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Calculation of Perturbation Energy.

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$\Psi_I = (\psi, 0)$ $\qquad$ $\Psi_{II} = (0, \psi)$

$$B_1 = \frac{e}{c}\frac{\mu}{r^2}\begin{pmatrix} 0 & 0 & 0 & -ie^{i\varphi} \\ 0 & 0 & ie^{-i\varphi} & 0 \\ 0 & -ie^{i\varphi} & 0 & 0 \\ ie^{-i\varphi} & 0 & 0 & 0 \end{pmatrix}\sin\theta, \quad C_1 = \frac{e}{c}\frac{\mu}{r^2}\left\{\begin{pmatrix} 0 & 0 & ie^{-i\varphi} & 0 \\ 0 & 0 & 0 & -ie^{-i\varphi} \\ ie^{-i\varphi} & 0 & 0 & 0 \\ 0 & -ie^{-i\varphi} & 0 & 0 \end{pmatrix}\sin\theta + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -2i & 0 \\ 0 & 0 & 0 & 0 \\ -2i & 0 & 0 & 0 \end{pmatrix}\cos\theta\right\}$$

$$B_2 = \frac{e}{c}\frac{\mu}{r^2}\left\{\begin{pmatrix} 0 & 0 & -ie^{i\varphi} & 0 \\ 0 & 0 & 0 & ie^{i\varphi} \\ -ie^{i\varphi} & 0 & 0 & 0 \\ 0 & ie^{i\varphi} & 0 & 0 \end{pmatrix}\sin\theta + \begin{pmatrix} 0 & 0 & 0 & 2i \\ 0 & 0 & 0 & 0 \\ 0 & 2i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\cos\theta\right\} \quad C_2 = \frac{e}{c}\frac{\mu}{r^2}\begin{pmatrix} 0 & 0 & 0 & ie^{i\varphi} \\ 0 & 0 & -ie^{-i\varphi} & 0 \\ 0 & ie^{i\varphi} & 0 & 0 \\ -ie^{-i\varphi} & 0 & 0 & 0 \end{pmatrix}\sin\theta$$

$$\psi\begin{cases} \psi_1 = -iF_k P_{k+1}^u & \psi_2 = -iF_k P_{k+1}^{u+1} \\ \cancel{\psi_3 = -iF_k P_{k+1}^{u+1}} & \\ \psi_3 = (k+u+1)G_k P_k^u & \psi_4 = (-k+u)G_k P_k^{u+1} \end{cases}$$

$$\tilde{\psi}\begin{cases} \tilde{\psi}_1 = iF_k \tilde{P}_{k+1}^u & \tilde{\psi}_2 = iF_k \tilde{P}_{k+1}^{u+1} \\ \tilde{\psi}_3 = (k+u+1)G_k \tilde{P}_k^u & \tilde{\psi}_4 = (-k+u)G_k \tilde{P}_k^{u+1} \end{cases}$$

$$\begin{pmatrix} \tilde{\Psi}_I H_1 \Psi_I & \tilde{\Psi}_I H_1 \Psi_{II} \\ \tilde{\Psi}_{II} H_1 \Psi_I & \tilde{\Psi}_{II} H_1 \Psi_{II} \end{pmatrix} = \begin{pmatrix} \int \tilde{\psi} B_1 \psi & \int \tilde{\psi} C_1 \psi \\ \int \tilde{\psi} B_2 \psi & \int \tilde{\psi} C_2 \psi \end{pmatrix}$$

$$= 0.$$

$$\left(\begin{array}{l} \tilde{\Psi}_I(u) H_1 \Psi_I(u+1) \qquad \Psi_I \\ \tilde{\Psi}_{II}(u+1) H_1 \Psi_I(u) \end{array}\right.$$

$$\tilde{\Psi}_I(u) H \Psi_I(u+1) = \tilde{\psi}(u) B_1 \psi(u+1) = 0$$

$$\psi(u+1) = \begin{cases} \tilde{\psi}_1 = -iF_k P_{k+1}^{u+1} & \psi_2 = -iF_k P_{k+1}^{u+2} \\ \psi_3 = (k+u+2) P_k^{u+1} & \psi_4 = (-k+u+1) G_k P_k^{u+2} \end{cases}$$

$$\tilde{\Psi}_{II}(u) H \Psi_{II}(u+1) = \tilde{\psi}(u) B_1 C_2 \psi(u+1) = 0.$$

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$$\tilde{\Psi}_{I}(u) H_{I}\, \Psi_{II}(u+1) = \tilde{\psi}_{(u)}\, \psi(u+1)$$

$$= i F_k G_k (k+u+2)\, \tilde{P}^{u}_{k+1} P^{u+1}_{k}\; i e^{-i\varphi} \sin\theta$$
$$+ i F_k G_k (-k+u+1)\, \tilde{P}^{u+1}_{k+1} P^{u+2}_{k}\; (-i) e^{-i\varphi} \sin\theta$$
$$- i F_k G_k (k+u+1)\, \tilde{P}^{u}_{k} P^{u+1}_{k+1}\; i e^{-i\varphi} \sin\theta$$
$$- i F_k G_k (-k+u)\, \tilde{P}^{u+1}_{k} P^{u+2}_{k+1} (-i) e^{-i\varphi} \sin\theta$$
$$+ i F_k G_k (k+u+2)\, \tilde{P}^{u+1}_{k+1} P^{u+1}_{k+1}\; (-2i) \cos\theta$$
$$- i F_k G_k (-k+u)\, \tilde{P}^{u+1}_{k} P^{u+1}_{k+1} (-2i) \cos\theta .$$

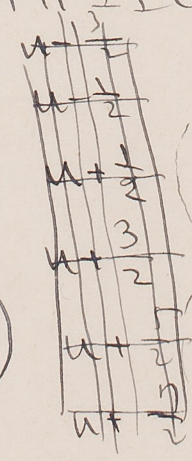
$$\tilde{\Psi}_{I}(u) H_{I}\, \Psi_{I}(u+1) = \tilde{\psi} B_2 \psi = 0$$

$$\tilde{\Psi}_{I}(u) H_{I}\, \Psi_{I}(u+2) = (-k+u) G_k \tilde{P}^{u+1}_{k} (-i) F_k P^{u+2}_{k+1}\, i e^{-i\varphi} \sin\theta$$
$$+ \cancel{k}\, i F_k \tilde{P}^{u+1}_{k+1} (k+u+3) P^{u+2}_{k}\; i e^{-i\varphi} \sin\theta$$

$$\psi(u+2) = \begin{cases} \psi_1 = -i F_k P^{u+2}_{k+1} & \psi_2 = -i F_k P^{u+3}_{k+1} \\ \psi_3 = (k+u+3) P^{u+2}_{k} & \psi_4 = (-k+u+1) G_k P^{u+3}_{k} \end{cases}$$

$$\tilde{\Psi}_{II}(u) H_I\, \Psi_{II}(u+2) \cancel{=} \tilde{\Psi}_{I}(u) H_I\, \Psi_{I}(u+2)$$

$\Psi_{I}(u-1)$ $\Psi_{II}(u-1)$
$\Psi_{I}(u)$ $\Psi_{II}(u)$
$\Psi_{I}(u+1)$ $\Psi_{II}(u+1)$
$\Psi_{I}(u+2)$ $\Psi_{II}(u+2)$
$\Psi_{I}(u+3)$ $\Psi_{II}(u+3)$



$\Psi_{I}(u-2) - \Psi_{II}(u-1)$
$\boxed{\Psi_{I}(u-1) - \Psi_{II}(u)}$ u
$\Psi_{I}(u) - \Psi_{II}(u+1)$
$\boxed{\Psi_{I}(u+1) - \Psi_{II}(u+2)}$ u+2
$\Psi_{I}(u+2) - \Psi_{II}(u+3)$
$\boxed{\Psi_{I}(u+3)}\,\boxed{\Psi_{II}(u+4)}$ u+4

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$$\tilde{\Psi}_I H_1 \Psi_I = \tilde{\Psi} B_1 \Psi = i F_k \tilde{P}^u_{k+1} \cdot (-k+u) G_k P^{u+1}_k \, i e^{-i\varphi} \sin\theta$$
$$+ i F_k \tilde{P}^{u+1}_{k+1} (k+u+1) G_k P^u_k (-i) e^{i\varphi} \sin\theta$$
$$+ (k+u+1) G_k \tilde{P}^u_k (-i) F_k P^{u+1}_{k+1} \, i e^{-i\varphi} \sin\theta$$
$$+ (-k+u) G_k \tilde{P}^{u+1}_k (-i) F_k P^{u}_{k+1} (-i) e^{-i\varphi} \sin\theta$$

$\frac{e}{c} \frac{\mu}{r^2}$

$$= (k-u) F_k G_k \cdot \left(\tilde{P}^u_{k+1} P^{u+1}_k \, e^{-i\varphi} + \tilde{P}^{u+1}_k P^u_{k+1} \right) e^{+i\varphi}) \sin\theta$$
$$+ (k+u+1) F_k G_k \left(\tilde{P}^{u+1}_{k+1} P^u_k \, e^{i\varphi} + \tilde{P}^u_k P^{u+1}_{k+1} \right) e^{-i\varphi}) \sin\theta$$

$$\therefore \; = \int \frac{e}{c} \frac{\mu}{r^2} F_k G_k r^2 dr \cdot \Big[2(k-u) \frac{4\pi}{(2k+3)(2k+1)} (k+u+1)!\,(k-u+1)!$$
$$2(k+u+1) \frac{4\pi}{(2k+3)(2k+1)} (k+u+2)!\,(k-u)! \Big]$$

$$= \int \frac{e}{c} \frac{\mu}{r^2} F_k G_k dr \cdot \frac{8\pi}{(2k+3)(2k+1)} (k+u+1)!\,(k-u)! \left[-(k-u)(k-u+1) + (k+u+1)(k+u+2) \right]$$

$$\begin{array}{l} (k+u)^2 + 3(k+u) + 2 \\ -(k-u)^2 - (k-u) \\ \hline 4ku + 2k + 4u + 2 \\ = 2(2ku + k + 2u + 1) \\ = 2(k+1)(2u+1) \end{array}$$

$$= \int \frac{e}{c} \mu F_k G_k dr \cdot \frac{16\pi (k+1)}{(2k+3)(2k+1)} (2u+1)(k+u+1)!\,(k-u)!$$

$$= \frac{\int \frac{e}{c} \mu F_k G_k dr}{\int (F_k^2 + G_k^2) r^2 dr} \cdot \frac{4(k+1)}{(2k+3)(2k+1)} (2u+1)$$

$$\tilde{\Psi}_I(u) H_1 \Psi_{II}(u+1) = \tilde{\Psi}(u) C_1 \Psi(u+1)$$

$$= F_k G_k \Big\{ \frac{4\pi (k+u+2)}{(2k+3)(2k+1)} (k+u+1)!\,(k-u+1)!$$
$$+ (k-u-1) \frac{4\pi}{(2k+3)(2k+1)} (k+u+2)!\,(k-u)!$$
$$+ (k+u+1) \frac{4\pi}{(2k+3)(2k+1)} (k+u+2)!\,(k-u)!$$
$$+ (k-u) \frac{4\pi}{(2k+3)(2k+1)} (k+u+3)!\,(k-u-1)!$$
$$+ 2(k+u+2) \frac{4\pi}{(2k+3)(2k+1)} (k+u+2)!\,(k-u)!$$
$$+ 2(k-u) \frac{4\pi}{(2k+3)(2k+1)} (k+u+2)!\,(k-u)!$$

