DEPARTMENT OF PHYSICS
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DATE
NO.

constant coefficient

$$\vec{F} = \sum_{l\cdot m} \begin{pmatrix} A^x_{lm} \\ A^y_{lm} \\ A^z_{lm} \end{pmatrix} F_l^m(r).\ P_l^m(\theta\cdot\varphi).$$

$$\vec{U} = \sum_{l\cdot m} \begin{pmatrix} B^x_{lm} \\ B^y_{lm} \\ B^z_{lm} \end{pmatrix} U_l^m(r)\ P_l^m(\theta\,\varphi).$$

$$\vec{G} = \sum_{l\cdot m} \begin{pmatrix} C^x_{lm} \\ C^y_{lm} \\ C^z_{lm} \end{pmatrix} G_l^m(r).\ P_l^m(\theta\cdot\varphi).$$

$$U_0 = \sum_{l\cdot m} D_{lm}\ R_l^m(r).\ P_l^m(\theta\,\varphi)$$

$$P_l^m(\theta\cdot\varphi) = \cancel{l}\,(l-m)!\ P_l^m(\cos\theta)\, e^{im\varphi}.$$

と仮定して.

$$\frac{i}{\hbar c}(V-E)\vec{F} - \mathrm{curl}\,\vec{G} - \kappa\vec{U} = 0$$

$$\frac{i}{\hbar c}(V-E)\vec{U} + \mathrm{grad}\,U_0 + \kappa\vec{F} = 0$$

$$\mathrm{div}\,\vec{F} + \kappa U_0 = 0$$

$$\mathrm{curl}\,\vec{U} - \kappa\vec{G} = 0$$

に代入すれば

$G_{l}^{m}\,P_{l+1}^{m-1}$ の如き項は $G_{l-1}^{m+1}\,P_l^m$ と書き代へて. P_l^mで~~括~~くくって. ~~書けば~~ $P_l^m(\theta\,\varphi)$ のみをのぞけば

I.

$$\frac{i}{\hbar c}(V-E)\,A^x_{lm} F_l^m - \frac{C^z_{l-1,m-1}}{2i(2l-1)}\left\{\frac{dG^{m-1}_{l-1}}{dr} - \frac{l-1}{r}G^{m-1}_{l-1}\right\} + \frac{C^y_{l-1,m}}{2l-1}\left\{\frac{dG^m_{l-1}}{dr} - \frac{l-1}{r}G^m_{l-1}\right\}$$

$$-\frac{C^z_{l-1,m+1}}{2i(2l-1)}\left\{\frac{dG^{m+1}_{l-1}}{dr} - \frac{l-1}{r}G^{m+1}_{l-1}\right\} + \frac{(l+2-m)(l+1-m)}{2i(2l+3)}\,C^z_{l+1,m-1}\left\{\frac{dG^{m-1}_{l+1}}{dr} + \frac{l+2}{r}G^{m-1}_{l+1}\right\}$$

$$+\frac{(l+2)(l+m+1)(l+1-m)}{2l+3}\,C^{(y)}_{l+1,m}\left\{\frac{dG^m_{l+1}}{dr} + \frac{l+2}{r}G^m_{l+1}\right\}$$

$$+\frac{(l+m+2)(l+m+1)}{2i(2l+3)}\,C^z_{l+1,m+1}\left\{\frac{dG^{m+1}_{l+1}}{dr} + \frac{l+2}{r}G^{m+1}_{l+1}\right\} - K\,B^x_{lm}\,U_l^m = 0$$

$$\cancel{\frac{C^z_{l-1,m-1}}{2i} - C^y_{l-1,m} + \frac{C^z_{l-1,m+1}}{2i}}$$

今 $$\frac{1}{(2l-1)}\left\{\frac{C^z_{l-1,m-1}}{2i}G^{m-1}_{l-1} - C^y_{l-1,m}G^m_{l-1} + \frac{C^z_{l-1,m+1}}{2i}G^{m+1}_{l-1}\right\} = \xi^{(1)}_{lm}$$

$$\frac{1}{2l+3}\left\{-\frac{1}{2i}(l+2-m)(l+1-m)C^z_{l+1,m-1}G^{m-1}_{l+1} - (l+2)(l+m+1)(l+1-m)C^{(y)}_{l+1,m}G^m_{l+1}\right.$$
$$\left. - \frac{1}{2i}(l+m+2)(l+m+1)C^{(z)}_{l+1,m+1}G^{m+1}_{l+1}\right\} = \xi^{(2)}_{l,m}$$ とすると

x成分

(1) $$\frac{i}{\hbar c}(V-E)A^x_{lm}F_l^m - \left(\frac{d}{dr} - \frac{l-1}{r}\right)\xi^{(1)}_{lm} - \left(\frac{d}{dr} + \frac{l+2}{r}\right)\xi^{(2)}_{l,m} - K B^x_{lm}U_l^m = 0$$

