Department of Physics,
Osaka Imperial University,
Osaka, Japan, March 30.

Dear sir,

I believe that you already received a letter from me and S. Sakata entitled "The Mass and the Mean Life Time of the Mesotron." After having sent it out, we noticed that there was an error in the calculation, so that we made a thorough revision of it.

I enclose the corrected letter under the same heading. I beg you will kindly adopt this instead of the previous one.

I remain,

Yours very sincerely,

Hideki Yukawa

Nordheim, Phys. Rev. 55 (1939), 506

$\tau_0 = 1.6 \times 10^{-9}$

$\mu = 200m$

$\frac{g^2}{\hbar c} = 0.3$

$G_F = 5.5 \times 10^{-12}$
$= \frac{4m^4c^2}{\hbar^6} g_F^2$

(16) ms factor が落ちてゐる

∴ 正しくは

$\tau_0 = 2.6 \times 10^{-8}$

Letter to the Editor (Corrected)

The Mass and the Mean Life Time of the Mesotron.

According to the new field theory of the nuclear forces and the β-decay, the mesotron with the charge $-e$ (or $+e$) can transform into a negative (or a positive) electron and an antineutrino (or a neutrino) even in vacuum, the mean life time of the former due to this process being proportional to the energy.[1] In the previous papers,[2] the proper life time τ_0, i.e. the mean life time at rest, of the mesotron was calculated by assuming the interaction of it with the light particle, which was equivalent to that in Fermi's theory of β-decay. The result was

$$\frac{1}{\tau_0} = \frac{g'^2}{\hbar c}\frac{m_u c^2}{\hbar}\left(\frac{2}{3}\lambda_1^2 + \frac{1}{3}\mu_1^2\right) \qquad (1)$$

where g' was a small constant with the dimension of the charge, m_u the proper mass of the mesotron and λ_1, μ_1 the dimensionless constants of the order of 1. Thus we obtained a value

$$\tau_0 = 1.3 \times 10^{-7} \text{ sec} \qquad (2)$$

by taking

$$m_u = 200\,m, \qquad g' = 4 \times 10^{-17}, \qquad \lambda_1 = \mu_1 = 1.$$

Similar calculations were made for the case, which was equivalent to K.-U.'s theory of β-decay, but the value of τ_0 thus obtained was smaller by a factor of the order of $(m/m_u)^2$ than (1).

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1) Bhabha, Nature 141(1938), 117.
2) Yukawa, Sakata and Taketani, Proc. Phys.-Math. Soc. Japan 20(1938), 319; Yukawa, Sakata, Kobayasi and Taketani, ibid., 993. The latter will be referred to as IV.
3) IV, §6.

On the other hand, the proper life time was determined by several authors[4] from the experiment on the cosmic ray according to the suggestion of Euler and Heisenberg.[5] Their results all point to a value

$$\tau_0 = 2\sim 4\times 10^{-6}\ \text{sec}, \tag{3}$$

which is in qualitative agreement with the theoretical value (2) corresponding to Fermi interaction, but is in contradiction with that corresponding to K.-U. interaction. On account of the importance of the problem it will be worth while to consider the former case more closely.

According to Kemmer,[6] we assume that the interaction between two heavy particles at a distance r in S state is given approximately by

$$V_{12} = \vec{\tau}^{(1)}\vec{\tau}^{(2)}\left\{\frac{1}{8} + \frac{5}{24}(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})\right\}\frac{g^2 e^{-\kappa r}}{r} \tag{4}$$

where $\kappa = m_U c/\hbar$ and g is the constant with the dimension of the charge. $\vec{\sigma}^{(i)}$, $\vec{\tau}^{(i)}$ are spin vector and isotopic spin vector of the i-th particle respectively. The numerical values of the constant

$$a = g^2 M/\hbar^2 \kappa = g^2/\hbar c \cdot M/m_U, \tag{5}$$

which changes slowly with m_U, can be determined for several values of m_U by using the results of numerical solution of the deuteron wave equation with the potential of the form (4),[7] where M is the mass of the heavy particle.