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$$= F_k G_k \frac{4\pi}{(2k+3)(2k+1)} (k+u+2)!\,(k-u)!\,\Big[(k-u+1)+(k-u-1)$$

$$+(k+u+1)+(k+u+3)+2(k+u+2)+2(k-u)\Big]$$

$$= \qquad [8k+8]$$

$$= F_k G_k \cdot \frac{4\pi\cdot 8(k+1)}{(2k+3)(2k+1)} \frac{(k+u+2)!\,(k-u)!}{4\pi\,(k+u+1)!\,(k-u-1)!\,\sqrt{(k+u+2)(k-u)}\int(F_k^2+G_k^2)\,r^2dr}$$

$$= \frac{F_k G_k}{\int(F_k^2+G_k^2)\,r^2dr}\;\frac{8(k+1)}{(2k+3)(2k+1)}\sqrt{(k+u+2)(k-u)}$$

$$\begin{vmatrix} (2u+1)-\varepsilon & 2\sqrt{(k+u+2)(k-u)} \\ 2\sqrt{(k+u+2)(k-u)} & -(2u+3)-\varepsilon \end{vmatrix} = 0$$

$$\{(2u+1)-\varepsilon\}\{(2u+3)+\varepsilon\} + 4(k+u+2)(k-u) = 0$$

$$\varepsilon^2 + 2\varepsilon - (2u+1)(2u+3) - 4(k+u+2)(k-u) = 0$$

$$\varepsilon = -1 \pm \sqrt{1+(2u+1)(2u+3)+4(k+u+2)(k-u)}$$

$$= -1 \pm \sqrt{1+4u^2+8u+3+4k^2-4u^2+8k-8u}$$

$$= -1 \pm \sqrt{4k^2+8k+4}$$

$$= -1 \pm 2(k+1) = 2k+1 \quad \text{or} \quad -(2k+3).$$

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Eigenfunction: $\beta_{11}\Psi_I(u) + \beta_{12}\Psi_{II}(u+1)$

$\beta_{21}\Psi_I(u) + \beta_{22}\Psi_{II}(u+1)$

$$\{(2u+1)-(2k+1)\}\beta_{11} + 2\sqrt{(k+u+2)(k-u)}\,\beta_{12} = 0$$

$$\{(2u+1)+(2k+3)\}\beta_{21} + 2\sqrt{(k+u+2)(k-u)}\,\beta_{22} = 0$$

$$2\sqrt{(k+u+2)(k-u)}\,\Psi_I(u) + 2(k-u)\Psi_{II}(u+1)$$

Ψ_A $\cancel{or}$ $\left\{\begin{array}{l}\sqrt{k+u+2}\,\Psi_I(u)+\sqrt{k-u}\,\Psi_{II}(u+1) \\ \sqrt{k-u}\,\Psi_I(u)-\sqrt{k+u+2}\,\Psi_{II}(u+1)\end{array}\right.$ $\begin{array}{l}2k+1 \\ \cancel{-(2k+3)} \\ -(2k+3)\end{array}$

$2(k+u+2)$

Ψ_B

	$+1$	-3
$k=0$;	$\Psi_{II}(-1)$	
	$\cancel{\Psi_I}\,\Psi_{II}(0)$	$\Psi_I(-1)-\Psi_{II}(0)$
	$\Psi_I(0)$	
	$+3$	-5
$k=1$;	$\Psi_{II}(-2)$	
	$\Psi_I(-2)+\sqrt{3}\,\Psi_{II}(-1)$	$\sqrt{3}\,\Psi_I(-2)-\Psi_{II}(-1)$
	$\cancel{\sqrt{2}}\,\Psi_I(-1)+\sqrt{2}\,\Psi_{II}(0)$	$\cancel{\sqrt{2}}\,\Psi_I(-1)-\sqrt{2}\,\Psi_{II}(0)$
	$\sqrt{3}\,\Psi_I(0)+\Psi_{II}(1)$	$\Psi_I(0)-\sqrt{3}\,\Psi_{II}(1)$
	$\Psi_I(1)$	

$$\frac{\cancel{4k}(k+1)\ \cancel{2k+1}\qquad 4(k+1)}{(2k+1)(2k+3)}$$

$k=1$; $\dfrac{4\cdot 4}{3\cdot 5} = \dfrac{16}{15}$

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$$\frac{e}{c}\mu \frac{\int F_k G_k \, dr}{\int (F_k^2 + G_k^2) r^2 dr} \doteqdot \frac{e}{c}\mu \frac{8 Z^3 m^3 e^6 \mu}{c\, h^6 n^3} \cdot \frac{k}{2k(2k+1)(2k+2)} \cdot \frac{h}{2mc}$$

$k \neq 0.$

$$= \frac{e}{c}\mu \frac{8 Z^3 m^3 e^6 \mu}{h^6 n^3} \frac{1}{2(2k+1)(2k+2)} \cdot \frac{h}{2mc}$$

$$= A.$$

$$\Delta E = A \cdot \frac{4(k+1)\,4(k+1)}{(2k+3)(2k+1)}$$

$k=1$

$$\frac{8k(k+1)^2}{(2k+1)(2k+1)} = \frac{32}{5 \cdot 3} = 2 \cdot \frac{16}{15}$$

$$\frac{4k(k+1)}{(2k+1)} = \frac{8}{3} = 2$$

$$\frac{16}{2 \cdot 15}$$

$k=0$

$$\frac{4}{3}$$

$$\frac{k}{n^3\, 2k(2k+1)(2k+2)} \frac{4(k+1)\,4(k+1)}{(2k+3)(2k+1)}$$

$k=0$: $= \frac{1}{n^3} \frac{4 \cdot 4}{2 \cdot 2 \cdot 3} = \frac{1}{n^3} \frac{4}{3}$

$k=1$: $= \frac{1}{n^3} \frac{4 \cdot 2 \cdot 4 \cdot 2}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 3} = \frac{1}{n^3} \cdot \frac{8}{45}$

$$\frac{45}{8} \times \frac{4}{3} = \frac{15}{2} = 7.5$$

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$\kappa = 0$:

$$\left(A^2 + \frac{\gamma}{r}\right)F + \frac{dG}{dr} = 0$$

$$\frac{d}{dr}.\quad \left(B^2 - \frac{\gamma}{r}\right)G + \frac{dF}{dr} + \frac{2}{r}F = 0$$

$$\left(B^2 - \frac{\gamma}{r}\right)\frac{dG}{dr} + \frac{\gamma}{r^2}G + \frac{d^2F}{dr^2} + \frac{2}{r}\frac{dF}{dr} - \frac{2}{r^2}F = 0$$

$$-\left(B^2 - \frac{\gamma}{r}\right)\left(A^2 + \frac{\gamma}{r}\right)F - \frac{\gamma}{r^2}\,\frac{\frac{dF}{dr} + \frac{2}{r}F}{B^2 - \frac{\gamma}{r}} + \frac{d^2F}{dr^2} + \frac{2}{r}\frac{dF}{dr} - \frac{2}{r^2}F = 0$$

$$-\left(B^2 - \frac{\gamma}{r}\right)^2\left(A^2 + \frac{\gamma}{r}\right)F - \frac{\gamma}{r^2}\frac{dF}{dr} - \frac{2\gamma}{r^3}F + \left(B^2 - \frac{\gamma}{r}\right)\frac{d^2F}{dr^2}$$

$$+\left(B^2 - \frac{\gamma}{r}\right)\frac{2}{r}\frac{dF}{dr} - \left(B^2 - \frac{\gamma}{r}\right)\frac{2}{r^2}F = 0$$

$$\left(A^2 + \frac{\gamma}{r}\right)GF + \left(B^2 - \frac{\gamma}{r}\right)GF + \frac{d}{dr}\left(G^2 + F^2\right) + \frac{2\cdot F^2}{r} = 0$$

$$\left(A^2 + B^2\right)\int_0^\infty GF\,dr = -2\int \frac{F^2}{r}\,dr$$

$$\int_0^\infty GF\,dr = \frac{-2}{A^2 + B^2}\int \frac{F^2}{r}\,dr \qquad \int \frac{F^2}{r}\,dr$$

A^2

$$\int_0^\infty GF\,dr = \int_0^\infty \left(A^2 + \frac{\gamma}{r}\right)^{-1} \cdot G\frac{dG}{dr}\,dr \qquad \left(A^2 + \frac{\gamma}{r}\right)^{-1} = x$$

$$= \left(A^2 + \frac{\gamma}{r}\right)^{-1} G^2(0) - \int_0^\infty G^2 \frac{d}{dr}\left(A^2 + \frac{\gamma}{r}\right)^{-1} dr \qquad A^2 + \frac{\gamma}{r} = \frac{1}{x}$$

$$r = \frac{\gamma}{\left(\frac{1}{x} - A^2\right)}$$

$$= \int_0^\infty G^2 \cdot \left(A^2 + \frac{\gamma}{r}\right)^{-2} \cdot \frac{\gamma}{r^2}\,dr$$

$$\left(A^2 + \frac{\gamma}{r}\right)^2 \frac{r^2}{\gamma} = \left(A^2 + \frac{2A^2\gamma}{r} + \frac{\gamma^2}{r^2}\right)\frac{r^2}{\gamma}$$

$$= \frac{A^2}{\gamma} + 2A^2 r \quad = \gamma\cdot\left(\frac{A}{\gamma}r + 1\right)^2 \qquad \int_0^{\frac{1}{A^2}} dx\, G^2\left(\frac{\gamma}{\frac{1}{x} - A^2}\right)$$

$$A^2\int_0^\infty GF\,dr + \gamma\int_0^\infty GF\,dr = G^2\Big|_0 (r=0)$$

$$r^2 (A^2 + B^2)\int r^{2} FG\,dr + \int r\left(F\frac{dG}{dr} + F\frac{dF}{dr}\right)dr + 2\int F^2\,dr = 0$$

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$$-\int_0^\infty G^2\, \cancel{\gamma \cdot \left(\frac{A}{\gamma} r + 1\right)^2} \left(1 + \frac{\gamma}{rA^2}\right)^{-2} \cdot \frac{\gamma}{A^4 r^2}\, dr$$

$$= -\int_0^\infty \left(1 - \frac{2\gamma}{rA^2} + \frac{2\cdot 3}{2}\left(\frac{\gamma}{r^2 A^2}\right)^2 - \cdots\right) G \frac{\gamma}{A^4 r^2}\, dr$$

$$= -\frac{\gamma}{A^4}\int_0^\infty \frac{G\,dr}{r^2} + \frac{\gamma^2}{A^6}\int_0^\infty \frac{G\,dr}{r^3}$$

$$\Downarrow$$

$$\infty$$

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Selection Rule

$k=0 \rightleftarrows k=1$; $k=2$

$$\left\{ \begin{array}{l} \Phi_{II}(-1) \\ \Phi_I(-1)+\Phi_{II}(0) \\ \Phi_I(0) \end{array} \right. \qquad \begin{array}{l} \Phi_{II}(-2) \\ \Phi_I(-2)+\sqrt{3}\,\Phi_{II}(-1) \\ \sqrt{2}\,\Phi_I(-1)+\sqrt{2}\,\Phi_{II}(0) \\ \sqrt{3}\,\Phi_I(0) + \Phi_{II}(1) \\ \Phi_I(1) \end{array}$$

$$\left\{ \Phi_I(-1)-\Phi_{II}(0) \right. \qquad \begin{array}{l} \sqrt{3}\,\Phi_I(-2) - \Phi_{II}(-1) \\ \sqrt{2}\,\Phi_I(-1) - \sqrt{2}\,\Phi_{II}(0) \\ \Phi_I(0) - \sqrt{3}\,\Phi_{II}(1) \end{array}$$

