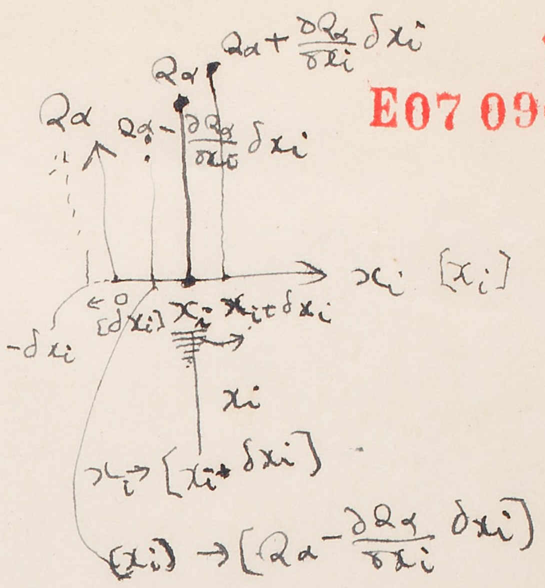
§1. Allgemeine Methode und Impulssätze.

$\underline{59}$ 3–4
(S 168)

$F(Q_\alpha)$

$F\left(Q_\alpha - \frac{\partial Q_\alpha}{\partial x_i}\delta x_i\right) =$

$0 \to \delta x_i$.



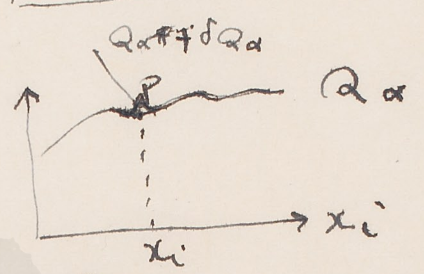
$F \to F - \int dV \sum$

$$F(Q_\alpha + \delta Q_\alpha) = F + \int \sum_\alpha \frac{\delta F}{\delta Q_{\alpha;P}} \delta Q_{\alpha;P}\, dV$$

$$= F - \int \sum_\alpha \frac{\delta F}{\delta Q_\alpha} \delta \frac{\partial Q_\alpha}{\partial x_i} \delta x_i \cdot dV \quad \frac{\partial Q_\alpha}{\partial x_i}\delta x_i$$

$$1 - \delta x_i \int dV \sum \frac{\partial Q_\alpha}{\partial x_i} \frac{\delta}{\delta Q_\alpha}$$

$$\frac{\delta}{\delta Q_\alpha} \to \frac{2\pi i}{h} P_\alpha$$



$Q_\alpha = r, \theta, \varphi$.

$\left\{ \frac{x}{r} P_r + \quad P_\theta + \quad P_\varphi \right\}$

Impulssätze: $\int dV \sum_\alpha \frac{\partial Q_\alpha}{\partial x_i} P_\alpha = \text{const}.$

$P_\alpha \to \frac{\partial q'_\alpha}{\partial q_\alpha} p'_\alpha$

§2. Erhaltung der Ladung $\psi_\rho \to \psi_\rho e^{i\alpha}$ $\psi_\rho^* \to \psi_\rho^* e^{-i\alpha}$

$\psi_\rho \to \psi_\rho + i\delta\alpha\, \psi_\rho$ $\quad \psi_\rho^* \to \psi_\rho^* - i\delta\alpha\, \psi_\rho^*$

$$F \to F + i\delta\alpha \int dV \frac{\delta F}{\delta \psi_\rho} \psi_\rho$$

$$1 + i\delta\alpha \int dV\, \psi_\rho \frac{\delta}{\delta\psi_\rho}.$$

$$\int dV\, \psi_\rho \psi_\rho^* = \text{constant}.$$

Elektronen und Protonen $(\psi_\rho^{(e)}, \psi_\rho^{(p)})$

$$\int dV \left(-\psi_\rho^{*(e)} \psi_\rho^{(e)} + \psi_\rho^{*(p)} \psi_\rho^{(p)}\right) = \text{const}$$

