Atom ノ Nucleus ノ (ガ magnetic moment ヲ持ツ為ニ) optical spectra ニ及ボス影響ハ所謂 hyperfine structure トシテ
アラハレテ居ルガ、之ニ関シテハ量子力学的研究ガ最近 Hargreaves[†] ニヨッテナサレタガ、
其ノ結果ニ依レバ Pauli-Darwin ノ spin electron ノ波動方程式ヲ満足スルモノト見テ計
算サレ、~~Dirac ノ relativistic ナ結果~~ ニ於テ殊ニ重要ナ S-level ニ就テノ separation
~~ヲ与ヘテ居ルガ~~、サレテ居ル。 relativistic ナ波動方程式ヲ使フ必要ガアラウ。

之ニ対シテハ計算ニハ矢張リ Dirac ノ relativistic ナ波動方程式ヲ用ヰテ計算シテ見
ソレデ(計算シテ) ノ問題ヲ最初カラ Dirac ノ ~~計算モシタ。之ニ ト思ハレル~~
方法ト ~~同ジデアル~~ デアル。併シ理論的ニモ完全デアルト計算 ...
デアルノデアル。

其ノ後、最近 Fermi ガ同様ノ計算ヲヤッテ居ルコトヲ知ッタノデ、之レト重複スル
部分ハ省イテ、特ニ ~~興味アル~~ 重要ナ問題デアルト思ハレル angular momentum
ノ conservation ト、一般的ナ selection rule ヲ出シテ total ノ conservation ト ...

1. 記号ハ Fermi ト同様デアッテ同ジ notation ヲ使ッテ Dirac ノ
rel. Hamiltonian ハ

$$H = eV - c\alpha\left(p - \frac{e}{c}\mathfrak{A}\right) - (\rho+1)mc^2$$

但シ

$$\rho = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} \quad \alpha_x = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} \quad \alpha_z = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{vmatrix} \quad \alpha_y = \begin{vmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{vmatrix}$$

$\rho^2 = \alpha_x^2 = \alpha_y^2 = \alpha_z^2 = 1 \quad \alpha_x\alpha_y + \alpha_y\alpha_x = 0, \quad \alpha_x\rho + \rho\alpha_x = 0$ etc.

之等ハ又 $\alpha = \rho_1\sigma$, $\rho = \rho_3$ トシテアラハサレル。 $\rho_i\rho_k + \rho_k\rho_i = 2\delta_{ik}$

~~alkali atom ノ場合等ハ~~

electric potential V ガ radius r ノミニ depend シテ居ル場合 alkali atom ヲ、
陽子デ ~~orbital~~ vector potential $\mathfrak{A} = 0$ ノ場合ニハ orbital angular
momentum $\mathfrak{M} = [r\,p]$[†] ([ ] ハ vector product ヲアラハスモノトスル)
ト spin angular momentum ノ ~~和~~ $\frac{h}{4\pi}\sigma$ トノ和 $M^* = \mathfrak{M} + \frac{h}{4\pi}\sigma$ ハ
conserve スル即チ、

$$MH - HM = 0$$

デアル。

ソコデ (Fermi ト同様ニ) nucleus ガ magnetic moment μ ヲ持ッテ居ルトシ、ソノ component ヲ
μ_x, μ_y, μ_z ~~トスル~~ トスル。ソノ Eigenwert ガ

$$\mu, \quad \frac{\mu}{k}(k-1), \quad \frac{\mu}{k}(k-2), \quad \cdots \quad -\frac{\mu}{k}(k-1), \quad -\mu.$$

デ、

$$\mu_y\mu_z - \mu_z\mu_y = i\frac{\mu}{k}\mu_x$$

$$\text{or} \quad [\mu\,\mu] = i\frac{\mu}{k}\mu$$

~~故~~ 此内ハ ヲ満足シ、$\mathbf{r}, \mathbf{p}, \boldsymbol{\alpha}, \beta$（従ッテ $\boldsymbol{\rho}, \boldsymbol{\sigma}$）ト commute スル才 (?) 仮定オク。

nucleus ノ magnetic ~~field~~ moment ~~カラ来~~ ニヨル field ハ

vector potential $\quad \mathbf{U} = \dfrac{[\boldsymbol{\mu}\,\mathbf{r}]}{r^3}$

