\delta(r,r') \\ &\psi_\mu(r,t)\,\psi_\nu(r',t) + \psi_\nu(r',t)\,\psi_\mu(r,t) = 0 \\ &\psi_\mu^\dagger(r,t)\,\psi_\nu^\dagger(r',t) + \psi_\nu^\dagger(r',t)\,\psi_\mu^\dagger(r,t) = 0 \end{aligned}\right\} \quad (11)$$

according to the fact that they obey Fermi's statistics, and

$$\psi_\mu(r,t)\,\chi_i(r',t) \pm \chi_i(r',t)\,\psi_\mu(r,t) = 0 \qquad (12)$$

etc

Equation of motion for any field ~~dynamical~~ quantity ~~of ψ_μ, χ_i $\psi_\nu^\dagger$, χ_i, $\chi_j^\dagger$~~
can be written in the form

$$i\hbar\frac{\partial F}{\partial t} = F\bar{H} - \bar{H}F \qquad (13)$$

Now ~~If~~ we put the expression ~~of~~ (4) for $\bar{H}$ and ~~the~~ any one of ψ_μ, χ_i, $\chi_j^\dagger$ for F, The equations obtained ~~expression are~~ identical with ~~the expression~~ (2) and ~~their~~ conjugate equations.

Next put $F = \psi_\mu$ or $\psi_\nu^\dagger$, the result differs from ~~in~~ (1), since in ~~this case~~ $\bar{H}$ ~~[illegible]~~ there appears the terms linear in ψ and ~~or~~ $\psi^\dagger$, thus

$$\left.\begin{aligned} \frac{i\hbar}{c}\frac{\partial\psi(r,t)}{\partial t} &= \psi\bar{H} - \bar{H}\psi \\ &= \left\{-\frac{e}{c}A_0 - \rho_1\left(\sigma, p + \frac{e}{c}A\right) - \rho_3 mc\right\}\psi(r,t) \\ &\quad + \psi(r,t)\int \chi^\dagger(\gamma\psi^\dagger + \gamma^\dagger\psi)\chi\,dv - \int\chi^\dagger(\gamma\psi^\dagger+\gamma^\dagger\psi)\chi\,dv\,\psi(r,t) \end{aligned}\right\} \quad (14)$$

If ~~the~~ electrons obey Bose's statistics, + sign in (11) ~~is~~ are to be changed into − sign and the equations (14) ~~are~~ become identical with (14).