仝様に

(2) $$\frac{i}{\hbar c}(V-E)A^y_{lm}F_l^m - \left(\frac{d}{dr} - \frac{l-1}{r}\right)\eta^{(1)}_{l,m} - \left(\frac{d}{dr} + \frac{l+2}{r}\right)\eta^{(2)}_{l,m} - K B^y_{lm}U_l^m = 0$$

y成分

(3) $$\frac{i}{\hbar c}(V-E)A^z_{lm}F_l^m - \left(\frac{d}{dr} - \frac{l-1}{r}\right)\zeta^{(1)}_{l,m} - \left(\frac{d}{dr} + \frac{l+2}{r}\right)\zeta^{(2)}_{l,m} - K B^z_{l,m}U_l^m = 0$$

但し $$\frac{-1}{(2l-1)}\left[\frac{C^z_{l-1,m-1}}{2}G^{m-1}_{l-1} - C^x_{l-1,m}G^m_{l-1} - \frac{C^z_{l-1,m+1}}{2}G^{m+1}_{l-1}\right] = \eta^{(1)}_{l,m}$$

z成分

$$\frac{1}{2l+3}\left[\frac{(l+2-m)(l+1-m)}{2}C^z_{l+1,m-1}G^{m-1}_{l+1} + (l+m+1)(l+1-m)C^x_{l+1,m}G^m_{l+1}\right.$$
$$\left. - \frac{1}{2}(l+m+2)(l+m+1)G^{m+1}_{l+1}C^z_{l+1,m+1}\right] = \eta^{(2)}_{l,m}$$

II.

$$\frac{1}{2(2l-1)}\left[\left(C^{y}_{l-1\cdot m-1}-\frac{1}{i}C^{x}_{l-1\cdot m-1}\right)G^{m-1}_{l-1}-\left(C^{y}_{l-1\cdot m+1}+\frac{1}{i}C^{x}_{l-1\cdot m+1}\right)G^{m+1}_{l-1}\right]=\zeta^{(1)}_{l\cdot m}$$

$$\frac{1}{2(2l+3)}\left[(l+m+2)(l+m+1)\left(C^{y}_{l+1\cdot m+1}+\frac{1}{i}C^{x}_{l+1\cdot m+1}\right)G^{m+1}_{l+1}+(l+2-m)(l+1-m)\left(\frac{1}{i}C^{x}_{l+1\cdot m-1}-C^{y}_{l+1\cdot m-1}\right)G^{m-1}_{l+1}\right]$$

$$=\zeta^{(2)}_{l\cdot m}.$$

又 $\frac{i}{\hbar c}(V-E)\vec{U}+\mathrm{grad}\,U_0+\kappa\vec{F}=0$ からは

$$\frac{i}{\hbar c}\cancel{(V-E)B^{x}_{lm}U^{m}_{l}+\left(\frac{d}{dr}-\frac{l-1}{r}\right)J^{(1)}_{lm}+\left(\frac{d}{dr}+\frac{l+2}{r}\right)K^{m}_{l}+k A^{x}_{lm}F^{m}_{l}=0},$$

(4) $$\frac{i}{\hbar c}(V-E)B^{x}_{lm}U^{m}_{l}+\left(\frac{d}{dr}-\frac{l-1}{r}\right)J^{(1)}_{lm}+\left(\frac{d}{dr}+\frac{l+2}{r}\right)J^{(2)}_{lm}+k A^{x}_{lm}F^{m}_{l}=0$$

(x成分)

(5) $$\frac{i}{\hbar c}(V-E)B^{y}_{lm}U^{m}_{l}+\left(\frac{d}{dr}-\frac{l-1}{r}\right)K^{(1)}_{lm}+\left(\frac{d}{dr}+\frac{l+2}{r}\right)K^{(2)}_{lm}+k A^{y}_{lm}F^{m}_{l}=0$$

(y成分)

(6) $$\frac{i}{\hbar c}(V-E)B^{z}_{lm}U^{m}_{l}+\left(\frac{d}{dr}-\frac{l-1}{r}\right)\frac{1}{2l-1}D_{l-1\cdot m}R^{m}_{l-1}$$

$$+\frac{(l+1+m)(l+1-m)}{2l+3}\cdot\left(\frac{d}{dr}+\frac{l+2}{r}\right)D_{l+1\cdot m}R^{m}_{l+1}+k A^{z}_{lm}F^{m}_{l}=0.$$