(x, y)

$k,$: $\sqrt{k+u+2}\,\Phi_I(u) + \sqrt{k-u}\,\Phi_{II}(u+1)$ $\qquad \sqrt{k-u}\,\Phi_I(u) - \sqrt{k+u+2}\,\Phi_{II}(u)$

$k-1$; $\sqrt{k+u+1}\,\Phi_I'(u) + \sqrt{k-u-1}\,\Phi_{II}'(u+1)$ $\qquad \sqrt{k-u-1}\,\Phi_I'(u) - \sqrt{k+u+1}\,\Phi_{II}'(u)$

$$\sqrt{k+u'+1}\sqrt{k-u}\,\underset{(k,u)}{\Phi_I(u)}\underset{(k-1,u)}{\Phi_I'(u')} - \sqrt{k+u+2}\sqrt{k-u'-1}\,\underset{(k,u+1)}{\Phi_{II}(u+1)}\underset{(k-1,u'+1)}{\Phi_{II}'(u'+1)}$$

$$= \frac{\sqrt{(k+u'+1)(k-u)}}{\sqrt{(k+u+1)!\,(k-u)!}\ \sqrt{(k+u')!\,(k-u'-1)!}} \quad (k+u)!\,(k-u-1)!$$

$$- \frac{\sqrt{(k+u+2)(k-u'-1)}}{\sqrt{(k+u+2)!\,(k-u-1)!}\,(k+u'+1)!\,(k-u'-2)!} \quad (k+u+1)!\,\cancel{(k-u-2)!}$$

$u = u'$

$$\downarrow \quad = \frac{1}{(k+u)!\,(k-u-1)!} - \frac{1}{(k+u+1)!\,(k-u-2)!}$$

$$= \frac{k+u+1 - k+u+1}{(k+u+1)!\,(k-u-1)!}$$

$u = u'+1.$

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$$\frac{\cancel{k+u'+1}\sqrt{k-u}\,\sqrt{k+u'+1}\cdot k - \sqrt{(k-u'-1)}\sqrt{k-u}}{\sqrt{(k+u+1)!\,(k-u)!\,(k+u'+1)!\,(k-u'-1)!}}$$

$$= \frac{2(u'+1)}{\sqrt{(k+u+1)!\,(k-u-1)!\,(k+u'+1)!\,(k-u'-1)!}}$$

$k=1$,

$$\frac{2(u'+1)}{\sqrt{(u+2)!\,\cancel{(u+1)}\,-u!\,(u'+2)!\,-u'!}}$$

$$\frac{k+u'+1-(k-u'-1)}{\sqrt{(k+u+1)!\,(k-u-1)!\,(k+u'+1)!\,(k-u'-1)!}}$$

$u'=-1$ $\quad = 0.$

Ξ

$$\sqrt{(k+u+2)(k-u'-1)}\,\Xi_I(u)\,\Xi_I'(u') - \sqrt{(k-u)(k+u'+1)}\,\Xi_{II}(u+1)\,\Xi_{II}'(u'+1)$$

$$= \frac{\sqrt{(k+u+2)(k-u'-1)}}{\sqrt{(k+u+1)!\,(k-u)!\,(k+u'\cancel{+1})!\,(k-u'-1)!}} - \frac{\sqrt{\cancel{(k+u+2)}(k-u)(k+u'+1)}}{\sqrt{(k+u+2)!\,(k-u-1)!\,(k+u'+1)!\,(k-u'-2)!}}$$

$$= \frac{k+u+2-(k-u)}{\sqrt{(k+u+2)!\,(k-u)!\,(k+u'\cancel{+1})!\,(k-u'-2)!}}$$

$$= \frac{2(u+1)}{\sqrt{\ \ }}$$

$k=1$

$$\frac{2(u+1)}{\sqrt{(u+3)!\,(1-u)!\,\cancel{(u+2)}!\,(u'+1)!\,(-u'-1)!}}$$

$u=-2,-1,0,1$

$u'=-1,\ 0.$

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Selection Rule.

k: $\sqrt{k+u+2}\,\Psi_I(k,u)+\sqrt{k-u}\,\Psi_{II}(k,u+1)$, 1)

$k-1$: $\sqrt{k+u'+1}\,\Psi_I(k-1,u')+\sqrt{k-u'-1}\,\Psi_{II}(k-1,u'+1)$ 2)

k: $\sqrt{k-u}\,\Psi_I(k,u)-\sqrt{k+u+2}\,\Psi_{II}(k,u+1)$ 3)

$k-1$: $\sqrt{k-u'-1}\,\Psi_I(k,u')-\sqrt{k+u'+1}\,\Psi_{II}(k,u'+1)$ 4)

1) – 4)

$$\sqrt{(k+u+2)(k-u'-1)}\,\tilde{\Psi}_I(k,u)\Psi_I(k,u')-\sqrt{k-u}\sqrt{k+u'+1}\,\tilde{\Psi}_{II}(k,u+1)\Psi_{II}(k-1,u'+1)$$

$u=u'$

$\cancel{i F_k P_{k+1}^u\,(-i)F_{k-1}P}$

$$=(k+u'+2)\tilde{\Psi}_I(k,u)\Psi_I(k-1,u')-(k-u')\tilde{\Psi}_{II}(k,u+1)\times\Psi_{II}(k-1,u'+1)$$

$u=u'$ $\qquad z=r\cos\theta$

$$=(k+u+2)\Big\{iF_k\tilde{P}_{k+1}^u(-iF_{k-1})P_k^u+i\tilde{F}_k\tilde{P}_{k+1}^{u+1}(-iF_{k-1})P_k^{u+1}$$
$$+(k+u+1)G_k\tilde{P}_k^u(k+u)G_{k-1}P_{k-1}^u$$
$$+(k-u)G_k\tilde{P}_k^{u+1}(k-u-1)G_{k-1}P_{k-1}^{u+1}\Big\}\cos\theta$$
$$-(k-u)\Big\{iF_k\tilde{P}_{k+1}^{u+1}(-iF_{k-1})P_k^{u+1}+iF_k\tilde{P}_{k+1}^{u+2}(-iF_{k-1})P_k^{u+2}$$
$$+(k+u+2)G_k\tilde{P}_k^{u+1}(k+u+1)G_{k-1}P_{k-1}^{u+1}$$
$$+(k-u)G_k\tilde{P}_k^{u+2}(k-u-2)G_{k-1}P_{k-1}^{u+2}\Big\}\cos\theta$$

$$=4\pi F_kF_{k-1}\Big\{\frac{(k+u+2)\{(k+u+1)!(k-u+1)!+(k+u+2)!(k-u)!\}}{(2k+3)(2k+1)}$$
$$-\frac{(k-u)\{(k+u+2)!(k-u)!+(k+u+3)!(k-u-1)!\}}{\text{''}}\Big\}$$

$$+4\pi G_kG_{k-1}\Big\{\frac{(k+u+2)(k+u+1)(k+u)\cdot(k+u)!(k-u)!+(k-u)(k-u-1)(k+u+1)!(k-u-1)!}{(2k+1)(2k-1)}$$
$$-(k-u)\big[(k+u+2)(k+u+1)\cdot(k+u+1)!(k-u-1)!+(k-u-1)(k-u-2)(k+u+2)!\times(k-u-2)!\big]\,\text{''}$$

$\neq 0$

$u' = u \pm 1 \qquad \sin\theta\, e^{i\varphi}$

$$(k+u+1)\Big\{ iF\tilde{P}^{u}_{k+1}(-iF_{k-1})P^{u-1}_{k} + iF_k \tilde{P}^{u+1}_{k+1}(-iF_{k-1})P^{u}_{k}$$
$$+(k+u+1)G_k \tilde{P}^{u}_{k}(k+u-1)G_{k-1}P^{u-1}_{k-1}$$
$$+(k-u)G_k \tilde{P}^{u+1}_{k}(k-u)G_{k-1}P^{u}_{k-1}\Big\}\sin\theta\, e^{i\varphi}$$

$$-(k-u)\Big\{ iF_k \tilde{P}^{u+1}_{k+1}(-iF_{k-1})P^{u}_{k} + iF_k \tilde{P}^{u+2}_{k+1}(-iF_{k-1})P^{u+1}_{k}$$
$$+(k+u+2)G_k \tilde{P}^{u+1}_{k}(k+u)G_{k-1}P^{u}_{k-1}$$
$$+(k-u-1)G_k \tilde{P}^{u+2}_{k}(k-u-1)G_{k-1}P^{u+1}_{k-1}\Big\}\sin\theta\, e^{i\varphi}$$

$$= \frac{4\pi}{(2k+3)(2k+1)}F_k F_{k-1}\Big\{(k+u+1)\{(k+u+1)!(k-u+1)! + (k+u+2)!(k-u)!\}$$
$$-(k-u)\{(k+u+2)!(k-u)! + (k+u+3)!(k-u-1)!\}\Big\}$$

$\cancel{(k+u+1)(k-u+1)} - (k+u+3)(k+u+2)$

$$+\frac{4\pi\, G_k G_{k-1}}{(2k+1)(2k-1)}\Big\{(k+u+1)\{(k+u+1)!(k-u)!(k+u-1) + (k+u+2)!(k-u-1)!(k-u)\}$$
$$-(k-u)\{(k+u+2)!(k-u-1)!(k+u) + (k+u+2)!(k-u-1)!(k-u-1)\}\Big\}$$

$k+u-1+k-u-k-u-k+u+1)$

$= 0,$

$u' = u+1 \qquad \sin\theta\, e^{-i\varphi}$

$= 0.$

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2) − 3)

$$(k+u'+1)\,\widetilde{\Phi}_1(k,u)\,\Phi_1(k-1,u') - (k-u'-1)\,\Phi_{II}(k,u+1)\,\Phi_{II}(k-1,u'+1)$$

$=$

$u=u'$ $\quad z = r\cos\theta$

$$= \frac{(k+u+1)}{4\pi\sqrt{\Gamma_k\Gamma_{k-1}}\,(2k+3)(2k+1)} \Big\{(k+u+1)!\,(k-u+1)! + (k+u+2)!\,(k-u)!\Big\} - (k-u-1)\Big\{(k+u+2)!\,(k-u)! + (k+u+3)!\,(k-u-1)!\Big\}$$

$=$

$$\cancel{(k+u+1)(k-u+1)(k-u) - (k-u-1)(k+u+3)(k+u+2)}$$

$$= (k+u+1)\big\{(k-u+1)(k-u) - (k-u-1)(k+u+3)\big\} - (k-u-1)(k+u+3)$$

$$\cancel{(k-u)^2 - k + u} - k^2 + u$$

$$k^2 - 2ku + u^2 - k + u - k^2 + u^2 - 2k$$

$=$

$u' = u+1$

$= 0$

$$\Phi_A(k) \qquad\qquad \Phi_B(k)$$

$$k:\quad \sqrt{k+u+2}\;\Phi_I(u) + \sqrt{k-u}\;\Phi_{II}(u+1) \qquad \sqrt{k-u}\;\Phi_I(u) - \sqrt{k+u+2}\;\Phi_{II}(u+1)$$

$$k-1:\quad \sqrt{k+u'+1}\;\Phi_I'(u') + \sqrt{k-u'-1}\;\Phi_{II}'(u'+1) \qquad \sqrt{k-u'-1}\;\Phi_I'(u') - \sqrt{k+u'+1}\;\Phi_{II}'(u'+1)$$

$$\Phi_A(k-1) \qquad\qquad \Phi_B(k-1)$$

$$\begin{array}{ccc} \Phi_A(k+1) & & \Phi_B(k+1) \\ | & & | \\ \Phi_A(k) & & \Phi_B(k) \\ | & & | \\ \Phi_A(k-1) & & \Phi_A(k-1) \end{array}$$

$k=1$
$\downarrow$
$k=0$

$\cancel{\Phi_A(-2)}$ $\quad\Phi_A$

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$\frac{2m(2m+1)(2m+2)}{4\cdot 6}$

Σu^2

$\sum_{-k}^{k-1} u = 4k^2 - \frac{k}{3} + 2k$

$= \frac{12k^2+5k}{3}$

$4k^2 x$

$3m^2+3m+1 = (m+1)^3 - m^3$

$3(m-1)^2+3(m-1)+1 = m^3-(m-1)^3$

$3\cdot 1^2+3\cdot 1+1 = 2^3-1^3$

$3\sum_{m=1}^{m} m^2 + 3\sum_{m=1}^{m} m + 1 = (m+1)^3 - 1$

$3\sum m^2 + \frac{3m(m+1)}{2} + m = (m+1)^3 - 1$

$1+2^2=5$

$\frac{2(8+6+3)}{6}$

$2m^3$

$(m+1)$

$2m^3+6m^2+6m+2$

$-2-3m^2-3m-2m$

$\sum m^2 = \frac{2(m+1)^3-2-3m(m+1)-2m}{6} = \frac{m(2m^2+3m+1)}{6}$

$= \frac{m(2m+1)(m+1)}{6}$

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Intensity Ratio

G = g₁₂ ... F₇ neglect ...