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4) Blackett, Phys. Rev. 54(1938), 973; Nature 142(1938), 992; Rossi, ibid., 993; Ehrenfest and Freon, C. R. 207(1938), 853; Johnson and Pomerantz, Phys. Rev. 55(1939), 104; Clay, Jonker and Wiersma, Physica 6(1939), 174.
5) Euler, Naturwiss. 26(1938), 382; Zeits. f. Phys. 110(1938), 450; Euler and Heisenberg, Erg. exakt. Naturwiss. 17(1938), 1. See also Ferretti, Nuovo Cimento 15(1938), 421.
6) Kemmer, Proc. Camb. Phil. Soc. 34(1938), 354. See further IV, §2.
7) Sachs and Goeppert-Mayer, Phys. Rev. 53(1938), 991; Wilson, Proc. Camb. Phil. Soc. 34(1938), 365.

Next, the constant g' is determined by comparing the mean life time of the β-radioactive nucleus obtained in the previous paper[8]) with the corresponding formula in Fermi's theory. The result is

$$\frac{g'^2}{\hbar c} = \frac{\pi}{5}\frac{G^2}{a}\frac{M}{m}\left(\frac{m_u}{m}\right)^3 \tag{6}$$

for the case i) $\lambda_1 = 0$, $\mu_1 = 1$, and

$$\frac{g'^2}{\hbar c} = \frac{\pi}{2}\frac{G^2}{a}\frac{M}{m}\left(\frac{m_u}{m}\right)^3 \tag{7}$$

for the case ii) $\lambda_1 = 1$, $\mu_1 = 0$, where

$$G^2 = 0.9\times 10^{-4}\cdot\frac{\hbar}{mc^2} = 1.2\times 10^{-26}$$

according to Bethe and Critchfield.[9])

Hence, (1) reduces to

$$\frac{1}{\tau_0} = \frac{\pi}{15}\frac{G^2}{a}\left(\frac{m_u}{m}\right)^4\frac{mc^2}{\hbar} \tag{8}$$

for the case i) and to

$$\frac{1}{\tau_0} = \frac{\pi}{3}\frac{G^2}{a}\left(\frac{m_u}{m}\right)^4\frac{mc^2}{\hbar} \tag{9}$$

for the case ii). These expressions show that the proper life time of the mesotron increases rapidly with the decreasing mass m_u, the numerical values for several cases being summarized in Table 1.

Table 1.

	m_u	100 m	125 m	160 m	200 m	
τ_0	case i)	10^{-6}	0.4×10^{-6}	1.2×10^{-7}	0.5×10^{-7}	sec
	case ii)	0.2×10^{-6}	0.8×10^{-7}	0.2×10^{-7}	0.1×10^{-7}	sec

Thus, the theoretical life time is always a little too short to account for the cosmic ray phenomena. However, the discrepancy between theory and experiment diminishes, if we take a value of m_u nearer to 100 m than to 200 m and take

9) Bethe and Critchfield, Phys. Rev. 54(1938), 248.

μ_1 large compared with λ_1.[10)]

Hideki Yukawa

Shoichi Sakata

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March 9.

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10) According to Breit and Knipp (Phys. Rev. 54(1938), 652), the K-electron capture of ^{7}Be can be explained satisfactory, only if the condition ~~μ_1~~ $\lambda_1 < \mu_1$ is fulfilled. See further Betne and Critchfield, loc. cit.

Abstract

It is shown that the mean life time of the mesotron due to spontaneous disintegration depends critically on its mass and that the discrepancy between theory and experiment becomes smaller as the mass decreases, although the theoretical life time is always smaller than the experimental value $2 \sim 4 \times 10^{-6}$ sec.

Letter to the Editor

The Mass and the Mean Life Time of the Mesotron.