(z成分)

但し

$$\frac{1}{2(2l-1)}\left[D_{l-1\cdot m-1}R^{m-1}_{l-1}-D_{l-1\cdot m+1}R^{m+1}_{l-1}\right]=J^{(1)}_{lm}$$

$$\frac{1}{2(2l+3)}\left[(l+m+2)(l+m+1)D_{l+1\cdot m+1}R^{m+1}_{l+1}-(l+2-m)(l+1-m)D_{l+1\cdot m-1}R^{m-1}_{l+1}\right]=J^{(2)}_{lm}$$

$$\frac{1}{2i(2l-1)}\left[D_{l-1\,m-1}R^{m-1}_{l-1}+D_{l-1\,m+1}R^{m+1}_{l-1}\right]=K^{(1)}_{lm}$$

$$\frac{-1}{2i(2l+3)}\left[(l+m+2)(l+m+1)D_{l+1\cdot m+1}R^{m+1}_{l+1}+(l+2-m)(l+1-m)D_{l+1\cdot m-1}R^{m-1}_{l+1}\right]=K^{(2)}_{lm}$$

又 $\mathrm{div}\,\vec{F} + \kappa U_0 = 0$ からは

$\cancel{\frac{d}{dr}}\ \cancel{d}$

$$(7)\quad \boxed{\left(\frac{d}{dr} - \frac{l-1}{r}\right) M_{lm}^{(1)} + \left(\frac{d}{dr} + \frac{l-2}{r}\right) M_{lm}^{(2)} + \kappa D_{lm} R_{lm} = 0.}$$

こゝに．

$$M_{lm}^{(1)} = \frac{1}{2(2l-1)}\left[A^x_{l-1.m-1} F_{l-1}^{m-1} - A^x_{l-1.m+1} F_{l-1}^{m+1}\right] + \frac{1}{2i(2l-1)}\left[A^y_{l-1.m-1} F_{l-1}^{m-1} + A^y_{l-1.m+1} F_{l-1}^{m+1}\right] + \frac{A^z_{l-1.m}}{2l-1} F_{l-1}^{m}.$$

$$M_{lm}^{(2)} = \frac{1}{2(2l+3)}\Big[(l+m+2)(l+m+1)\left(A^x_{l+1.m+1} - \frac{1}{i}A^y_{l+1.m+1}\right) \cancel{R_{l+1}} F_{l+1}^{m+1} - (l+2-m)(l+1-m)\left(A^x_{l+1.m-1} + \frac{1}{i}A^y_{l+1.m-1}\right) F_{l+1}^{m-1}\Big] + \frac{(l+1+m)(l+1-m)}{2l+3} A^z_{l+1.m} R_{l+1}^{m}$$

及び $\mathrm{curl}\,\vec{\cancel{G}} - \kappa \vec{G} = 0$ から

$$(8)\quad \left(\frac{d}{dr} - \frac{l-1}{r}\right) X_{lm}^{(1)} + \left(\frac{d}{dr} + \frac{l+2}{r}\right) X_{lm}^{(2)} - \kappa C^x_{lm} G_l^m = 0.\quad (x\text{成分})$$

$$(9)\quad \left(\frac{d}{dr} - \frac{l-1}{r}\right) Y_{lm}^{(1)} + \left(\frac{d}{dr} + \frac{l+2}{r}\right) Y_{lm}^{(2)} - \kappa C^y_{lm} G_l^m = 0.\quad (y\text{成分})$$

$$(10)\quad \left(\frac{d}{dr} - \frac{l-1}{r}\right) Z_{lm}^{(1)} + \left(\frac{d}{dr} + \frac{l+2}{r}\right) Z_{lm}^{(2)} - \kappa C^z_{lm} G_l^m = 0.\quad (z\text{成分})$$