1) − 2).

$$\sqrt{k+u+2}\,\sqrt{k+u'+1}\;\frac{(k+u+1)(k+u')\tilde{P}_k^{u}P_{k-1}^{u'} + (k-u)(k-u'-1)\tilde{P}_k^{u+1}P_{k-1}^{u'+1}}{4\pi\sqrt{(k+u+1)!\,(k-u)!}\sqrt{(k+u')!\,(k-u'-1)!}}$$

$$+\sqrt{k-u}\,\sqrt{k-u'-1}\cdot\frac{-(k+u+2)(k+u'+1)\tilde{P}_k^{u+1}P_{k-1}^{u'+1} - (k-u-1)(k-u'-2)\tilde{P}_k^{u+2}P_{k-1}^{u'+2}}{4\pi\sqrt{(k+u+2)!\,(k-u-1)!\,(k+u'+1)!\,(k-u'-2)!}}$$

$$=\sqrt{\frac{(k+u+2)(k+u'+1)}{(k+u+1)!\,(k-u)!\,(k+u')!\,(k-u'-1)!}}\Big\{$$

$u = u'$, $z = r\cos\theta$

$$=\sqrt{\frac{(k+u+2)(k+u+1)}{(k+u+1)!\,(k-u)!\,(k+u)!\,(k-u-1)!}}\left\{\frac{(k+u+1)!\,(k-u)!(k+u) + (k+u+1)!\,(k-u)!(k-u-1)}{(2k+1)(2k+1)}\right\}$$

$$+\sqrt{\frac{(k+u)(k-u-1)}{(k+u+2)!\,(k-u-1)!\,(k+u+1)!\,(k-u-2)!}}\left\{\frac{(k+u+2)!\,(k-u-1)!\,(k+u+1) + (k+u+2)!\,(k-u-1)!\,(k-u-2)}{(2k+1)(2k-1)}\right\}$$

$$=\frac{k+u}{(k+u)!\,(k-u-1)!}\sqrt{\frac{k+u+2}{k-u}}\left\{\frac{(k+u+1)!\,(k-u)!}{2k+1}\right\}$$

$$+\frac{1}{(k+u+1)!\,(k-u-2)!}\sqrt{\frac{k-u}{k+u+2}}\left\{\frac{(k+u+2)!\,(k-u-1)!}{2k+1}\right\}$$

$$=\sqrt{\frac{k+u+2}{k-u}}\,\frac{(k+u+1)(k-u)}{2k+1}+\sqrt{\frac{k-u}{k+u+2}}\,\frac{(k+u+2)(k-u-1)}{2k+1}$$

$$=\sqrt{\frac{k+u+2}{k}}\sqrt{\frac{k-u}{k+u+2}}\left\{\frac{(k+u+2)(k+u+1)(k-u)}{(k-u)(2k+1)}+\frac{(k+u+2)(k-u-1)}{(2k+1)}\right\}$$

$$=\sqrt{\frac{k-u}{k+u+2}}\cdot\frac{(k+u+2)\cdot 2k}{(2k+1)}=\sqrt{(k-u)(k+u+2)}\cdot\frac{2k}{2k+1}.$$

$$\left(\frac{2k}{2k+1}\right)^2\sum_{u=-k}^{k-1}(k-u)(k+u+2)=\frac{k\{2k^2+1\}}{3}$$

$$=\left(\frac{2k}{2k+1}\right)^2\sum_{u=-k}^{k-1}(k^2-u^2+2k-2u)=\left(\frac{2k}{2k+1}\right)^2\{2k^3+4k^2+2k\}$$

$$\sum_{u=-k}^{k-1}u^2=\frac{(k-1)(2k-1)k+k(k+1)(k+1)}{6}$$

$$=\frac{(2k)^2}{3(2k+1)^2}\{8k^3+12k^2+7k\}$$

$2k^2-3k+1$, $+3k^2+3k+1$

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$$\left(\frac{2(k+1)}{2k+1}\right)^2 \sum_{u=-k}^{k-1} (k-u-1)(k+u+1)$$

$$\sum(k^2-u^2-2u-1) = 2k^3 - \frac{k(2k^2+1)}{3} + 2k^2 - 2k$$

$$= \frac{4k^3-k}{3}$$

$$= \frac{k(2k+1)(2k-1)}{3}$$

$$\left(\frac{1}{2k+1}\right)^2 \sum 4(u+1)^2 = \frac{4}{(2k+1)^2} \sum (u^2+2u+1)$$

$$= \frac{4}{(2k+1)^2} \left\{ \frac{k(2k^2+1)}{3} - 2k + 2k \right.$$

$$= \frac{2k(2k^2+1)}{(2k+1)^2 \cdot 3}$$

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3)-4)　$u=u'$, $z=r\cos\theta$.

$$\sqrt{\frac{(k-u)(k-u-1)}{(k+u+2)(k+u+1)}}\cdot\sqrt{\frac{k+u+2}{k-u}}\ \frac{(k+u+1)(k-u)}{2k+1}$$

$$+\sqrt{\frac{(k+u+2)(k+u+1)}{(k-u)(k-u-1)}}\sqrt{\frac{k-u}{k+u+2}}\ \frac{(k+u+2)(k-u-1)}{2k+1}$$

$$=\sqrt{(k-u-1)(k+u+1)}\ \frac{k-u}{2k+1}+\sqrt{(k-u-1)(k+u+1)}\ \frac{k+u+2}{2k+1}$$

$$=\sqrt{(k-u-1)(k+u+1)}\cdot\frac{2(k+1)}{2k+1}$$

2)-3).

$$\sqrt{\frac{(k+u+1)(k-u)}{(k+u+2)(k+u+1)}}\sqrt{\frac{k+u+2}{k-u}}\ \frac{(k+u+1)(k-u)}{2k+1}$$

$$-\sqrt{\frac{(k-u-1)(k+u+2)}{(k-u)(k-u-1)}}\sqrt{\frac{k-u}{k+u+2}}\ \frac{(k+u+2)(k-u-1)}{2k+1}$$

$$=\frac{\cancel{(k+u+2)}\,2(u+1)}{2k+1}$$

$$=\frac{2(u+1)}{2k+1}.$$

$(k+u+1)(k-u)$
$-\{(k+u+1)+1\}\{(k-u)-1\}$
$=(k+u+1)-(k-u)+1$
$=2(u+1)$

$+2(k-u)$
$-k-u$

k^2-u^2+k-u
$-k^2+u^2-2(k-u)$
$+k+u+2$
$=$

1)-4)

$$\sqrt{\frac{(k+u+2)(k-u-1)}{(k+u+2)(k+u+1)}}\sqrt{\frac{k+u+2}{k-u}}\ \frac{(k+u+1)(k-u)}{2k+1}$$

$$-\sqrt{\frac{(k-u)(k+u+1)}{(k-u)(k-u-1)}}\sqrt{\frac{k-u}{k+u+2}}\ \frac{(k+u+2)(k-u-1)}{2k+1}$$

$$=\sqrt{(k+u+2)(k-u)(k+u+1)(k-u-1)}\ \frac{1}{2k+1}$$

$$-\sqrt{(k+u+2)(k-u)(k+u+1)(k-u-1)}\ \frac{1}{2k+1}$$

$=0$.

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1)−2)　　$u' = u-1$, $\cancel{z = r\cos\theta}$, $x+iy = e^{i\varphi}\sin\theta$

$$\sqrt{\frac{(k+u+2)(k+u)}{(k+u+1)!(k-u)!(k+u-1)!(k-u)!}}\left\{\frac{(k+u+1)!(k-u)!(k+u-1)+(k+u+1)!(k-u)!(k-u-2)}{(2k+1)(2k-1)}\right\}$$

$$+\sqrt{\frac{(k-u)(k-u)}{(k+u+2)!(k-u-1)!(k+u)!(k-u-1)!}}\left\{\frac{(k+u+2)!(k-u-1)!(k+u)+(k+u+2)!(k-u-1)!(k-u-1)}{(2k+1)(2k-1)}\right\}$$

$$=\frac{\sqrt{\frac{k+u+2}{k+u+1}}}{(k+u-1)!(k-u)!}\left\{\frac{(k+u+1)!(k-u)!}{2k+1}\right\}$$

$$+\frac{(k-u)\sqrt{(k+u+2)(k+u+1)}}{(k+u)!(k-u-1)!}\left\{\frac{(k+u+2)!(k-u-1)!}{2k+1}\right\}$$

$$=\sqrt{\frac{k+u+2}{k+u+1}}\;\frac{1}{2k+1}\sqrt{(k+u+2)(k+u+1)}(k+u)\cdot\frac{1}{2k+1}$$

$$+\sqrt{(k+u+2)(k+u+1)}\,(k-u)\;\frac{1}{2k+1}$$

$$=\sqrt{(k+u+2)(k+u+1)}\;\frac{2k}{2k+1}$$

3)−4)

$$\sqrt{\frac{(k-u)(k-u)}{(k+u+2)(k+u+1)}}\cdot\sqrt{(k+u+2)(k+u+1)}\;\frac{k+u}{2k+1}$$

$$+\sqrt{\frac{(k+u+2)(k+u)}{(k-u)(k-u)}}\;\sqrt{(k+u+2)(k+u+1)}\;\frac{k-u}{2k+1}$$

$$=\sqrt{(k-u)(k-u-1)}\;\frac{k+u}{2k+1}+\frac{(k+u+2)(k+1)\sqrt{k-u}}{\sqrt{k-u-1}\,(2k+1)}$$

$$=\sqrt{\frac{(k-u)}{(k-u-1)}}\cdot\frac{(k-u-1)(k+u)+(k+u+2)(k+u+1)}{2k+1}$$

$$=\sqrt{(k+u)(k+u+1)}\;\frac{(k-u)}{2k+1}+\sqrt{(k+u+1)(k+u)}\;\frac{k+u+2}{2k+1}$$

$$=\sqrt{(k+u+1)(k+u)}\cdot\frac{2(k+1)}{2k+1}$$

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2) − 3) $\sqrt{\frac{(k-u)(k+u)}{(k+u+2)(k+u)}}\sqrt{(k+u+2)(k+u+1)}\,\frac{k+u}{2k+1}$

