Letter to the Editor

On the Nuclear Transformation with the Absorption of the Orbital Electron

According to the present theory of β-disintegration, the nucleus of atomic number Z transforms into its isobar Z-1 with the emission of a positron and a neutrino, if the difference ΔW of proper energies of these isobares is larger than $mc^2+\mu c^2$, where m and μ are the masses of the electron and the neutrino respectively. On the contrary, the isobar Z-1 transforms into Z with the emission of an electron and an anti-neutrino, if ΔW is smaller than $-mc^2-\mu c^2$. The isobar Z can transform into Z-1 also by absorbing one of the orbital electrons and emitting a neutrino at the same time, if ΔW is larger than $-E+\mu c^2$, where E is the total energy of the orbital electron.

Thus, two isobares with consecutive atomic numbers are both stable, only if ΔW lies between $-mc^2-\mu c^2$ and $-mc^2+\mu c^2$. This condition can be fulfilled very rarely, if the neutrino mass is small compared with the electron mass. Since the existence of several [redacted] such pairs of stable nuclei was confirmed by experiment recently[1], it will be worth while to give a brief account of the results of our [redacted] previous calculations on this subject.[2] It will be interesting, moreover, to determine the ratio of the probabilities of the positron emission and the electron absorption above considered, when ΔW is larger than $mc^2 + \mu c^2$.

First, the mean life time τ of the nucleus Z due to the absorption of either of two K-electrons with $E = mc^2\sqrt{1-\alpha^2 Z^2}$ was calculated for the allowed transition, where α was the fine structure constant. If the neutrino mass is assumed to be zero, τ is approximately proportional to

$$\frac{(\alpha Z)^{2\gamma+1}}{(\Delta w+\gamma)^2} \quad \text{or} \quad \frac{(\alpha Z)^{2\gamma+1}}{(\Delta w+\gamma)^4},$$

according as the coupling scheme of Fermi or Konopinski-Uhlenbeck is adopted, where

$$\Delta w = \frac{\Delta W}{mc^2}, \qquad \gamma = \sqrt{1-\alpha^2 Z^2}.$$

The numerical values for several cases are shown in Table 1.[3)]

Table 1.

Z	αZ	τ(Fermi)	τ(K.-U.)
1	1/137	$2740\,(\Delta w+1)^{-2}$ years	$1860\,(\Delta w+1)^{-4}$ years
2	2/137	$170\,(\Delta w+1)^{-2}$ years	$120\,(\Delta w+1)^{-4}$ years
14	0.1	$200\,(\Delta w+1)^{-2}$ days	$130\,(\Delta w+1)^{-4}$ days
27	0.2	$25\,(\Delta w+1)^{-2}$ days	$16\,(\Delta w+1)^{-4}$ days
69	0.5	$12\,(\Delta w+0.87)^{-2}$ hours	$8\,(\Delta w+0.87)^{-4}$ hours

The apparent discrepancy between these results and the existence of stable pairs of heavy nuclei can be removed, only if we assume

i) the difference of nuclear spins to be large in every case, or

ii) the neutrino mass to be comparable with the electron mass, or

iii) the wave functions of the electron in the neighborhood of the nucleus to be much smaller than those calculated by Dirac's theory.

The extreme case $Z=1$ in Table 1, which corresponds to the transformation of the hydrogen atom into the neutron, will not occur actually, according to the recent data of mass defects, whereas the case $Z=2$ has some practical importance indicating the spontaneous transformation of He^3 into H^3 by absorbing one of the K-electrons.

Next, the ratio σ of the probabilities of the positron emission and the K-electron absorption was calculated on similar assumptions as above, when ΔW is larger than $mc^2+\mu c^2$, the numerical results for $\mu=0$ being summarized in Table 2.

Table 2.

Z	αZ	ΔW	σ (Fermi)	σ (K.-U.)
14	0.1	2	2.9	0.15
14	0.1	5	250.	36.
27	0.2	2	0.2	0.022
27	0.2	5	21.	3.1
82	0.6	2	0.8×10^{-3}	2.5×10^{-5}
82	0.6	5	0.1	0.016

Thus, for ordinary radio-elements emitting positrons, for which Z is small and ΔW is about 5 or more, the ratio σ is so large that the order of the mean life time calculated by assuming the positron emission alone are not changed by the additional contribution of ~~the process of~~ the absorption of the orbital electron. On the contrary, for large values of Z, the latter process will occur far more frequently than the former as long as Δw is not too large compared with 1. ~~It is~~ will be ~~urgently needed to verify these theoretical conclusions, experiment.~~ It will be possible to test these conclusions by experiment.

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1) Bainbridge and Jordan, Phys. Rev. **50**, 282, 1936.

2) Proc. Phys.-Math. Soc. Japan **17**, 467, 1935; **18**, 128, 1936.
See also Bethe and Bacher, Rev. Mod. Phys. **8**, 82, 1936. Similar calculations were made recently by Møller, Phys. Rev. **51**, 84, 1937.
Extension of the calculation to the case of forbidden transitions was made by Lamb, Phys. Rev. 50, 388, 1936. (Abstract)

3) For the numerical calculation in the case of U.-K. coupling, the same coupling constant as that of Bethe and Bacher (l.c. p.193) was employed.