$$\psi^{(e)} \to \psi_\rho^{(e)} e^{i\alpha} \quad \left(\psi_\rho^{*(e)} \to \psi_\rho^{*(e)} e^{-i\alpha}\right)$$

$$\psi_\rho^{(p)} \to \psi_\rho^{(p)} e^{-i\alpha} \quad \left(\psi_\rho^{*(p)} \to \psi_\rho^{*(p)} e^{i\alpha}\right)$$

$$\psi_\rho^{(e)} \psi_\sigma^{(p)} F_{ik} S_{\rho\sigma}^{ik} + \psi_\rho^{*(e)} \psi_\sigma^{*(p)} F_{ik} S_{\rho\sigma}^{*ik}$$

Zerstrahlungsprozesse.

§3. Die Transformation $\Phi_\nu \to \Phi_\nu + \frac{\partial \chi}{\partial x_\nu}$; $\psi_\rho \to e^{-\frac{2\pi i}{h}\frac{e}{c}\chi} \cdot \psi_\rho$.

(Fermi: Rendiconti d. R. Acc. dei Lincei (6) 9, 1. Hälfte, S. 881, 1929)

$$\Phi_i \to \Phi_i + \delta\frac{\partial \chi}{\partial x_i} \qquad \psi_\rho \to \psi_\rho - \frac{2\pi i}{h}\frac{e}{c}\delta\chi \cdot \psi_\rho$$

$$\int dV\, \chi \left(-\frac{1}{c}\frac{\partial E_i}{\partial x_i} - \frac{e}{c}\sum_\rho \psi_\rho^* \psi_\rho\right)$$

$\operatorname{div} E + e\sum_\rho \psi_\rho^* \psi_\rho = \text{const} = C.$ (beliebige Raumfunktion)

$$\overline{C} = \int X(\operatorname{div} E + e\sum_\rho \psi_\rho^* \psi_\rho)\, dv = \int X \cdot C\, dv$$

$$\left.\begin{aligned} [\overline{C}, \psi_\rho] &= -eX\psi_\rho, \\ [\overline{C}, \psi_\rho^*] &= eX\psi_\rho^* \\ [\overline{C}, \Phi_k] &= \frac{hc}{2\pi i}\frac{\partial X}{\partial x_k} \end{aligned}\right\}$$

何となれば、

$$\int X \cancel{e}\, d(\operatorname{div} E + e\sum_\rho \psi_\rho^* \psi_\rho)\, dv \cdot \psi_{\rho'}$$

$$-\psi_{\rho'}\int \quad \text{〃} \quad \text{〃} \quad \text{〃} \quad ;$$

$$\psi_\rho^*\psi_{\rho'} - \psi_{\rho'}\psi_\rho^* = -\delta_{\rho\rho'}\,\delta(r, r')$$

$$f \to f + \frac{2\pi i}{hc}[\delta\overline{C}, f]$$

$$f_1 f_2 \to \left\{f_1 + \frac{2\pi i}{hc}[\delta\overline{C}, f_1]\right\}\left\{f_2 + \frac{2\pi i}{hc}[\delta\overline{C}, f_2]\right\}$$

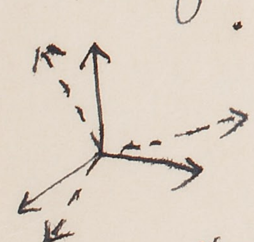
$$= f_1 f_2 + \frac{2\pi i}{hc}\{f_1[\delta\overline{C}, f_2] + [\delta\overline{C}, f_1]f_2\}$$

$$\delta\{f_1(\overline{C}f_2 - f_2\overline{C}) + (\overline{C}f_1 - f_1\overline{C})f_2\}$$

$$= [\delta\overline{C}, f_1 f_2]$$

$$f \to e^{\frac{2\pi i}{hc}\overline{C}} \cdot f \cdot e^{-\frac{2\pi i}{hc}\overline{C}}$$