$-^{*}\sqrt{\frac{(k+u+2)(k-u)}{(k-u)(k-u)}}\sqrt{(k+u+2)(k+u+1)}\,\frac{k-u}{2k+1}$

$=\sqrt{(k-u)(k+u+1)}\,\frac{k+u}{2k+1}$

$-\sqrt{(k-u)(k+u+1)}\,\frac{k+u+2}{2k+1}$

$=-\sqrt{(k-u)(k+u+1)}\cdot\frac{2}{2k+1}$,

3) − 4) $\sqrt{\frac{(k+u+2)(k-u)}{(k+u+2)(k+u)}}\sqrt{(k+u+2)(k+u+1)}\,\frac{k+u}{2k+1}$

$-\sqrt{\frac{(k-u)(k+u)}{(k-u)(k-u)}}\sqrt{(k+u+2)(k+u+1)}\,\frac{k-u}{2k+1}$

$=\sqrt{(k+u+2)(k+u+1)}\,\frac{\sqrt{(k+u)(k-u)}-\sqrt{(k+u)(k-u)}}{2k+1}=0$

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Intensity Ratio　Σ;　$u \rightleftarrows u'$

1) ~ 2)　$\left(\frac{2k}{2k+1}\right)^2 \sum_{u=-k}^{k-1} (k-u)(k+u+2) = \cancel{\left(\frac{2k}{2k+1}\right)^2 \frac{1}{3}\{8k^3+12k^2+7k\}}$

$= \cancel{\frac{(2k)^3}{(2k+1)^2} \frac{8k^2+12k+7}{6}}$　$4k^3$

$k=1$　$= \frac{(2k)^2}{(2k+1)^2} \frac{12k^2+5k}{3}$　2　1　2　5

$2 \cdot 2 + 3 = 7.$

$\frac{28}{9}$　$= \frac{(2k)^2 \cdot k(2k+1)(2k+5)}{(2k+1)^2 \; 3}$

$= \frac{(2k)^3(2k+5)}{6(2k+1)} = \mathrm{I}$

3) − 4)　$\left(\frac{2(k+1)}{2k+1}\right)^2 \sum_{u=-k}^{k-1} (k-u-1)(k+u+1)$

1 + 1 − 1
1 2 1 + 1

$\frac{16 \times 3}{9}$　$= \frac{(2(k+1))^2}{(2k+1)^2} \cdot \frac{k(2k+1)(2k-1)}{3}$

$= \frac{(2k+2)^2 \cdot 2k \cdot 2k-1}{6(2k+1)} = \mathrm{II}$

$\cancel{\Sigma}$ 2) − 3)　$\sum \frac{2(u+1)}{2k+1} = \frac{2 \cdot 2k \cdot (2k^2+1)}{3 \cdot (2k+1)^2} = \mathrm{III},$

1) − 4)　0.

$k=1.$　$\mathrm{I} : \mathrm{II} : \mathrm{III} = (2k)^3(2k+5) : (2k+2)^2 2k(2k-1) :$

$: \frac{4 \cdot 2k \cdot (2k^2+1)}{2k+1}$

16 × 2 ×

$= 56 : 32 : 8.$

$\mathrm{I} + \mathrm{III} : \mathrm{II} = 2 : 1.$

$\cancel{4}k^2 + 8k + 4$
$+ 8k^2 + 8k + 2$
$\frac{2k(2k-1)}{6(k+1)}(2k+2)^2 - 4(2k+1)\}$
$12k^2 + 16k + 6$
$= 2\{6k^2 + 8k + 3\}$

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$x+iy \qquad u' = u-1$

1) −2) $\left(\frac{2k}{2k+1}\right)^2 \sum (k+u+2)(k+u+1) = \frac{(2k)^2(2k+2)}{3(2k+1)} = \frac{(2k)^2(2k-1)(4k+5)}{3}$

$\sum (k^2+2ku+u^2+3k+3u+2)$

$= 2k^3 - 2k^2 + \frac{k(2k^2+1)}{3} + 6k^2 - 6k + 4k$

$= \frac{8k^3 + 12k^2 + 4k}{3}$

$= \frac{(2k-1)(4k+5)}{3} = \frac{4k(2k+1)(k+1)}{3}$

$4k^2 + 6k + 2$

3) −4) $\left(\frac{2k+2}{2k+1}\right)^2 \sum_{u=-k}^{k-1} (k+u+1)(k+u) = \frac{2k(2k+2)^2(2k-1)}{3(2k+1)}$

$\sum (k^2+u^2+2ku + k+u)$

$= 2k^3 - 2k^2 + \frac{k(2k^2+1)}{3} + 2k^2 - k$

$= \frac{8k^3 - 6k^2 - 5k}{3}$

$= \frac{k(4k-5)(2k+1)}{3}$

$= \frac{8k^3 - 2k}{3}$

$= \frac{2k(2k+1)(2k-1)}{3} = \frac{2 \cdot 2k(2k+1)^2(2k-1)}{6(2k+1)}$

2) −3) $\left(\frac{2}{2k+1}\right)^2 \sum (k-u)(k+u+1) = \frac{2 \cdot 2k(2k+2)}{3(2k+1)}$

$\sum (k^2 - u^2 + k - u) = 2k^3 - \frac{k(2k^2+1)}{3} + 2k^2 + k$

$= \frac{4k^3 + 6k^2 + 2k}{3} \qquad \frac{2k^2+3k+1}{(2k+1)(k+1)}$

$= \frac{2k(2k+1)(k+1)}{3}$

$k =$

$\frac{2}{3}\frac{2k}{(2k+1)^2}\{2k^2+1 + (2k+1)(2k+2)\}$

$2k^2 + 1 + 4k^2 + 6k + 2$

$= 6k^2 + 6k + 3$

$= 3(2k^2+2k+1) \qquad \frac{2(2k)(2k^2+2k+1)}{(2k+1)^2}$

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Total Intensity

1)−2)

$$\frac{(2k)^3}{6(2k+1)}\{2k+5+4k+4\} = \frac{(2k)^3(2k+3)}{2(2k+1)} = \mathrm{I}$$

3)−4)

$$\frac{2k(2k-1)(2k+2)^2}{2(2k+1)} = \mathrm{II} \doteq \frac{2k(2k-1)(\cancel{2k}\ 2(2k+1)(2k+3)}{6(2k+1)}$$

$$= \frac{2k(2k-1)(2k+3)}{3}$$

2)−3)

$$\frac{2(2k)(2k^2+2k+1)}{(2k+1)^2} = \mathrm{III}$$

$$(1)-2)) + (2)-3)) = \frac{(2k)\cdot\{4k^2(2k+3)(2k+1) + 4(2k^2+2k+1)\}}{2(2k+1)^2}$$

$4k^4+8k^3+3k^2$
$+2k^2+2k+1$

$\doteq$

$k=1$

$$\mathrm{I} = \frac{2\cdot2\cdot2\cdot5}{2\cdot3} = \frac{20}{3} \qquad 5 \qquad 15$$

$$\mathrm{II} = \frac{2\cdot1\cdot16}{2\cdot3} = \frac{16}{3} \qquad 4 \qquad 12$$

$$\mathrm{III} = \frac{2\cdot2\cdot5}{3\cdot3} = \frac{20}{9} \qquad \frac{5}{3} \qquad 5$$

$$\mathrm{I}+\mathrm{III} = \frac{20\cdot4}{9} = \frac{80}{9}$$

$$\mathrm{II} = \frac{48}{9}$$

$80:48 = 10:6 = 5:3$

I : II : III

$= (2k)^2(2k+3)(2k+1)$
$:(2k-1)(2k+2)^2(2k+1)$
$: 4(2k^2+2k+1)$

$=$

j	j
1)	2)
3)	4)

$f=j-\frac{3}{2}$ — $f=j-\frac{1}{2}$

$f=j-\frac{1}{2}$ — $f=j+\frac{1}{2}$

f

j	−1	1	−2	2	−3	3
	S	P	P	D	D	F
k	0	1	1	2	2	2
ja	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{5}{2}$

$\underline{-k-1}$ $\qquad \psi_1 = -i(k+u) F_{-k-1} P_{k-1}^{u} \qquad \psi_2 = -i(-k+u+1) F_{-k-1} P_{k-1}^{u+1}$

$\qquad \psi_3 = G_{-k-1} P_k^u \qquad \psi_4 = G_{-k-1} P_k^{u+1}$.

$\underline{k+1} \rightarrow -k-2$,

$\psi_1 = -i(k+1+u) F_{-k-2} P_k^u \cdot \quad \psi_2 = -i(-k+u) F_{-k-2} P_k^{u+1}$

$j = 1 \qquad \psi_3 = G_{-k-2} P_{k+1}^u \qquad \psi_4 = G_{-k-2} P_{k+1}^{u+1}$

$k=1$

$\tilde{\varphi}_1 = i(k+1+u) F_{-k-2} \tilde{P}_k^u \qquad \tilde{\varphi}_2 = i(-k+u) F_{-k-2} \tilde{P}_k^{u+1}$

$\tilde{\varphi}_3 = G_{-k-2} \tilde{P}_{k+1}^u \qquad \tilde{\varphi}_4 = G_{-k-2} \tilde{P}_{k+1}^{u+1}$

$\tilde{\Psi}_I H_I \Psi_I$

$= \tilde{\Psi} B_1 \Psi = \cancel{G_{-k-2} \tilde{P}_{k+1}^{u+1} (-i)(k+1+u) F_{-k-2} P_k^u} \cdot i e^{-i\varphi}$

$+ \cancel{G_{-k-2} \tilde{P}_{k+1}^u (-i)(-k+u) F_{-k-2} P_k^{u+1}} (-i) e^{-i\varphi}$

$+ \cancel{i(-k+u)}$

$= +i(k+1+u) F_{-k-2} \tilde{P}_k^u \cdot G_{-k-2} P_{k+1}^{u+1} \quad i e^{-i\varphi}$

$+ i(-k+u) F_{-k-2} \tilde{P}_k^{u+1} \cdot G_{-k-2} P_{k+1}^u \quad -i e^{i\varphi}$

$+ G_{-k-2} \tilde{P}_{k+1}^u \cdot (-i)(-k+u) F_{-k-2} P_k^{u+1} \quad i e^{-i\varphi}$

$+ G_{-k-2} \tilde{P}_{k+1}^{u+1} \cdot (-i)(k+1+u) F_{-k-2} P_k^u \quad -i e^{i\varphi}$

$= F_{-k-2} G_{-k-2} \Big\{ -(k+u+1) \frac{4\pi}{(2k+3)(2k+1)} (k+u+2)!(k-u)!$

$- (-k+u) \frac{4\pi}{(2k+3)(2k+1)} (k+u+1)!(k-u+1)!$

$- \frac{(-k+u) \, 4\pi \, (k+u+1)!(k-u+1)!}{(2k+3)(2k+1)}$

$- \frac{(k+u+1) \, 4\pi \, (k+u+2)!(k-u)!}{(2k+3)(2k+1)}$

$= \frac{-4\pi}{(2k+3)(2k+1)} \; 4(k+u+1)!(k-u)!(k+1)(2u+1)$

2r

$$= -\frac{\int \frac{e}{c}\mu F_{-k-2}G_{-k-2}\,dr}{\int (F_{-k-2}^2 + G_{-k-2}^2) r^2 dr} \cdot \frac{4(k+1)}{(2k+3)(2k+1)}(2u+1)$$