~~カラ来テ~~ ヲアタヘルデアラウハサレル。

~~スルト~~ ソコデ乃、~~M~~ $\mathbf{M}H - H\mathbf{M}$ ヲ考ヘテ見ルニ。$\mathbf{M}$ ハ $\mathbf{U}$ 以外ノ項トハ commute スルカラ

$$\mathbf{M}H - H\mathbf{M} = \frac{e}{c}\Big\{\mathbf{M}\Big(\frac{\boldsymbol{\alpha}[\boldsymbol{\mu}\,\mathbf{r}]}{r^3}\Big) - \Big(\frac{\boldsymbol{\alpha}[\boldsymbol{\mu}\,\mathbf{r}]}{r^3}\Big)\mathbf{M}\Big\}$$

然ルニ $\quad \mathfrak{m}\{\boldsymbol{\alpha}[\mathbf{p}\,\mathbf{r}]\} - \{\boldsymbol{\alpha}[\mathbf{p}\,\mathbf{r}]\}\ \ [\boldsymbol{\alpha}\boldsymbol{\mu}]\,\mathbf{r} = \mathbf{r}[\boldsymbol{\alpha}\boldsymbol{\mu}] = \boldsymbol{\mu}[\mathbf{r}\,\boldsymbol{\alpha}]$

デ $\quad \mathfrak{m}\{\mathbf{r}[\boldsymbol{\alpha}\boldsymbol{\mu}]\} - \{\mathbf{r}[\boldsymbol{\alpha}\boldsymbol{\mu}]\}\mathfrak{m} = \frac{1}{2}\frac{hi}{2\pi}[\mathbf{r}[\boldsymbol{\sigma}\mathbf{p}]]$

トナル。*

* 何故カトイフト、

$$\{y p_z - z p_y\}\{x(\alpha_y\mu_z - \alpha_z\mu_y) + y(\alpha_z\mu_x - \alpha_x\mu_z) + z(\alpha_x\mu_y - \alpha_y\mu_x)\}$$

$$- \{\ \ \}\{y p_z - z p_y\}$$

$$= \frac{h}{2\pi i}\{y(\alpha_x\mu_y - \alpha_y\mu_x) - z(\alpha_z\mu_x - \alpha_x\mu_z)\} = \frac{h}{2\pi i}[\mathbf{r}[\boldsymbol{\sigma}\boldsymbol{\mu}]]_x,$$

$= \cancel{\frac{h}{2\pi i}}\{$

又 $\quad \frac{1}{2}\frac{h}{2\pi}\{\boldsymbol{\sigma}\{\mathbf{r}[\boldsymbol{\sigma}\boldsymbol{\mu}]\} - \{\boldsymbol{\sigma}[\boldsymbol{\mu}\,\mathbf{r}]\}\frac{1}{2}\frac{h}{2\pi}\boldsymbol{\sigma} = \frac{h}{2\pi i}\{\boldsymbol{\sigma}[\boldsymbol{\mu}\,\mathbf{r}]\}$ †

† 何故ナラ $\quad \frac{1}{2}\frac{h}{2\pi}\sigma_x\{\sigma_x[\boldsymbol{\mu}\,\mathbf{r}]_x + \sigma_y[\ \ ]_y + \sigma_z[\ \ ]_z\} - \{\sigma_y[\ \ ]_z - \sigma_z[\ \ ]_y\}\sigma_x$

~~$= 2i\sigma_z[\boldsymbol{\mu}\,\mathbf{r}]_z - \sigma$~~

$$= \frac{1}{2}\frac{h}{2\pi}(-2i)\{\sigma_y[\ \ ]_z - \sigma_z[\ \ ]_y\} = \frac{h}{2\pi i}[\boldsymbol{\sigma}[\boldsymbol{\mu}\,\mathbf{r}]]_x,$$

$$\therefore\quad \mathbf{M}H - H\mathbf{M} = \frac{e}{c}\frac{h}{2\pi i}\frac{\{[\mathbf{r}[\boldsymbol{\sigma}\boldsymbol{\mu}]] + [\boldsymbol{\sigma}[\boldsymbol{\mu}\,\mathbf{r}]]\}}{r^3}$$