According to the new field theory of the nuclear forces and the β-decay, the mesotron with the charge $-e$ (or $+e$) can transform into a negative (or a positive) electron and an antineutrino (or a neutrino) even in vacuum, the mean life time of the former due to this process being proportional to the energy.[1] In the previous papers,[2] the proper life time τ_0, i.e. the mean life time at rest, of the mesotron was calculated by assuming the interaction of ~~t~~it with the light particle, which was equivalent to that in Fermi's theory of β-decay. The result was

$$\frac{1}{\tau_0} = \frac{g'^2}{\hbar c}\cdot\frac{m_u c^2}{\hbar}\left(\frac{2}{3}\lambda_1^2 + \frac{1}{3}\mu_1^2\right), \tag{1}$$

where g' is a small constant ~~of~~ with the dimension of the charge, m_u the proper mass of the mesotron and λ_1, μ_1 are dimensionless constant of the order of 1. Thus we obtained a value

$$\tau_0 = 1.3\times10^{-7}\mathrm{sec} \tag{2}$$

by taking

$$m_u = 200\ m, \qquad g' = 4\times10^{-17}, \qquad \lambda_1 = \mu_1 = 1.$$

Similar calculations were made for the case, which was equivalent to K.-U.'s theory of β-decay, but the value of τ_0 thus obtained was smaller by a factor $(m/m_u)^2$ than (1).

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1) Bhabha, Nature 141(1938), 117.
2) Yukawa, Sakata and Taketani, Proc. Phys.-Math. Soc. Japan 20(1938), 319; Yukawa, Sakata, Kobayasi and Taketani, ibid., 993. The latter will be referred to as IV.
3) IV, §6.

On the other hand, the proper life time was determined by several authors[4]) from the experiments on the cosmic ray according to the suggestion of Euler and Heisenberg[5]). Their results all point to a value

$$\tau_0 = 2\sim 4\times 10^{-6}\ \text{sec}, \qquad (3)$$

which is in qualitative agreement with the theoretical value (2) corresponding to Fermi interaction, but is in contradiction with that corresponding to K.-U. interaction. On account of the importance of the problem it will be worth while to consider the matter more closely. The discrepancy of a factor 10 in the former case seems to be due to ~~x~~ inaccuracies in the theoretical estimation (former case) rather than to experimental errors as will be shown presently.

According to Kemmer,[6]) we assume that the interaction between two heavy particles at a distance r in S state is given approximately by

$$V_{12} = \vec{\tau}^{(1)}\vec{\tau}^{(2)}\left\{\frac{1}{8} + \frac{5}{24}(\vec{\sigma}^{(1)}\vec{\sigma}^{(2)})\right\}\frac{g^2 e^{-\kappa r}}{r} \qquad (4)$$

where $\kappa = m_u c/\hbar$ and g is the constant with the dimension of the charge. $\vec{\sigma}^{(i)}, \vec{\tau}^{(i)}$ are spin vector and isotopic spin vector of the i-th particle respectively. The numerical values of

$$a = g^2 M/\hbar^2 \kappa = g^2/\hbar c \cdot M/m_u, \qquad (5)$$

which changes slowly with m_u, ~~as shown in Table I,~~ can be determined for several values of m_u by using the results of numerical solution of the deuteron ~~problem~~ wave equation with the potential of the form (4),[7]) where M is the mass of the heavy particle.