$k \to -k-2 \quad = -2k-2u$

$$\widetilde{\Psi}^{(u)}_{I\!I} H_1 \Psi^{(u+1)}_{I\!I} = \widetilde{\Psi}^{(u)} G_1 \Psi^{(u+1)}$$

$$= G_{-k-2}\tilde{P}^{u}_{k+1}\; i e^{i\varphi}(-i)(k+u+2)F_{-k-2}P_k^{u+1}$$

$$+ G_{-k-2}\tilde{P}^{u+1}_{k+1}\; -ie^{-i\varphi}(-i)(-k+u+1)F_{-k-2}P_k^{u+1} \Big\} \sin\theta$$

$$+ i(k+u+1)F_{-k-2}\tilde{P}^{u}_{k}\, i e^{-i\varphi} \quad G_{-k-2}\,P^{u+1}_{k+1}$$

$$+ i(-k+u)F_{-k-2}\tilde{P}^{u+1}_{k}\, i e^{-i\varphi} \quad G_{-k-2}\,P^{u+2}_{k+1}$$

$$+ i(k+u)F_{-k-2}\tilde{P}^{u+1}_{k}(-2i) \quad G_{-k-2}P^{u+1}_{k+1} \quad \cos\theta$$

$$+ G_{-k-2}\tilde{P}^{u+1}_{k+1}(-2i)(-i)(k+u+2)F_{-k-2}P_k^{u+1} \quad \cos\theta$$

$$= G_{-k-2}F_{-k-2}\Big\{-(k+u+2)\frac{4\pi}{(2k+3)(2k+1)}(k+u+1)!\,(k-u+1)!$$

$$+ (-k+u+1)\frac{4\pi}{(2k+3)(2k+1)}(k+u+2)!\,(k-u)!$$

$$-(k+u+1)\frac{4\pi}{(2k+3)(2k+1)}(k+u+2)!\,(k-u)!$$

$$+(-k+u)\quad \frac{4\pi}{\;,,\;}\quad (k+u+3)!\,(k-u-1)!$$

$$+2(-k+u) \quad ,, \quad (k+u+2)!\,(k-u)!$$

$$-\cancel{4}\,2(k+u+2) \quad ,, \quad (k+u+2)!\,(k-u)! \Big\}$$

$$= G_{-k-2}F_{-k-2}\frac{4\pi}{(2k+3)(2k+1)}(k+u+2)!\,(k-u)!$$

$$\{-k+u-1 \;-k+u+1 \;-k-u-1 \;-k-u-3$$

$$\pm 2k \pm 2u \;\pm 2k \pm 2u \pm \cancel{4}\; -4\}$$

$$= 0 \qquad = -8k-8 \quad = -8(k+1)$$

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$$= -\frac{F_{-k-2}\,G_{-k-2}}{\int (F_{-k-2}^2 + G_{-k-2}^2)\, r^2 dr} \; \frac{8(k+1)}{(2k+3)(2k+1)} \sqrt{(k+u+2)!\,(k-u)!}$$

$$\tilde{\Phi}_{II}(u)\, H_1\, \Phi_1(u+1) = \tilde{\Psi}(u)\, B\,\Psi(u+1)$$
$$= 0$$

このstring spreechを行つた答と固有値2

$$-(2k+1) \qquad 2k+3$$

が得られ。

$$\Phi_{II}(-\cancel{k-1}) \to -(2k\cancel{+3})$$

$$\Phi_{I}(-k) \to -(2k+1)$$

$$2k+3$$

$$-(2k+1)$$

$$\sqrt{k+u+2}\;\Phi_{I}(u) + \sqrt{k-u}\;\Phi_{II}(u+1); \quad \sqrt{k-u}\;\Phi_{I}(u) - \sqrt{k+u+2}\;\Phi_{II}(u+1)$$

2)' 4)'

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Intensity Ratio of the Intercombination of
K and $(-K-2)$ (or ~~$j-1$ and $j+1$.~~ $-\tilde{j}$ and $\tilde{j}$)

1) − 2′)

$$\sqrt{(k+u+2)}\,\tilde{\Psi}_1(k,u)\,\Psi_1^{*}(-k-2,u') + \sqrt{(k-u)}\,\tilde{\Psi}_{II}(k,u)\,\Psi_{II}^{*}(-k-2,u'+1)$$

$$= \sqrt{(k+u+2)}\,(k+u+1)\,G_k\tilde{P}_k^{u}\,G_{-k-2}P_{k+1}^{u'} + (-k+u)\,G_k\tilde{P}_k^{u+1}P_{k+1}^{u'+1}$$

$$+\sqrt{(k-u)}\,(k+u+2)\,G_k\tilde{P}_k^{u+1}\,G_{-k-2}P_{k+1}^{u'+1} + (-k+u+1)\,G_k\tilde{P}_k^{u+2}P_{k+1}^{u'+2}$$

$z = r\cos\theta,\qquad u' = u$

$$= \frac{G_kG_{-k-2}}{(2k+3)(2k+1)}\left\{\frac{(k+u+2)(k+u+1)\,4\pi\,(k+u+1)!\,(k-u+1)!}{4\pi\,(k+u+1)!\,(k-u)!} + \frac{(k+u+2)(-k+u)\,4\pi\,(k+u+2)!\,(k-u)!}{4\pi\,(k+u+1)!\,(k-u)!}\right.$$

$$\left. + \frac{(k-u)(k+u+2)\,4\pi\,(k+u+2)!\,(k-u)!}{4\pi\,(k+u+2)!\,(k-u-1)!} + \frac{(-k+u+1)(k-u)\,(k+u+3)!\,(k-u-1)!}{4\pi\,(k+u+2)!\,(k-u-1)!}\right\}$$

$$= \frac{G_kG_{-k-2}}{(2k+3)(2k+1)}\left\{(k+u+2)(k+u+1)(k-u+1) - (k+u+2)(k+u+2)(k-u)\right.$$
$$-(k+u+2)\,2(u+1)\;(k-u)$$
$$\left.+ (k+u+2)(k-u)(k-u) - (k+u+3)(k-u)(k-u-1)\right\}$$

$$= \frac{G_kG_{-k-2}}{(2k+3)(2k+1)}\left\{(k+u+2)(2u+1) + (k-u)(2u+3)\right\}$$

$(k+u+2)\{\,k^2-u^2+2k+1-2ku-2k+2u^2+2u\}$

$= (k+u+2)\{(k-u)^2+2u+1\}$

$(k+1)$

$(k-u)\{(k+u+2)(k-u) - (k+u+3)(k-u-1)\}$

$+(k+u+2)(2u+1)$

$(k+1)^2 - (u+1)^2 \quad - \quad (k+1)^2 + (u+2)^2$

$+\;4u+4$

$2u+1$

$(k-u)\{(k+1)^2-(u+1)^2 - (k+1)^2 + (u+2)^2\}$

$2u+4$
$2u-1$

$(k+u+2)(2u+1)$
$+\ (k-u)(2u+3)\quad = 2u+3$
$= ~~4k+4u-2k+4u~~$
$= 4ku+2u+4k+2$

$2(2k+1)(u+1)$

$(k+u+2)\{(k+1)^2-u^2$
$-(k+1)^2+(u+1)^2\}$

$=(k+u+2)(2u+1)$
$+(k-u)(2u+3)$

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$$= \frac{G_k G_{-k-2}}{(2k+3)} 2(u+1) \;!!!$$

3) — 4′)

$$\sqrt{k-u}\,\Psi_I(k,u)\sqrt{k-u'}\,\Psi_I^{(-k-2u)}(k,u') + \sqrt{k+u+2}\,\Psi_{II}(k,u+1)$$

$$\sqrt{k+u+2}\,\Psi_{II}(k,u'+1)$$

$u = u'$ $\quad z = r\cos\theta$

$$= \frac{G_k G_{-k-2}}{(2k+3)(2k+1)}\left\{\frac{k-u}{k+u+2}(k+u+2)(2u+1) + \frac{k+u+2}{k-u}(k-u)(2u+3)\right\}$$

$(k-u)(2u+1)$
$+(k+u+2)(2u+3)$
$= k(4u+4)$
$-2u^2 - u + 2u^2 + 7u + 6$
$= 6u + 6.$

$$= \;''\;\left\{k(4u+4) + 2u + 2(2u+3)\right\}$$

$$= \frac{G_k G_{-k-2}}{(2k+3)(2k+1)}\left\{2(2k+3)(2u+1)\right\}$$

$$= \frac{G_k G_{-k-2}}{(2k+1)} 2(u+1)$$

3) — 2′)

$$\sqrt{(k-u)}\sqrt{k+u+2}\;\cdots\; - \sqrt{k+u+2}\sqrt{k-u}\;\cdots$$

$$= \frac{G_k G_{-k-2}}{(2k+3)(2k+1)}\left[\sqrt{\frac{k-u}{k+u+2}}\left\{(k+u+2)(2u+1)\right\} - \sqrt{\frac{k+u+2}{k-u}}\left\{(k-u)(2u+3)\right\}\right]$$

$$= -\frac{G_k G_{-k-2}}{(2k+3)(2k+1)} 2\sqrt{(k-u)(k+u+2)}$$

1) — 4′)

$$= \;''\;\left\{\sqrt{\frac{k-u}{k+u+1}}\;\cdots\;\right\} - \sqrt{\frac{k+u+2}{k-u}}\left\{\;\cdots\;\right\}$$

$$= \frac{G_k G_{-k-2}}{(2k+3)(2k+1)}\frac{1}{\sqrt{(k-u)(k+u+2)}}\left\{(k+u+2)^2(2u+1) - (k-u)^2(2u+3)\right\}$$

$\{2ku + 2k + 2u \;\cdots\; - k^2 - u^2 + 2ku\}$

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$$\cancel{=}4(k+1)(u+1)(2u+1)-2(k-u)^2$$

$$= \qquad -2(k+1)^2+4(k+1)(u+1)^2-2(u+1)^2$$

$$4(k+1)(u+1)(2u+2)$$

$$=8(k+1)(u+1)^2$$

$$(u+1)^2\,2\{4k+3\}$$

$$\cancel{4ku+4k+4u+4}$$

$$8ku^2+8ku+8u^2+4u$$

$$+4ku+4k+4u+4$$

$$-2k^2+4ku-2u^2$$

$$8ku^2-2k^2+16ku+6u^2+8u+4k+4$$

$$=$$

$$\frac{1}{9}\ \ 4$$

$$\boxed{\begin{matrix} j-\frac{1}{2} & j-\frac{1}{2} \\ j+\frac{1}{2} & j+\frac{1}{2}\end{matrix}}$$

$$\mathrm{I}=\sum_{k=-k-1}^{k}\frac{2(u+1)}{2k+3}=\frac{1}{2k+3}\sum k(k+1)\ \ (k+1)(k+2)+2(-k-1)$$

$$\frac{k(k+1)}{2}$$

$$=\frac{-4(k+1)}{2k+3} \qquad -2(k+1)+2(k+1)$$

$$2k^2+4k+3$$

$$\mathrm{I}=\frac{4}{(2k+3)^2}\sum_{u=-k-1}^{k}(u^2+2u+1)$$

$$=\frac{4}{(2k+3)^2}\,\frac{(k+1)(2k+1)^2+1)}{3}$$

$$\mathrm{II}=\frac{4}{(2k+1)^2}\,\frac{(k+1)(2(k+1)^2+1)}{3}$$

$$\frac{(2k+3)}{(2k+1)}\ \mathrm{IV}=\mathrm{III}=\frac{4}{(2k+3)^2(2k+1)^2}\sum_{u=-k-1}^{k}\{(k+1)^2-(u+1)^2\}=\frac{8(k+1)^3}{(2k+3)^2(2k+1)^2}-\frac{4(k+1)\{2(k+1)^2+1\}}{3}$$

$$=\frac{\frac{4}{3}(k+1)\{4(k+1)^2-1\}}{(2k+3)^2(2k+1)^2}=\frac{4}{3}\frac{(k+1)}{(2k+3)(2k+1)}$$

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Z-component.