$$\neq 0.$$

$\mathbf{U}$ ニ於ケル項トハ commute シ

ソコデ、$\frac{h}{2\pi}\boldsymbol{\mu}$ ヲ考ヘテ見ルト、

$$\frac{h}{2\pi}\boldsymbol{\mu}\{\boldsymbol{\mu}[\mathbf{r}\boldsymbol{\sigma}]\} - \frac{h}{2\pi}\{\boldsymbol{\mu}[\mathbf{r}\boldsymbol{\sigma}]\}\boldsymbol{\mu} = \frac{h}{2\pi i}\frac{\mu}{K}\{\boldsymbol{\mu}[\mathbf{r}\boldsymbol{\sigma}]\}$$

何故ナラ $\quad \frac{h}{2\pi}\mu_x\{\mu_x[\ \ ]_x + \mu_y[\ \ ]_y + \mu_z[\ \ ]_z\} - \frac{h}{2\pi}\{\ \ \}\mu_x$

$[\mathbf{A}[\mathbf{B}\mathbf{C}]] = \mathbf{B}(\mathbf{A}\mathbf{C}) - \mathbf{C}(\mathbf{A}\mathbf{B})$

$$= \frac{h}{2\pi}\,i\frac{\mu}{K}\{\mu_z[\ \ ]_y - \mu_y[\ \ ]_z\}$$

$$= \frac{h}{2\pi i}\frac{\mu}{K}$$

ソシテ然ルニ $[\mathbf{r}[\boldsymbol{\sigma}\boldsymbol{\mu}]] + [\boldsymbol{\sigma}[\boldsymbol{\mu}\,\mathbf{r}]] + [\boldsymbol{\mu}[\mathbf{r}\boldsymbol{\sigma}]] = 0$ ~~デアルカラシテ~~、ガ成立スルカラ、

$$\therefore\quad \Big(\mathfrak{m} + \cancel{\tfrac{h}{2}}\frac{h}{4\pi}\boldsymbol{\sigma} + \frac{h}{2\pi}\frac{K}{\mu}\boldsymbol{\mu}\Big)H - H\Big(\qquad\Big) = 0$$

即チ $\mathfrak{m} + \frac{h}{4\pi}\boldsymbol{\sigma} + \frac{h}{2\pi}\frac{K}{\mu}\boldsymbol{\mu}$ ガ conserve スルコトヲ ~~spin~~ angular momentum ト考ヘルコトガ ~~出来ル~~ $\frac{h}{2\pi}\frac{K}{\mu}\boldsymbol{\mu}$ ヲ nucleus ノ spin angular momentum ト考ヘ ~~nucleus~~ ノ total momentum ト考ヘルコトガ出来ル。

2, $K=\frac{1}{2}$, ノ時ニハ Eigenwert ハ $\mu, -\mu$.

$$\sigma=\begin{pmatrix}\sigma & 0\\ 0 & \sigma\end{pmatrix} \quad \rho=\begin{pmatrix}\rho & 0\\ 0 & \rho\end{pmatrix}$$

$$\mu=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

etc

3, $K=1$ $\mu, 0, -\mu$,

$$\mu_x=\mu\begin{vmatrix}0 & \frac{1}{\sqrt2} & 0\\ \frac{1}{\sqrt2} & 0 & \frac{1}{\sqrt2}\\ 0 & \frac{1}{\sqrt2} & 0\end{vmatrix} \quad \mu_y=\mu\begin{vmatrix}0 & -\frac{i}{\sqrt2} & 0\\ \frac{i}{\sqrt2} & 0 & -\frac{i}{\sqrt2}\\ 0 & \frac{i}{\sqrt2} & 0\end{vmatrix} \quad \mu_z=\mu\begin{vmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{vmatrix}$$

$$\sigma_x=\begin{pmatrix}\sigma_x & & \\ & \sigma_x & \\ & & \sigma_x\end{pmatrix} \quad \rho_i=\begin{pmatrix}\rho_i & 0 & 0\\ 0 & \rho_i & 0\\ 0 & 0 & \rho_i\end{pmatrix}$$