§4. Lorentz transformation



$$x_i' = x_i + \sum \varepsilon_{ik} x_k$$

$$x_i' x_i' = x_i x_i + \cancel{\sum \varepsilon_{il}\varepsilon_{ik}\, x_l x_k} + \sum \varepsilon_{ik}\, x_k x_i + \sum \varepsilon_{ikl}\, x_k x_l$$

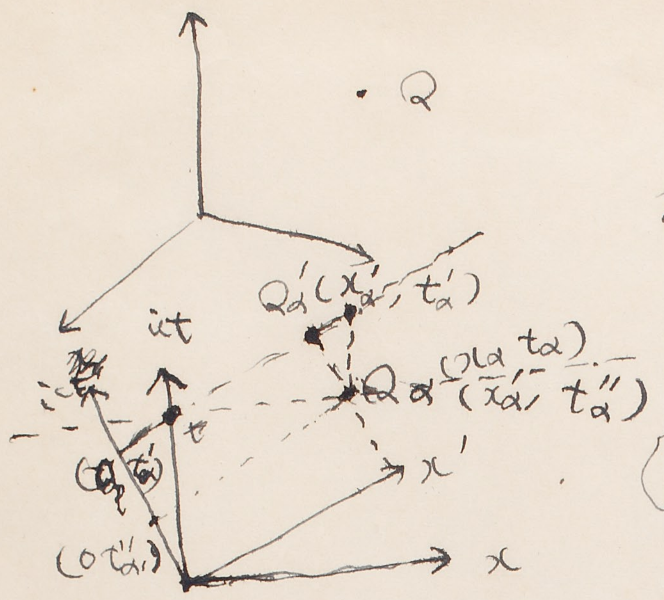
$$\cancel{x_l x_k : \sum_i \varepsilon_{il}\varepsilon_{ik} = 0}, \qquad x_k^2 : \quad 2\varepsilon_{ik} = 0$$

$$\cancel{x_k^2 : \sum_i \varepsilon_{ik}^2 = 0}. \qquad x_k x_l : \quad (\varepsilon_{lk} + \varepsilon_{kl}) = 0.$$

$$\begin{pmatrix} 0 & \varepsilon_{12} & \varepsilon_{13} \\ -\varepsilon_{12} & 0 & \varepsilon_{23} \\ -\varepsilon_{13} & -\varepsilon_{23} & 0 \end{pmatrix}$$

$$f(x_i) = f(x_i + \sum \varepsilon_{ik} x_k) \quad \sum \varepsilon_{ik}\left\{x_k \frac{\partial}{\partial x_i} - x_i \frac{\partial}{\partial x_k}\right\} f$$

$$= f(x_i) + \sum \frac{\partial f}{\partial x_i}\, \varepsilon_{ik} x_k$$



$$x_\mu \to x_\mu + \varepsilon s_{\mu\nu} x_\nu \quad (s_{\mu\nu} = -s_{\nu\mu})$$

$$\boxed{x_0 \to x_0 + \varepsilon s_{0\nu} x_\nu,}$$

$$Q_\alpha \to Q_\alpha - \varepsilon \cancel{s_{\mu\nu} x_\nu}\, s_{\mu\nu} x_\nu \frac{\partial Q_\alpha}{\partial x_\mu}$$
$$+ \varepsilon t_{\alpha\rho} Q_\rho$$

$$P_{\alpha 4} \to P_{\alpha 4} - \varepsilon t_{\rho\alpha} P_{\rho 4} - \varepsilon \frac{\partial H}{\partial \frac{\partial Q_\alpha}{\partial x_\kappa}} s_{4\kappa} x_\kappa - \varepsilon \frac{\partial P_{\alpha 4}}{\partial x_\mu} s_{\mu\nu} x_\nu$$

$$Q_\alpha \to Q_\alpha + \varepsilon \frac{2\pi}{hc} [\bar{\Lambda}, Q_\alpha]$$