$\mathrm{I} : \mathrm{II} : \mathrm{III} : \mathrm{IV} = \cancel{4}\,(2k+1)^2(2(k+1)^2+1) : (2k+3)^2(2(k+1)^2+1)$

$: (2k+3)(2k+1) : (2k+3)(2k+1)$

$k=0 \qquad = 3 : 27 : 3 : 3 = 1 : 9 : 1 : 1$

$k=1 \qquad = 81 : 25\times 9 : 5\times 3 : 5\times 3$

$\qquad\qquad = 27 : 75 : 5 : 5$

$x+iy$ comp.

1) ~ 2') $\quad u' = u+1$.

$$G_k G_{k-2}\Big[\sqrt{k+u+2}\sqrt{k+u+3}\left\{(k+u+1)\tilde{P}_k^u P_{k+1}^{u+1} - (k-u)\tilde{P}_k^{u+1} P_{k+1}^{u+2}\right\}$$
$$+\sqrt{k-u}\sqrt{k-u-1}\left\{(k+u+2)\tilde{P}_k^{u+1} P_{k+1}^{u+2} - (k-u-1)\tilde{P}_k^{u+2} P_{k+1}^{u+3}\right\}\Big]$$

$=$

$\{ \qquad\qquad \} = \dfrac{(k+u+1)}{\quad}$

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Amount of M

$$M^2 = \{\mathfrak{M} + \tfrac{1}{2}\tfrac{h}{2\pi}(\alpha+\beta)\}^2$$
$$= (\mathfrak{M} + \tfrac{1}{2}h/2\pi\,\alpha)^2 + \left(\tfrac{h}{2\pi}\right)^2\tfrac{3}{4} + \tfrac{h}{2\pi}\overset{(\mathfrak{M}\beta)}{(\beta\mathfrak{M})} + \left(\tfrac{h}{2\pi}\right)^2\tfrac{1}{2}\overset{(\alpha\beta)}{(\beta\alpha)}$$
$$= \left(j^2-\tfrac{1}{4}\right)\left(\tfrac{h}{2\pi}\right)^2 + \left(\tfrac{h}{2\pi}\right)^2\tfrac{3}{4} + \tfrac{h}{2\pi}(\mathfrak{M}\beta) + \left(\tfrac{h}{2\pi}\right)^2\tfrac{1}{2}(\alpha\beta)$$

$$(\mathfrak{M}\beta)\cdot\Psi_A = \frac{h}{2\pi i}\Big[\left(y\frac{\partial}{\partial z}-z\frac{\partial}{\partial y}\right)\left(\sqrt{k-u}\,\psi(u+1),\ \sqrt{k+u+2}\,\psi(u)\right)$$
$$+\left(z\frac{\partial}{\partial x}-x\frac{\partial}{\partial z}\right)\left(-i\sqrt{k-u}\,\psi(u+1),\ i\sqrt{k+u+2}\,\psi(u)\right)$$
$$+\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)\left(\sqrt{k+u+2}\,\psi(u),\ -\sqrt{k-u}\,\psi(u+1)\right)\Big]$$

$$= \frac{h}{2\pi i}\Big[-\left(\sin\varphi\frac{\partial}{\partial\theta}+\cos\varphi\cot\theta\frac{\partial}{\partial\varphi}\right)\left(\sqrt{k-u}\,\psi(u+1),\ \sqrt{k+u+2}\,\psi(u)\right)$$
$$+\left(\cos\varphi\frac{\partial}{\partial\theta}-\sin\varphi\,\mathrm{ctg}\,\theta\frac{\partial}{\partial\varphi}\right)\left(-i\sqrt{k-u}\,\psi(u+1),\ i\sqrt{k+u+2}\,\psi(u)\right)$$
$$+\frac{\partial}{\partial\varphi}\left(\sqrt{k+u+2}\,\psi(u),\ -\sqrt{k-u}\,\psi(u+1)\right)\Big]$$

$=$

$$(\psi(u)) = (P_{k+1}^{u},\ P_{k+1}^{u+1},\ P_k^u,\ P_k^{u+1})$$

$\cancel{\sin\varphi\frac{\partial}{\partial\theta}+\cos\varphi\cot\theta\frac{\partial}{\partial\varphi}}$

$$\frac{\partial P_k^u}{\partial\theta} = u\,\mathrm{ctg}\,\theta\,P_k^u - (k-u)P_k^{u+1}$$
$$\frac{\partial P_k^u}{\partial\varphi} = iu\,P_k^u$$
$$\sin\varphi\frac{\partial P_k^u}{\partial\theta}+\cos\varphi\,\mathrm{ctg}\,\theta\frac{\partial P_k^u}{\partial\varphi} = u\sin\varphi\,\mathrm{ctg}\,\theta\,P_k^u - (k-u)\sin\varphi\,P_k^{u+1}$$
$$+\cos\varphi\,\mathrm{ctg}\,\theta\,iu\,P_k^u = \cancel{u\,\mathrm{ctg}\,\theta\,e^{iu}P_k^u - (k-u)\sin\varphi\,P_k^{u+1}}$$

$$\cos\varphi\frac{\partial P_k^u}{\partial\theta} - \sin\varphi\,\mathrm{ctg}\,\theta\frac{\partial P_k^u}{\partial\varphi} = u\cos\varphi\,\mathrm{ctg}\,\theta\,P_k^u - (k-u)\cos\varphi\,P_k^{u+1}$$
$$-\sin\varphi\,\mathrm{ctg}\,\theta\,iuP_k^u =$$

$-i$
$+i$

$$-(\ )-i(\ ) = -2iu\,\mathrm{ctg}\,\theta\,e^{-i\varphi}P_k^u + i(k-u)\,e^{-i\varphi}P_k^{u+1} = A_u(k,u)$$
$$-(\ )+i(\ ) = 2iu\,\mathrm{ctg}\,\theta\,e^{i\varphi}P_k^u - i(k-u)\,e^{i\varphi}P_k^{u+1} = B_u(k,u)$$

$$\begin{cases}\sqrt{k-u}\,A_u(k,u+1) + iu\sqrt{k+u+2} & P_k^u\\ \sqrt{k+u+2}\,B_u(k,u) - i(u+1)\sqrt{k-u} & P_k^{u+1}\end{cases}$$

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1) $\sqrt{k-u}\, A(k+1, u+1) + iu\sqrt{k+u+2}\, P_{k+1}^{u}$

2) $\sqrt{k-u}\, A(k+1, u+2) + i(u+1)\sqrt{k+u+2}\, P_{k+1}^{u+1}$

3) $\sqrt{\;\;}\;\; '' \;(k, u+1) + iu\sqrt{k+u+2}\, P_{k}^{u}$

4) $\sqrt{\;\;}\;\; '' \;(k, u+2) + i(u+1)\sqrt{k+u+2}\, P_{k}^{u+1}$

5) $\sqrt{k+u+2}\, B(k+1, u) - \cancel{\;} - i(u+1)\sqrt{k-u}\, P_{k+1}^{u+1}$

6) $\;'' \;(k+1, u+1) \quad - i(u+2)\sqrt{k-u}\, P_{k+1}^{u+2}$

7) $\;'' \;(k, u) \quad - i(u+1)\sqrt{k-u}\, P_{k}^{u+1}$

8) $\;'' \;(k, u+1) \quad - i(u+2)\sqrt{k-u}\, P_{k}^{u+2}$

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1) $\{-2i(u+1)\operatorname{ctg}\theta\, e^{-i\varphi} P_{k+1}^{u+1} + i(k-u)e^{-i\varphi} P_{k+1}^{u+2}\}\sqrt{k-u}$

$+ iu\sqrt{k+u+2}\, P_{k+1}^{u}$

$$\tilde{P}_{k+1}^{u}\Big[\{-2i(u+1)\operatorname{ctg}\theta\, e^{-i\varphi} P_{k+1}^{u} + i(k-u)\, e^{-i\varphi} P_{k+1}^{u+2}\}\sqrt{k-u} + iu\sqrt{k+u+2}\, P_{k+1}^{u}\Big]$$

$$\tilde{P}_{k+1}^{u}\{-2i(u+1)\operatorname{ctg}\theta\, e^{-i\varphi} P_{k+1}^{u} + i(k-u)e^{-i\varphi} P_{k+1}^{u+2}\}$$

$$= \tilde{P}_{k+1}^{u}\Big\{-2i(u+1)\cos\theta \sin^{u+1}\theta \left(\frac{d}{d\cos\theta}\right)^{k+u+1} \frac{(\cos^2\theta-1)^{k+1}}{2^{k+1}(k+1)!} e^{i(u-1)\varphi}\,(k+1-u)!$$

$$+ i(k-u) \sin^{u+2}\theta \left(\frac{d}{d\cos\theta}\right)^{k+u+3} \frac{(\cos^2\theta-1)^{k+1}}{2^{k+1}(k+1)!} e^{i(u+1)\varphi}\,(k-u)!$$

$$= \tilde{P}_{k+1}^{u}\{\cancel{-2i(u+1)\cos\theta\, P_{k}^{u-1}}$$

$$= 0,$$

$\sqrt{k+u+2}\,\tilde{P}_{k+1}^{u}\quad (+iu)\sqrt{k+u+2}\, P_{k+1}^{u} = (k+u+2)(iu)\cdot\frac{4\pi}{(2k+3)}(k+u+1)!\,(k+1-u)!$

$\tilde{P}_{k+1}^{u+1}$	P_{k+1}^{u+1}	$(k+u+2)\; i(u+1)\,\frac{4\pi}{(2k+3)}$	$(k+u+2)!\,(k-u)!$
P_{k}^{u}	$\cdot\cdot$	$(\;\;''\;\;),\; iu$	$(k+u)!\,(k-u)!$
P_{k}^{u+1}	$\cdot\cdot$	$(\;\;''\;\;)\, i(u+1)$	$(k+u+1)!\,(k-u+1)!$
P_{k+1}^{u+1}	$\cdot\cdot$	$(k-u)\, i(u+1)$	$(k+u+2)!\,(k-u)!$
P_{k+1}^{u+2}	$\cdot\cdot$	$(u+2)$	$(k+u+3)!\,(k-u-1)!$
P_{k}^{u+1}	$\cdot\cdot$	$(u+1)$	$(k+u+1)!\,(k-u-1)!$
P_{k}^{u+2}	$\cdot\cdot$	$(u+2)$	$(k+u+2)!\,(k-u-2)!$

$\cancel{(k+u+1)(k-u)}$

$(k+u+2)(k-u-1)$

$\cancel{i\frac{4\pi}{2k+3}(k+u)!\,(k-u-2)!}$

$\cancel{\times\{u\{(k+u+2)(k+u+1)(k-u+1)+(k+u+2)^2(k+u}$

$$i\frac{4\pi}{2k+3}\Big\{u\,(k+u+2)\,(k+u)!\,(k-u)!\,[(k+u+1)(k+1-u)+1]$$

$$+(u+1)\,\cancel{(k+u+2)}\,(k+u+1)!\,(k-u-1)!\,[(k+u+2)(k-u)+1]$$

$$+(u+1)\,(k-u)\,(k+u+1)!\,(k-u-1)!\,[(k+u+2)(k-u)+1]$$

$$+(u+2)\,\cancel{(k+u+2)}\,(k+u+2)!\,(k-u-2)!\,[(k+u+3)(k-u-1)+1]$$

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$\mathfrak{e}^2 + \mathfrak{h}^2 - \alpha^k \{[\mathfrak{e}\mathfrak{h}] - [\mathfrak{h}\mathfrak{e}]\}_k$