$$\mu_x=\mu\begin{pmatrix}0 & \frac{1}{\sqrt2} & 0\\ \frac{1}{\sqrt2} & 0 & \frac{1}{\sqrt2}\\ 0 & \frac{1}{\sqrt2} & 0\end{pmatrix} = \frac{\mu}{\sqrt2}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}$$ etc $\cancel{\mu}\,\mu_0$

$$\frac{e}{c}\rho_1\frac{\sigma[\mu_0\, r]}{r^3} = \frac{e}{c}\mu\begin{pmatrix}\rho_1 & 0 & 0\\ 0 & \rho_1 & 0\\ 0 & 0 & \rho_1\end{pmatrix}\left\{x\begin{pmatrix}\sigma_2 & \frac{i}{\sqrt2}\sigma_3 & 0\\ -\frac{i}{\sqrt2}\sigma_3 & 0 & \frac{i}{\sqrt2}\sigma_3\\ 0 & -\frac{i}{\sqrt2}\sigma_3 & -\sigma_2\end{pmatrix}\right.$$

$$+y\begin{pmatrix}-\sigma_1 & \frac{\sigma_3}{\sqrt2} & 0\\ \frac{\sigma_3}{\sqrt2} & 0 & \frac{\sigma_3}{\sqrt2}\\ 0 & \frac{\sigma_3}{\sqrt2} & +\sigma_1\end{pmatrix} + z\left.\begin{pmatrix}0 & -\frac{i}{\sqrt2}\sigma_1-\frac{\sigma_2}{\sqrt2} & 0\\ \frac{i}{\sqrt2}\sigma_1-\frac{\sigma_2}{\sqrt2} & 0 & -\frac{i}{\sqrt2}\sigma_1-\frac{\sigma_2}{\sqrt2}\\ 0 & \frac{i}{\sqrt2}\sigma_1-\frac{\sigma_2}{\sqrt2} & 0\end{pmatrix}\right\}$$

$$= \frac{e}{c}\frac{\mu}{r^2}\begin{pmatrix}H_{11} & H_{12} & H_{13}\\ H_{21} & H_{22} & H_{23}\\ H_{31} & H_{32} & H_{33}\end{pmatrix}$$

$$H_{11}=\rho_1\{\sigma_2\cos\varphi\ [\text{illegible}] -\sigma_1\sin\varphi\}\sin\theta = i\begin{pmatrix}0 & 0 & 0 & -e^{i\varphi}\\ 0 & 0 & e^{i\varphi} & \\ 0 & -e^{-i\varphi} & & \\ e^{i\varphi} & & & \end{pmatrix}\sin\theta$$

i $+i$

$$H_{12}= [\text{illegible}] + \frac{i\rho_1\sigma_3}{\sqrt2}\, e^{-i\varphi}\sin\theta + \begin{pmatrix}2i & 0\\ 0 & -2i\end{pmatrix}\frac{\cos\theta}{\sqrt2}\ \} = \begin{pmatrix}0 & + e^{i\varphi}\sin\theta & 0\\ & -2 & -e^{-i\varphi}\sin\theta\\ +e^{-i\varphi}\sin\theta & 0 & \\ -2 & -e^{i\varphi}\sin\theta & 0\end{pmatrix}\frac{i}{\sqrt2}$$

$$H_{13}=0$$

$$H_{21} = \begin{pmatrix} 0 & -e^{+i\varphi}\sin\theta + 2 \\ & -2 0 + e^{+i\varphi}\sin\theta \\ -e^{+i\varphi}\sin\theta + 2 & 0 \\ 0 + e^{i\varphi}\sin\theta & \end{pmatrix} \frac{i}{\sqrt{2}} \qquad H_{22} = 0 \qquad H_{23} = H_{12} = \begin{pmatrix} 0 & +e^{-i\varphi}\sin\theta \quad 0 \\ & -2 - e^{-i\varphi}\sin\theta \\ +e^{-i\varphi}\sin\theta \quad 0 & 0 \\ -2 - e^{-i\varphi}\sin\theta & \end{pmatrix} \frac{i}{\sqrt{2}}$$