$$\bar{\Lambda} = \int \Lambda \, dV$$

$$\Lambda = \left(t_{\alpha\rho} Q_\rho - \frac{\partial Q_\alpha}{\partial x_\kappa} s_{\kappa\nu} x_\nu \right) P_{\alpha 4} - H s_{4\kappa} x_\kappa$$
$$= \left(t_{\alpha\rho} Q_\rho - \frac{\partial Q_\alpha}{\partial x_\mu} s_{\mu\nu} x_\nu \right) P_{\alpha 4} + L s_{4\kappa} x_\kappa$$

$$\frac{2\pi}{hc} [\bar{H}, F] = \frac{\partial F}{\partial x_4}, \qquad \frac{2\pi}{hc} [P_{\alpha 4}, Q'_\rho] = \delta_{\alpha\rho}\, \delta(r, r')$$

$$P_{\alpha 4} \to P_{\alpha 4} + \varepsilon \frac{2\pi}{hc} [\bar{\Lambda}, P_{\alpha 4}]$$

$$\frac{2\pi}{hc} [\bar{\Lambda}, P_{\alpha 4}] = - \left(\frac{\partial \Lambda}{\partial Q_\alpha} - \frac{\partial}{\partial x_i} \frac{\partial \Lambda}{\partial \frac{\partial Q_\alpha}{\partial x_i}} \right)$$

$$= - t_{\alpha\rho} P_{\rho 4} - \frac{\partial}{\partial x_i} (s_{i\nu} x_\nu P_{\alpha 4}) + \frac{\partial H}{\partial Q_\alpha} s_{4\kappa} x_\kappa - \frac{\partial}{\partial x_i} \left(\frac{\partial H}{\partial \frac{\partial Q_\alpha}{\partial x_i}} s_{4\kappa} x_\kappa \right)$$

$$\frac{2\pi}{hc} [\bar{\Lambda}, P_{\alpha 4}] = - t_{\rho\alpha} P_{\rho 4} - \frac{\partial P_{\alpha 4}}{\partial x_i} s_{i\nu} x_\nu - \frac{\partial P_{\alpha 4}}{\partial x_4} s_{4\kappa} x_\kappa - \frac{\partial H}{\partial \frac{\partial Q_\alpha}{\partial x_\kappa}} x_\kappa$$

$$F \to F + \varepsilon \frac{2\pi}{hc} [\bar{\Lambda}, F]$$

$$F \to S F S^{-1}$$

$$S = 1 + \varepsilon \frac{2\pi}{hc} \bar{\Lambda} + \cdots$$

$$\varphi \to S \varphi$$

$$J_\mu = \int \left(\frac{\partial Q_\alpha}{\partial x_\mu} P_{\alpha 4} - \delta_{\mu 4} L \right) dV$$

$J_\mu \to J_\mu + \varepsilon s_{\mu\nu} J_\nu$ (Energie-Impulsvektor $J_k = -ic\,G_k,\ J_4 = \overline{H}$)

$\overline{H} \to \overline{H} + \varepsilon s_{4k} J_k$

$\frac{2\pi}{hc}[\overline{\Lambda}, \overline{H}] = s_{4k} J_k$

$\frac{\partial F}{\partial x_4} = -\frac{2\pi}{hc}[F, \overline{H}]$

$F = \overline{\Lambda}$

$\frac{d\Lambda}{dx_4} = -\frac{2\pi}{hc}[\overline{\Lambda}, \overline{H}] - \int \frac{\partial \mathfrak{L}_\alpha}{\partial x_k} P_{\alpha 4} s_{k4}\, dV$

$= -\frac{2\pi}{hc}[\overline{\Lambda}, \overline{H}] + J_k s_{4k} = 0$

$\overline{\Lambda} = \mathrm{const}$

$s_{\mu\nu} = -s_{\nu\mu}$ (die $t_{\alpha\beta}$ sind durch die $s_{\mu\nu}$ eindeutig bestimmt)

$\frac{\partial T_{\mu\nu}}{\partial x} = 0$

§5. Lorentztransformationen und Eichinvarianz.

$C = \mathrm{div}\,\mathfrak{E} + e\sum_\rho \psi_\rho^* \psi_\rho = 0$

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正シイ。然ルニ φ 及 $\mathfrak{E}_\mu$ ハ之ト vertauschbar ナル。

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