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NO. 1

Letter to the Editor of the Physical Review

~~The~~ Density Matrix in the Theory of the Positron

In the usual theory of ~~the~~ the electron and the positron, ~~only~~ only one sort of them, ~~the electron for example~~, is considered at first, the existence of the other ~~i.e. the positron~~ being deduced as ~~the~~ necessary consequence of the theory. One can proceed, however, on the reverse way, accepting the existence of both at the beginning and only afterwards introducing ~~possible~~ theoretically possible relations between them. The mathematical formulation of the latter method will be as follows.

The quantised wave functions $\psi_-(x, k)$ and $\psi_+(x, k)$ of the electron and the positron satisfy Dirac's equations

$$\left\{ \frac{W \pm eV}{c} + \vec{\alpha}\left(\vec{p} \pm \frac{e}{c}\vec{A}\right) + \rho mc \right\} \psi_{\mp} = 0 \qquad (1)$$

respectively, where x denote position and time ~~of the particle~~ and k takes either of the values 1, 2, 3, 4. If we adopt a representation, in which all matrix elements of α's are real and those of ρ are pure imaginary, the wave functions ψ_-^* and ψ_+^*, which are complex conjugate to ψ_- and ψ_+ respectively, satisfy the same equations (1) for ψ_+ and ψ_- respectively, so that if the relations

$$\psi_- = \psi_+^* \quad , \quad \psi_+^{\cancel{*}} = \psi_-^* \qquad (2)$$

are assumed at an instant for all points, they will remain to hold good forever. These are obviously ~~a~~ mathematical expressions of the equivalence of the anti-

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electron and the positron on the one hand and that of the anti-positron and the electron on the other hand.

accepting (2), ~~Now~~ the resultant charge density ~~at a space time point x~~

$$-\frac{e}{2}\sum_k\{\psi_-^*(xk)\psi_-(xk)-\psi_+^*(xk)\psi_+(xk)\}, \quad (3)$$

at a space time point x, ~~can~~ due to both sorts of particles, can be reduced to

$$-\frac{e}{2}\sum_k\{\psi_-^*(xk)\psi_-(xk)-\psi_-(xk)\psi_-^*(xk)\}, \quad (4)$$

the factor $\frac{1}{2}$ in (3) and (4) being needed on account of the above equivalence. The density matrix, from which physical quantities such as ~~charge density and current densities~~ resultant density etc. can be derived, thus takes the form

$$R(xk, x'k') = \frac{1}{2}\{\psi_-^*(x'k')\psi_-(xk)-\psi_+^*(x'k')\psi_+(xk)\}$$

~~which~~ reduces to

$$= \frac{1}{2}\{\psi_-^*(x'k')\psi_-(xk)-\psi_-(x'k')\psi_-^*(xk)\}, \quad (\cancel{4})$$

by (2), in contrast to the symmetrical density matrix of Heisenberg[1]

$$\frac{1}{2}\{\psi_-^*(x'k')\psi_-(xk)-\psi_-(xk)\psi_-^*(x'k')\}, \quad (\cancel{4})'$$

the factor $\frac{1}{2}$ in (3) and (4) . . . .

In Dirac's approximation[2], in which particles are moving in a common field, it reduces to

$$\frac{1}{2}\left\{\sum_{occ}\psi_n^*(x'k')\psi_n(xk)-\sum_{unocc.}\psi_n(x'k')\psi_n^*(xk)\right\}, \quad (6)$$

in contrast to Dirac's

(1)
(2)

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$$\frac{1}{2}\left\{\sum_{occ}\psi_n^*(x'k')\psi_n(xk)-\sum_{unocc.}\psi_n^*(xk)\psi_n(x'k')\right\} \qquad (6)'$$

where ψ_n, ψ_n^* denote ~~ung~~ normalized (unquantized) wave functions of stationary states of the electron and the first and the second summations refer to the occupied and unoccupied states of the electron respectively.

In the case $V=0$ and $\vec{A}=0$, if we denote the wave function of a positive energy state by ψ_n, its complex conjugate ψ_n^* will be ~~also~~ the wave function of a negative energy state. Hence, if all the negative energy states of the electrons are occupied and all the pos. energy states are ~~unocc~~ empty, ~~each pair of term~~ (6) will be zero for any value of xk and $x'k'$, each ~~pair of terms in~~ term in one summation being compensated by the corresponding term in the other, whereas (6)' is not zero and becomes infinite for $x=x'$. Thus, if there are finite number of unoccupied negative energy states and ~~pos~~ occupied ~~positive~~ energy states, i.e. if there are finite number of electrons and positrons, the expression (6) will remain to be finite everywhere.

~~In the case, in which~~ If potentials are not zero, we should take gauge invariant density matrix introduced by Jordan[1]) instead of (5), ~~a similar~~ which ~~not~~ also reduces to zero in Dirac's approximation for empty space without

1)

~~although~~ the general convergency ~~problem~~ problem of external field. ~~If external field exists, the po~~ ~~(5) or (6) in general field is not solved~~ ~~the density matrix~~ ~~we find that~~ ~~was found~~ yet to be

For weak external field, we find that the density matrix ~~becomes~~ infinite for $x = x'$, ~~if we apply~~ applying ordinary perturbation theory. Thus ~~we need~~ the similar ~~the~~ subtraction technique as in the ordinary theory is ~~needed~~ ~~seems to~~ be needed in this case, ~~so~~ also, so that we are ~~far from~~ still far from complete theory of the positron. ~~Detailed accounts and~~ Further discussions will be made ~~in~~ elsewhere.

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April 21, 1936.

and ~~is~~ the ~~possible~~ application ~~of~~ to neutrino problems