$$H_{31} = 0 \qquad H_{32} = H_{21} = \begin{pmatrix} 0 & -e^{+i\varphi}\sin\theta + 2 \\ & 0 \quad +e^{+i\varphi}\sin\theta \\ -e^{+i\varphi}\sin\theta + 2 & 0 \\ 0 + e^{i\varphi}\sin\theta & \end{pmatrix} \frac{i}{\sqrt{2}} \qquad H_{33} = -H_{11} = \begin{pmatrix} & & e^{i\varphi} \\ & e^{i\varphi} & \\ -e^{i\varphi} & & \\ e^{i\varphi} & & \end{pmatrix} i \sin\theta$$

$$\psi_1 = -i F_K P_{K+1}^{u} \qquad \psi_2 = -i F_K P_{K+1}^{u+1}$$

$$\psi_3 = (k+u+1) G_K P_K^{u} \qquad \psi_4 = (-k+u) G_K P_K^{u+1}$$

$$\varphi(k, u+1)^{*} \quad \psi(k, u) \quad \psi(k, u+1)$$

$$H_{11} = A \cdot \frac{4(k+1)}{(2k+3)(2k+1)}(2u+1) \qquad H_{12} = \frac{1}{\sqrt{2}} A \frac{8(k+1)}{(2k+3)(2k+1)} \sqrt{(k+u+2)(k-u)}$$

$$\begin{pmatrix} -(2u+1)-\varepsilon & \sqrt{2}\sqrt{(k+u+2)(k-u)} & 0 \\ \sqrt{2}\sqrt{(k+u+2)(k-u)} & -\varepsilon & \sqrt{2}\sqrt{(k+u+2)(k-u)} \\ 0 & \sqrt{2}\sqrt{(k+u+2)(k-u)} & (2u+1)-\varepsilon \end{pmatrix}$$

$$-\varepsilon\{-(2u+1)-\varepsilon\}\{(2u+1)-\varepsilon\} + \{(2u+1)+\varepsilon\}\, 2(k+u+2)(k-u)$$

$$-2u(2u+2)$$

$$-4\{(2u+1)-\varepsilon\}\, 2(k+u+2)(k-u) = 0,$$

$$\varepsilon\{\varepsilon+(2u+1)\}\{\varepsilon-(2u+1)\} - 4\varepsilon(k+u+2)(k-u)$$

$$-\ 4(k+u+2)(k-u) = 0$$

$$-\ 4(\varepsilon(u+2)u = 0, \qquad (\varepsilon+1)$$

$$k = 0: \quad \varepsilon\{\varepsilon+(2u+1)\}\{\varepsilon-(2u+1)\} + 4(u+2)u\} \qquad (\varepsilon+1-2)$$

$$(\varepsilon+1)\{(\varepsilon+1)+2u\}\{(\varepsilon+1)-2(u+2)\} + 4(u+2)u\}$$

$$-\{(\varepsilon+1)+2u\}\{(\varepsilon+1)-2(u+1)\} = (\varepsilon+1)[(\varepsilon+1)^2 - 4(\varepsilon+1)] + [(\varepsilon+1)+2u] \times [(\varepsilon+1)-2(u+2)]$$

$$u=-1: \quad (\varepsilon+1)^2[(\varepsilon+1)-4] + \{(\varepsilon+1)-2\}\{(\varepsilon+1)-2\}$$

$$+\{(\varepsilon+1)^2 - 4(\varepsilon+1) + 4\}$$

$$= x^2(x-4) + (x-2)^2 = 0.$$

$$u=0:$$

$K=0$ ノ時ニハ nuclear spin $K=\frac{1}{2}$ ノ時ト同様..

$u=0$ ~~$u=-1$,~~

$$\begin{array}{c|cccccc}
 & \Psi_{III}(-1) & \Psi_{II}(-1) & \Psi_{I}(-1) & \Psi_{III}(0) & \Psi_{II}(0) & \Psi_{I}(0) \\
\hline
\Psi_{III}(-1) & \cancel{-}1 & 0 & 0 & 0 & 0 & 0 \\
\Psi_{II}(-1) & 0 & \cancel{-}1 & 0 & \sqrt{2} & 0 & 0 \\
\Psi_{I}(-1) & 0 & 0 & +1 & 0 & \sqrt{2} & 0 \\
\Psi_{III}(0) & 0 & \sqrt{2} & 0 & -1 & 0 & 0 \\
\Psi_{II}(0) & 0 & 0 & \sqrt{2} & 0 & -1 & 0 \\
\Psi_{I}(0) & 0 & 0 & 0 & 0 & 0 & 1
\end{array}$$