$= \mathfrak{e}^2 + \mathfrak{h}^2 - \rho_1 \{\sigma_1 \{e_y h_z - e_z h_y$

$(\sigma, \rho_2 \mathfrak{e} - \rho_3 \mathfrak{h})^2 = \sigma(\rho_2 e_1 - \rho_3 h_1)^2 + (\rho_2 e_2 - \rho_3 h_2)^2 + (\ )^2$

$\rho_1 \quad \rho_2 \quad \cancel{-2i\rho_1} + i\sigma_1 \{(\rho_2 e_2 - \rho_3 h_2)(\rho_2 e_3 - \rho_3 h_3)$

$- \{(\rho_2 e_3 - \rho_3 h_3)(\rho_2 e_2 - \rho_3 h_2)\}$

$= \sigma \rho\, \mathfrak{e}^2 + \mathfrak{h}^2 - i\rho_1 (e_1 h_1 - h_1 e_1)$

$+ \ldots \rho_1 [\quad]$

$+ \alpha^k$

$\overline{M}M - \alpha^k [\overline{M}M] = \overline{M}M$

$\rho_1 \sigma_1 \quad \rho_1 \sigma_3 - \rho_1 \sigma_2 \quad \rho_1 \sigma_1$

$= i \rho_1 \alpha_k$

$= i\sigma_3$

$(\alpha^k \overline{M})($

$(\overline{M}\alpha)(\alpha M) = (\overline{M}\alpha)(\alpha M)$

$= (\overline{M}_x \alpha_x + \overline{M}_y \alpha_y + \overline{M}_z \alpha_z)(\alpha_x M_x + \ldots)$

$= \overline{M}M +$

$e_x = -\dfrac{\mu x}{4\pi r^3}$

$e_y = -\dfrac{\mu y}{4\pi r^3}$

$e_z = -\dfrac{\mu z}{4\pi r^3}$

$\cancel{e_x^2 + e_y^2 + e_z^2}$

$x' = \dfrac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$

$t' = \dfrac{t - \frac{v}{c^2} x}{\sqrt{1 - \frac{v^2}{c^2}}}$

$t' = 0$

$\dfrac{e^2}{c^2} \cdot \dfrac{1}{l}$

$\dfrac{e^2}{mc^2}$

$\dfrac{M}{m} = \left(\dfrac{hc}{\pi e^2}\right)^2 \qquad \dfrac{m}{M} = (\pi\alpha)^2$

$\dfrac{hc}{2\pi e^2} = \alpha$

$\dfrac{137}{\pi} \quad 43.63$

$m l t^{-1}$

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覇剰

$(a+ib)(\tilde{a}-i\tilde{b}) + (a-ib)(\tilde{a}+i\tilde{b}) = 1$

$\boxed{a\tilde{a}+b\tilde{b}=\frac{1}{2}}$

$a\tilde{a}+b\tilde{b} \pm i(\tilde{a}b - a\tilde{b})$

$1 \geq H' = \frac{1}{2} + i(\tilde{a}b - a\tilde{b}) \geq 0$

$\tilde{a}b - a\tilde{b} = \pm\frac{1}{2}i$

$0, 1$

$\tilde{a}b - ab$

$\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}+\frac{i}{2}\right)\left(\frac{1}{2}-\frac{i}{2}\right)$

$(H'$

$(a+ib)(\tilde{a}-i\tilde{b})$

$+$

$(P+iQ)(P-iQ)$

$+(P-iQ)(P+iQ) = 1$

$P^2+Q^2 \cancel{\pm i(PQ-QP)} = \frac{1}{2}$

$PQ-QP = \frac{i}{2}$

$\cancel{P^2+Q^2=1}$

$pq+qp=\frac{1}{2}$

$pq = P^2+Q^2 \mp i(PQ-QP)$

$= \frac{1}{2} - i(PQ-QP)$

$Q = \begin{pmatrix} 0 & b \\ \tilde{b} & 0 \end{pmatrix}$

$P = \begin{pmatrix} 0 & a \\ \tilde{a} & 0 \end{pmatrix}$

$p = \begin{pmatrix} 0 & a+ib \\ \tilde{a}+i\tilde{b} & 0 \end{pmatrix}$

$q = \begin{pmatrix} 0 & a-ib \\ \tilde{a}-i\tilde{b} & 0 \end{pmatrix}$

$\frac{1}{2} \cdot \frac{i}{2}$

$\frac{i}{2} \quad -\frac{i}{2}$

$\frac{1}{2}\frac{i}{2} - \frac{1}{2}\left(-\frac{i}{2}\right)$

$\sigma_1\sigma_2+\sigma_2\sigma_1=1$

$\sigma_1\sigma_2 =$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

a

$(a-ib)(\tilde{a}+i\tilde{b})$

$|a-ib|$

$p = \begin{pmatrix} 0 & \frac{1}{2}+\frac{i}{2} \\ 1 & 0 \end{pmatrix}$

$q = \begin{pmatrix} 0 & 1 \\ \gamma & 0 \end{pmatrix}$

$p = P - Q$

$q =$

$(p-q)q + q(q+p) = 1$

$(p-q)q - q(p-q) = 1$

$(p-q)p - p(p-q) =$

$(p-q)p - p(p+q) = -1$

$\cancel{(p-q)(p+q) - (p-q)(p+q) = 0}$

$(p-q) \quad -(p-q)^2 + (p+q)^2 = 2$

$b^\dagger b + bb^\dagger = 1$

$b^\dagger b$

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$$= H - \sum \nu_k \Psi_k$$

$$\sum E_k \Psi_k$$

$$EH - HE = \sum h\nu_n$$

$$\sum p_r \cdots \sum q_r \cdots - \sum q_r \cdots \sum p_r$$

$$= \sum \int (\cdot\ \cdot)$$

$$H = \sum_n h\nu_n \left(N_n + \tfrac{1}{2}\right)$$

cos sin sin o

sin cos co

cos cos sin o

$$\Phi, \quad \int (EH - HE)\, dv$$

$$\alpha^2 + \beta^2 + \gamma^2 = \theta = k^2 - \tfrac{1}{4}$$

$$k = l + \tfrac{1}{2} \qquad = l(l+1)$$

$$\alpha^2 + \beta^2 + \gamma^2 = 0$$

$$\mu_x^2 + \mu_y^2 + \mu_z^2 = k(k+1)$$

$$\mu_x = \mu k \alpha.$$

$$(\mu k)^2 (\alpha\beta - \beta\alpha) = i \frac{\mu k}{k} \mu k \mu \alpha \gamma.$$

$$[\mu \times \mu] = i \frac{\mu k}{k} \mu.$$

$$i\mu$$

$$\mu_y \mu_z - \mu_z \mu_y = i \frac{k\mu}{\mu k} \mu_x \; !!!$$

$$\mu_x = \frac{\mu}{k} \alpha \qquad \left(\frac{\mu}{k}\right)^2 \alpha (\beta\gamma - \gamma\beta) = i \left(\frac{\mu}{k}\right)^2$$

$$10^8 \qquad \frac{20 \times 10^{-20}}{9 \times 10^{-27} \times 9 \times 10^{8}} = \frac{20 \times 10^{-20}}{80 \times 10^{-19}} = 2 \times 10^{-4}$$

10^7!!!

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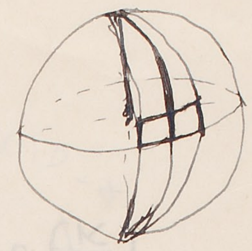


$$ds = dr^2 + r^2\sin^2\theta\, d\varphi^2 + r^2 d\theta^2$$

$$\Delta = \frac{1}{\sqrt{g_1 g_2 g_3}} \frac{\partial}{\partial x_1}\left\{\sqrt{\frac{g_2 g_3}{g_1}}\frac{\partial}{\partial x_1}\right\} + \cdots$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{1}{\sin\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)$$

$$\sin^2 it\cdot dx^2 \pm dt^2$$

$$r^2\sin^2 it\cdot dx^2 \pm r^2 dt^2$$

$$= \left(\frac{e^{-t}-e^{t}}{2i}\right)^2 dx^2 - r^2 dt^2$$

$$\boxed{\frac{d^2x}{ds^2} = \frac{1}{2}(1-\cosh 2t)\,dx^2 - dt^2,}$$

$$= \frac{1}{4}(2 - 2\cosh 2t)$$

$$x = f(u,v,w)$$

$$t = g(u,v,w)$$

$$= r^2\frac{1}{2}(1-\cosh 2t)\,dx^2 - r^2dt^2.$$

$$x^2 + y^2 + z^2 = r^2.$$

$$\frac{1}{r^2\sin^2 it}\frac{\partial^2}{\partial x^2} - \frac{1}{r^2 s}$$

$$z = r\cos\theta$$

$$= r\cos it,$$

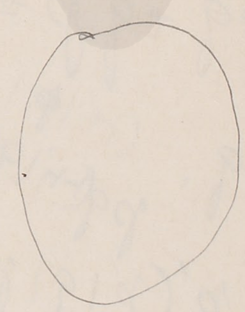
$$x^2 = r^2\cos^2 s\sin^2 it$$

$$y = ir\sin s\sin it$$

$$= r^2\cos^2 s\,\frac{1}{2}(1-\cosh 2t)$$

$$x = ir\cos s\sin it$$

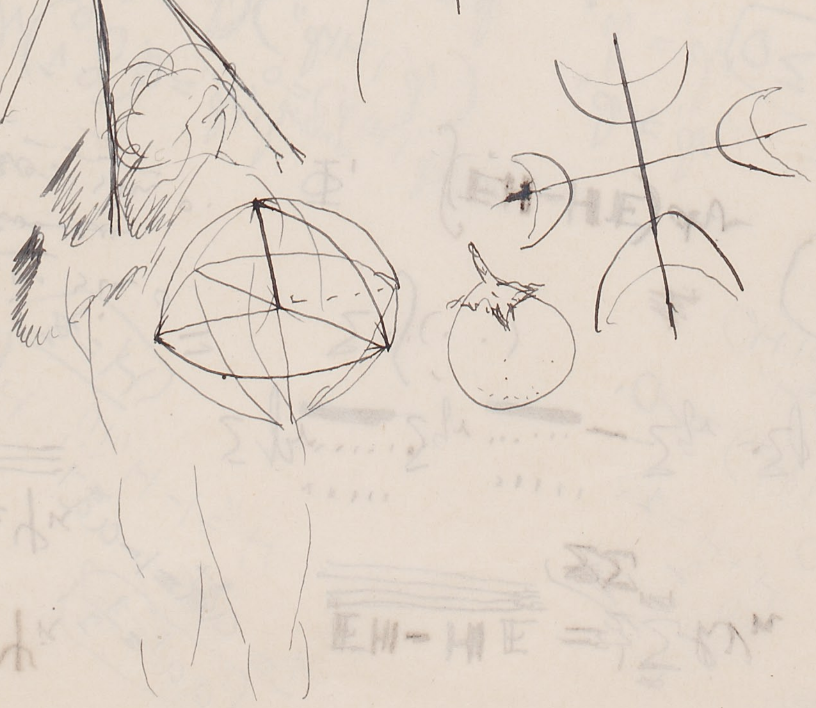
$$-(x^2+y^2) + z^2 = r^2$$



$$\sum_{n=1}(\pm C)$$

$$\frac{1}{N}\sum_{n=1}^{N} C_n \leq C.$$

$$C_n = \pm C.$$



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