Next, the constant g' is determined by comparing the mean life time of the -radioactive nucleus obtained in the previous paper[8] with the

4) Blackett, Phys. Rev. 54(1938), 973; Nature 142(1938), 992; Rossi, ibid., 993; Ehrenfest and Freon, C. R. 207(1938), 853; Johnson and Pomerantz, Phys. Rev. 55(1939), 104; Clay, Jonker and Wiersma, Physica 6(1939), 174.
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6) Kemmer, Proc. Camb. Phil. Soc. 34(1938), 354. See further IV, §2.
7) Sachs and Goeppert-Mayer, Phys. Rev. 53(1938), 991; Wilson, Proc. Camb. Phil. Soc. 34(1938), 365.
8) IV, §5.

the general trend

Thus, the general trend of the result shows that the ~~the theoretical life time~~ is ~~always~~ too short ~~by a factor~~. ~~The discrepancy~~ We can expect on the whole. ~~It seems necessary, however,~~ ~~to reformulate~~ for the

corresponding formula in Fermi's theory. The result is

$$\frac{g'^2}{\hbar c} = \frac{\pi}{5}\frac{G^2}{a}\frac{M}{m}\left(\frac{m_U}{m}\right)^3 \tag{6}$$

for the case

i) $\lambda_1 = 0$, $\mu_1 = 1$

and

$$\frac{g'^2}{\hbar c} = \frac{\pi}{2}\frac{G^2}{a}\frac{M}{m}\left(\frac{m_U}{m}\right)^3 \tag{7}$$

for the case

ii) $\lambda_1 = 1$, $\mu_1 = 0$

with

$$G = \frac{g_F \cdot m^2c}{\sqrt{2\pi^3}\,\hbar^3} = 1.1\times10^{-13} \longrightarrow G^2 = 0.9\times10^{-4}\cdot\frac{\hbar}{mc^2} = 1.2\times10^{-25}$$

according to Bethe and Critchfield[9]) (54(1938), 248)
where g_F stands for Fermi's constant $g = 4\times10^{-50}$ cm³ erg.

Hence, (1) reduces to

$$\frac{1}{\tau_0} = \frac{\pi}{15}\frac{G^2}{a}\left(\frac{m_U}{m}\right)^4\frac{mc^2}{\hbar} \tag{8}$$

for the case i) and to

$$\frac{1}{\tau_0} = \frac{\pi}{3}\frac{G^2}{a}\left(\frac{m_U}{m}\right)^4\frac{mc^2}{\hbar} \tag{9}$$

for the case ii). These expressions show that the proper life time of the mesotron increases rapidly with the decreasing mass m_U, the numerical ~~s~~ values for several cases being summarized in Table ~~I~~ 1.

Table 1.

	m_U	100 m	125 m	160 m	200 m	
τ_0	case i)	~~9.5~~ * $\times10^{-6}$	~~3.3~~ 0.4 $\times10^{-6}$	1.2×10^{-7}	0.~~5~~9 $\times10^{-7}$	sec
	case ii)	~~1.9~~ 0.2 $\times10^{-7}$	~~0.7~~ 0.8 $\times10^{-7}$	0.2,4 $\times10^{-7}$	0.~~1~~ $\times10^{-7}$	sec

~~Thus,~~ However, the ~~agreement~~ discrepancy between theory and experiment is not very ~~satisfactory,~~ serious, as long as we can take a value of m_U nearer to 100 m than to 200 m ~~intermediate between 100 m and 20~~ 200 m and take $\lambda_1 \ll \mu_1$.[10]) Complement agreement ~~between~~, however, can be expected

10) According to Breit and Knipp (Phys. Rev. 54(1938), 652), the K-electron capture of ^{7}Be can be explained satisfactory, only ~~if~~ the ~~latter~~ condition $\lambda_1 < \mu_1$ is fulfilled. See further Bethe and Critchfield, loc. cit.

Hideki Yukawa
Shoichi Sakata

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Osaka Imperial University,
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March 9.

Abstract

The mean life time of the mesotron due to spontaneous disintegration is shown to depend sensitively on its mass, and so that the satisfactory agreement is expected for the mass between 100 m and 160 m.

It is shown that the on its mass m_u and that the theoretical life time is always smaller than the experimental value $\tau_0 = 2 \sim 4 \times 10^{-6}$ sec, although the discrepancy ~~decreases~~ as m_u decreases becomes smaller