α_1

$$\begin{pmatrix} a_{11} & a_{12} \\ \end{pmatrix}\begin{pmatrix} 0 & 0 & \sqrt{2} & 0 \\ 0 & -1 & 0 & \sqrt{2} \\ \sqrt{2} & 0 & -1 & 0 \\ 0 & \sqrt{2} & 0 & 0 \end{pmatrix}\begin{pmatrix} \quad \end{pmatrix} = \begin{pmatrix} 0 & 0 & \sqrt{2} & 0 \\ 0 & -1 & 0 & \sqrt{2} \\ \sqrt{2} & 0 & -1 & 0 \\ 0 & \sqrt{2} & 0 & 0 \end{pmatrix}\begin{pmatrix} \quad \end{pmatrix}$$

$$\begin{pmatrix} -\varepsilon & 0 & \sqrt{2} & 0 \\ 0 & +1-\varepsilon & 0 & \sqrt{2} \\ \sqrt{2} & 0 & +1-\varepsilon & 0 \\ 0 & \sqrt{2} & 0 & -\varepsilon \end{pmatrix}$$

$a_{13}\sqrt{2} = \sqrt{2}\,a_{31}$

$-a_{22}\sqrt{2} = \sqrt{2}\,a$

$+a_{24}\sqrt{2} =$

$$= \varepsilon^2(1-\varepsilon)^2 - 2\varepsilon(-1+\varepsilon) - 2\varepsilon(-1+\varepsilon) + 4$$
$$= \varepsilon^2(1-\varepsilon)^2 - 4\varepsilon(\varepsilon-1) + 4$$
$$= \{\varepsilon(\varepsilon-1) - 2\}^2 = (\varepsilon+1)(\varepsilon-2)$$

$K=1$:

$\Psi_{III}(-2) \cdot\cdot\ \Psi(-1) \cdot\cdot\ \Psi(0) \cdot\cdot\ \Psi(1) \cdot\cdot$

Ψ_{III}
Ψ_{II}
Ψ_{I}
Ψ

$$\begin{array}{ccccccccc}
-3 & 0 & 0 & & & & & & \\
0 & 0 & \square & & & & & & \\
0 & 0 & 3 & 0 & \square & & & & \\
\square & & 1 & 0 & & & & & \\
 & & 0 & 0 & +1 & & & & \\
 & & & & 0 & +1 & 0 & 0 & \\
 & & & & 0 & 0 & 0 & \square & \\
 & & & & 0 & 0 & -1 & 0 & \square \\
 & & & & \square & 0 & 3 & 0 & 0 \\
 & & & & & \square & 0 & 0 & 0 \\
 & & & & & & 0 & 0 & -3
\end{array}$$

$$\begin{pmatrix} -\varepsilon & 0 & \sqrt{6} & 0 & & & & \\ 0 & 3-\varepsilon & 0 & \sqrt{8} & & & & \\ \sqrt{6} & 0 & -1-\varepsilon & 0 & 0 & & & \\ 0 & \sqrt{6} & 0 & -3-\varepsilon & 0 & \sqrt{8} & & \\ & & 0 & 0 & 1-\varepsilon & \sqrt{8} & 0 & \sqrt{6} \\ & & \sqrt{8} & & & -1-\varepsilon & 0 & \sqrt{6} \\ & & & \sqrt{8} & 0 & \sqrt{6} & -1-\varepsilon & 0 \\ & & & & \sqrt{6} & 0 & & 3-\varepsilon \end{pmatrix}$$

$$\varepsilon^4(\varepsilon-3)^2(\varepsilon-1)^2(\varepsilon+1)^2$$

$$\pm\ \varepsilon(\varepsilon-3)(\varepsilon+1)\varepsilon$$

$\sqrt{2}\cdot\sqrt{3}$　$\sqrt{2}\cdot\sqrt{3}$　$\sqrt{2}\cdot 2$

$\Psi - 1 \text{ or } 2$
$1+1$

$$\begin{pmatrix} 3-\varepsilon & \sqrt{6} & \sqrt{6} & 0 & 0 & 0 \\ \sqrt{6} & -\varepsilon & \sqrt{8} & & & 0 \\ 0 & \sqrt{8} & 1-\varepsilon & 0 & \sqrt{8} & 0 \\ 0 & \sqrt{8} & 0 & 1-\varepsilon & \sqrt{8} & 0 \\ 0 & 0 & \sqrt{8} & \sqrt{8} & -\varepsilon & \sqrt{6} \\ 0 & 0 & 0 & 0 & \sqrt{6} & 3-\varepsilon \end{pmatrix}$$

