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Note on the Theory of the Light Particle

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Abstract

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§1. Introduction

Experience shows the existence of two sorts of the light particle, i.e. the electron and the positron, ~~the usual way of This~~ with the same mass and the charges ~~only differing~~ just opposite to each other. The well known way of interpreting this fact is ~~first~~ to assume the existence of only one sort of the particle, the electron for example, ~~and~~ the existence of the other, i.e. the positron, being deduced ~~theoretically~~ as the ~~nat~~ necessary consequence of the theory[1]. One can, however, proceed in the reverse way, ~~assuming~~ accepting the existence of both at the beginning and only afterwards ~~th~~ assuming possible theoretically relations between them.

In this paper, the latter method ~~will be~~ is developed and ~~its~~ advantages over the former, ~~will be~~ ~~shown~~ ~~will be~~ ~~is~~ found to have some. Similar considerations ~~method~~ ~~will be~~ ~~is~~ are applied to the theory of the neutrino and compared with ~~these~~ recent ~~formulations~~ ~~of Jordan~~ photon theory of Jordan and Kronig[2].

1) ~~Dirac~~, Heisenberg
2)

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ψ_- and ψ_+ ~~involving~~ each with four components depending on space & time coordinates ~~and~~

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~~We consider first the quantised wave functions of $\psi_-(x,k)$ and $\psi_+(x,k)$~~
~~The quantised wave functions $\psi_-(x,k)$ and $\psi_+(x,k)$~~ of the electron and the positron separately, ~~which satisfy~~ Dirac's equations

$$\left\{\frac{W+eV}{c} + \vec{\alpha}\left(\vec{p} + \frac{e}{c}\vec{A}\right) + \beta mc\right\}\psi_- = 0, \quad (1)$$

$$\left\{\frac{W-eV}{c} + \vec{\alpha}\left(\vec{p} - \frac{e}{c}\vec{A}\right) + \beta mc\right\}\psi_+ = 0, \quad (2)$$

respectively, ~~where x represents the position~~

We consider first the quantised wave functions ψ_- and ψ_+, each with four components, of the electron and the positron separately, which ~~satisfy~~ depend on space time coordinates and satisfy

If we ~~denote~~ adopt a representation, in which all matrix elements of α's are real and those of β are pure imaginary, the wave functions ψ_-^* and ψ_+^*, which are complex conjugate to ψ_- and ψ_+ respectively, satisfy the ~~same~~ equations (2) and (1) for ψ_+ and ψ_- respectively, Same with so that if the relations

$$\psi_- = \psi_+^*, \quad \psi_-^* = \psi_+ \quad (3)$$

are assumed at an instant for all points, ~~they~~ will

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It should be noticed that while all the quantities obtained ~~on the assumption of complete~~ by adding the contributions from the electron and the positron should be divided by ~~two~~ 2 on account of the above equivalence.

continued to hold good forever. These ~~relations~~ are obviously a mathematical expressions of the equivalence of the anti-electron and the positron, ~~this~~ on the one hand and that of the anti-positron and the electron on the other hand. Using the ~~Now, the charge density~~ accepting ~~this the~~ (3), either of (ψ_-, ψ_-^*) ~~ψ_+^*~~ and (ψ_+, ψ_+^*) ~~can~~ can be eliminated from ~~the~~ later formulae completely.

The ~~charge~~ resultant charge density ~~for instance~~, due to both sorts of particles, for example,

$$-\frac{e}{2}\sum_{k=1,2,3,4}\{\psi_-^*(xk)\psi_-(xk)-\psi_+^*(xk)\psi_+(xk)\} \quad (4)$$

at a space time point x, can be reduced to

$$-\frac{e}{2}\sum_{k}\{\psi_-^*(xk)\psi_-(xk)-\psi_-(xk)\psi_-^*(xk)\},$$

which is the same as ~~that of~~ the symmetrical charge density introduced by Dirac and Heisenberg[27] only for mathematical purpose.

The density matrix, from which physical quantities can be derived, takes thus the form

~~$R(xk, x'k') =$~~

$$R(xk, x'k')=\frac{1}{2}\{\psi_-^*(x'k')\psi_-(xk)-\psi_+^*(x'k')\psi_+(xk)\}$$

27)

in contrast to Dirac's expression

$$\frac{1}{2}\left\{\sum_{occ}\psi_n^*(x'k')\psi_n(xk) - \sum_{unocc}\psi_n^*(x'k)\psi_n(xk)\right\} \quad (6')$$

in Dirac's theory),

$$= \frac{1}{2}\{\psi_-^*(x'k')\psi_-(xk) - \psi_-(x'k')\psi_-^*(xk)\},$$

in contrast to ~~slightly~~ ~~which differs~~ from the symmetrical density matrix ~~for~~ of Heisenberg

$$\frac{1}{2}\{\psi_-^*(x'k')\psi_-(xk) - \psi_-(xk)\psi_-^*(x'k')\}. \quad (5')$$

~~by the~~ ~~In~~ Dirac's approximation, in which ~~the~~ ~~electrons and~~ particles ~~positions~~ are moving in a common ~~field~~ self-consistent field, it reduces to

$$\frac{1}{2}\{\sum_{occ.}\psi_n^*(x'k')\psi_n(xk) - \sum_{unocc.}\psi_n(x'k')\psi_n^*(xk)\}, \quad (6)$$

~~to~~ where ~~the first~~ ψ_n, ψ_n^* denote unquantised and normalized ~~unquantised~~ ~~un~~ordinary wave functions of stationary states of the electron and the first and the second summations refer to the occupied and unoccupied states of the electron respectively.

~~If there exists no electromagnetic external field, wave functions of positive~~
~~If V and A are zero ψ's are ψ_n and~~ there ~~exist~~

In the case $V=0$ and $\vec{A}=0$, if we denote ~~ψ_n be~~ ~~a~~ ~~the~~ wave function of ~~the~~ a state ~~with positive energy,~~ positive energy by ψ_n,

i.e. if there are finite number of
electrons and positrons

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; if all the negative energy states of the electrons are occupied and all the pos. energy states are unoccupied,

its complex conjugate ψ_n^* will be ~~also the~~ wave function of a negative energy state, ~~so that~~ Hence, to each term in the first summation of (6) there appear the corresponding term in the second summation so as to ~~make~~ reduce the expression (6) ~~for~~ to zero for ~~any~~ value of $x\,k$ and $x'\,k'$, whereas (6') ~~has are~~ not zero and ~~has a singularity~~ becomes infinite for $x = x'$. ~~Thus for~~ ~~φ~~

If there are finite number of ~~p~~ unoccupied ~~posit~~ negative energy states and occupied positive energy states, the expression (6) ~~will~~ will remains to be finite ~~ev for eu~~ everywhere.