$\sqrt{2}\,\sqrt{3}$.

$$\left(-\varepsilon + \frac{8\sqrt{6}^2}{1+\varepsilon}\right)(-1-\varepsilon)^2 = \{\varepsilon(1+\varepsilon) - 6\}^2$$
$$= (\varepsilon-2)^2(\varepsilon+3)^2$$

$$\left(\begin{array}{ccc|ccc} 3-\varepsilon & \sqrt{6} & 0 & 0 & 0 & 0 \\ \sqrt{6} & -\varepsilon & 0 & \sqrt{8} & 0 & 0 \\ 0 & 0 & 1-\varepsilon & 0 & \sqrt{8} & 0 \\ \hline 0 & \sqrt{8} & 0 & 1-\varepsilon & 0 & 0 \\ 0 & 0 & \sqrt{8} & 0 & -\varepsilon & \sqrt{6} \\ 0 & 0 & 0 & 0 & \sqrt{6} & 3-\varepsilon \end{array}\right) = \begin{array}{l} (3-\varepsilon)^2\varepsilon^2(1-\varepsilon)^2 \\ -6\varepsilon(\varepsilon-3)(1-\varepsilon)^2 \\ \quad (\varepsilon-3)\cdot 8\cdot\varepsilon(\varepsilon-1) \\ +\varepsilon(\varepsilon-3)\cdot 8\cdot\varepsilon(\varepsilon-1) \\ \pm(\varepsilon-3)8\,\varepsilon^2(\varepsilon-1) \\ \pm(\varepsilon-3)\cdot 8\cdot(\varepsilon-1)\,6 \\ +(\varepsilon-3)\cdot 8\cdot(\varepsilon-1)\,6 \end{array}$$

$\varepsilon = 2,$

$$\begin{pmatrix} 1-\varepsilon & \sqrt6 & 0 & 0 & 0 & 0 \\ \sqrt6 & -2-\varepsilon & 0 & \sqrt8 & 0 & 0 \\ 0 & 0 & -1-\varepsilon & 0 & \sqrt8 & 0 \\ 0 & \sqrt8 & 0 & -1-\varepsilon & 0 & 0 \\ 0 & 0 & \sqrt8 & 0 & -2-\varepsilon & \sqrt6 \\ 0 & 0 & 0 & 0 & \sqrt6 & 1-\varepsilon \end{pmatrix} = \begin{pmatrix} \varepsilon-1 & \sqrt6 & 0 & 0 & 0 & 0 \\ -\sqrt6 & \varepsilon+2 & 0 & -\sqrt8 & 0 & 0 \\ 0 & 0 & \varepsilon+1 & 0 & -\sqrt8 & 0 \\ 0 & -\sqrt8 & 0 & \varepsilon+1 & 0 & 0 \\ 0 & 0 & -\sqrt8 & 0 & \varepsilon+2 & \sqrt6 \\ 0 & 0 & 0 & 0 & -\sqrt6 & \varepsilon-1 \end{pmatrix}$$

$$= (\varepsilon-1)^2(\varepsilon+2)^2(\varepsilon+1)^2 \underset{-6}{\cancel{+}} (\varepsilon-1)(\varepsilon+2)(\varepsilon+1)^2$$
$$-8(\varepsilon-1)^2(\varepsilon+2)(\varepsilon+1)$$
$$-8(\varepsilon-1)^2(\varepsilon+1)(\varepsilon+2)$$
$$+8\cdot6(\varepsilon-1)(\varepsilon+1)$$
$$+8\cdot8(\varepsilon-1)^2$$
$$+8\cdot6(\varepsilon-1)(\varepsilon+1)$$
$$-6(\varepsilon+1)^2(\varepsilon+2)(\varepsilon-1)$$
$$+6\cdot6(\varepsilon+1)^2$$
$$+8\cdot6(\varepsilon+1)(\varepsilon-1)$$

$1-(\varepsilon-2)$

$$= (\varepsilon-1)^2(\varepsilon+2)^2(\varepsilon+1)^2 - \cancel{20}\,12\,(\varepsilon-1)(\varepsilon+2)(\varepsilon+1)^2$$
$$-16(\varepsilon-1)^2(\varepsilon+2)(\varepsilon+1) + \cancel{48}\,96\,(\varepsilon-1)(\varepsilon+1)$$
$$+64(\varepsilon-1)^2 + 36(\varepsilon+1)^2$$
$$= (\varepsilon-1)(\varepsilon+2)(\varepsilon+1)^2\{(\varepsilon+2)(\varepsilon-1)-12\}$$

4
24
−32
−48
+64
36
0

$\varepsilon-$ $\varepsilon^6+4\varepsilon^5-24\varepsilon^4-60\varepsilon^3+225\varepsilon^2 \quad \varepsilon-$

$=0,\quad (\varepsilon^2-1)(\varepsilon^2-1)(\varepsilon^2+4\varepsilon+4) \quad 4-2=2$

$(\varepsilon^4-2\varepsilon^2+1)$

$$\varepsilon^4-\varepsilon^3-25\varepsilon^2+75\varepsilon$$
$$\varepsilon+5)\ \varepsilon^5+4\varepsilon^4-30\varepsilon^3-50\varepsilon^2+129\varepsilon-72$$
$$\varepsilon^5+5\varepsilon^4$$
$$-\varepsilon^4-30\varepsilon^3$$
$$-5\varepsilon^3$$
$$-25\varepsilon^3-50\varepsilon^2$$
$$-125\varepsilon^2$$
$$75\varepsilon^2+129\varepsilon$$
$$75\varepsilon^2+375\varepsilon$$
$$504\varepsilon-72$$

+4
+36
+16
−128
+72

2
−12
−16

−88
−36
−16

−7
−12
+48
+96
+64
+36

$(\varepsilon^2-1)(\varepsilon^2+3\varepsilon+2)$
$(\varepsilon^2-1)(\varepsilon^2+\varepsilon-2)$

36
29

180
26
104
3^2
7^2

129
96
225

12!!!

$(\varepsilon-2)^3(\varepsilon+3)^2$

ε^2!!

$$\varepsilon^3-\varepsilon^2-\cancel{34}\,\varepsilon+45$$
$$\varepsilon+5)\ \varepsilon^4+4\varepsilon^3-24\varepsilon^2-60\varepsilon^2+225\varepsilon$$
$$5\varepsilon^3$$
$$-\varepsilon^3-24\varepsilon^2$$
$$-5\varepsilon^2$$
$$-21\varepsilon^2-60\varepsilon^2$$
$$-105\varepsilon^2$$
$$45\varepsilon^2+225\varepsilon$$

$$\varepsilon^2+4\varepsilon$$
$$\varepsilon+5)\ \varepsilon^3-\varepsilon^2-21\varepsilon+45$$
$$\varepsilon^3-\varepsilon^2$$
$$+4\varepsilon^2-19\varepsilon$$
$$+20$$

$$\varepsilon^2-6\varepsilon+9$$
$$\varepsilon+5)\ \varepsilon^3-\varepsilon^2-21\varepsilon+45$$
$$+5\varepsilon^2$$
$$-6\varepsilon^2-21$$
$$-30$$
$$9\varepsilon+45$$

$$(\varepsilon-3)^2(\varepsilon+5)^2\varepsilon^2$$
$$= (\varepsilon-5)(\varepsilon+3)^2(\varepsilon-2)^2$$

$(\varepsilon-2)^4(\varepsilon+3)^6(\varepsilon-5)^2$!!!

Intensität !!!!!!!!!!!!!!!

$\mu = 1$

$l = 1 \quad s = 1/2, \quad 6 \quad 4 \quad 2$

$3/2 + 1 = \frac{5}{2}, \frac{3}{2}, \frac{1}{2}$!!!

$3/2 + \lambda = \lambda + 3/2, \lambda + 1/2, \lambda - 1/2, \lambda - 3/2.$