こゝに

$$X_{lm}^{(1)} = \frac{1}{2l-1}\left\{\frac{1}{2i} B^z_{l-1.m-1} U_{l-1}^{m-1} - B^y_{l-1.m} U_{l-1}^{m} + \frac{1}{2i} B^z_{l-1.m+1} U_{l-1}^{m+1}\right\}$$

$$X_{lm}^{(2)} = \frac{-1}{2l+3}\left\{\frac{1}{2i}(l+2-m)(l+1-m) B^z_{l+1.m-1} U_{l+1}^{m-1} + (l+2)(l+m+1)(l+m-1) B^y_{l+1.m} U_{l+1}^{m} + \frac{1}{2i}(l+m+2)(l+m+1) B^{(z)}_{l+1.m+1} U_{l+1}^{m+1}\right\}$$

$$Y_{lm}^{(1)} = -\frac{1}{2l-1}\left\{\frac{1}{2} B^z_{l-1.m-1} U_{l-1}^{m-1} - B^x_{l-1.m} U_{l-1}^{m} - \frac{1}{2} B^z_{l-1.m+1} U_{l-1}^{m+1}\right\}$$

$$Y_{lm}^{(2)} = \frac{1}{2l+3}\left\{\frac{1}{2}(l+2-m)(l+1-m) B^z_{l+1.m-1} U_{l+1}^{m-1} + (l+m+1)(l+1-m) B^x_{l+1.m} U_{l+1}^{m} - \frac{1}{2}(l+m+2)(l+m+1) U_{l+1}^{m+1} B^z_{l+1.m+1}\right\}$$

$$\left\{\begin{aligned} Z_{lm}^{(1)} &= \frac{1}{2(2l-1)}\left\{\left(B_{l-1.m-1}^{y} - \frac{1}{i}B_{l-1.m-1}^{x}\right)U_{l-1}^{m-1} - \left(B_{l-1.m+1}^{y} + \frac{1}{i}B_{l-1.m+1}^{x}\right)U_{l-1}^{m+1}\right\} \\ Z_{l.m}^{(2)} &= \frac{1}{2(2l+3)}\left\{(l+m+2)(l+m+1)\left(B_{l+1.m+1}^{y} + \frac{1}{i}B_{l+1.m+1}^{x}\right)U_{l+1}^{m+1}\right. \\ &\qquad \left. + (l+2-m)(l+1-m)\left(\frac{1}{i}B_{l+1.m-1}^{x} - B_{l+1.m-1}^{y}\right)U_{l+1}^{m-1}\right\} \end{aligned}\right.$$

$$\frac{i}{\hbar c}(V-E)A_{lm}^{x} = \alpha^{x} \qquad \frac{i}{\hbar c}(V-E)A_{lm}^{y} = \alpha^{y} \qquad \frac{i}{\hbar c}(V-E)A_{lm}^{z} = \alpha^{z}$$

$$\frac{i}{\hbar c}(V-E)B_{lm}^{x} = \beta^{x} \qquad \text{〃} \quad \beta^{y} \qquad \text{〃} \quad \beta^{z}$$

① $\alpha^{x}F_{l}^{m} - \left(\frac{d}{dr} - \frac{l-1}{r}\right)\xi_{l.m}^{(1)} - \left(\frac{d}{dr} + \frac{l+2}{r}\right)\xi_{l.m}^{(2)} - (\kappa B_{lm}^{x})U_{l}^{m} = 0.$

② $\alpha^{y}F_{l}^{m} - \left(\frac{d}{dr} - \frac{l-1}{r}\right)\eta_{lm}^{(1)} - \left(\frac{d}{dr} + \frac{l+2}{r}\right)\eta_{lm}^{(2)} - (\kappa B_{lm}^{y})U_{l}^{m} = 0.$

③ $\alpha^{z}F_{l}^{m} - \left(\frac{d}{dr} - \frac{l-1}{r}\right)\zeta_{lm}^{(1)} - \left(\frac{d}{dr} + \frac{l+2}{r}\right)\zeta_{lm}^{(2)} - (\kappa B_{lm}^{z})U_{l}^{m} = 0.$

④ $\beta^{x}U_{l}^{m} + \left(\frac{d}{dr} - \frac{l-1}{r}\right)J_{lm}^{(1)} + \left(\frac{d}{dr} + \frac{l+2}{r}\right)J_{lm}^{(2)} + (\kappa A_{lm}^{x})F_{l}^{m} = 0$

⑤ $\beta^{y}U_{l}^{m} + \left(\frac{d}{dr} - \frac{l-1}{r}\right)K_{lm}^{(1)} + \left(\frac{d}{dr} + \frac{l+2}{r}\right)K_{lm}^{(2)} + (\kappa A_{lm}^{y})F_{l}^{m} = 0$

⑥ $\beta^{z}U_{l}^{m} + \left(\frac{d}{dr} - \frac{l-1}{r}\right)\left(\frac{D_{l-1.m}R_{l-1}^{m}}{2l-1}\right) + \left(\frac{d}{dr} + \frac{l+2}{r}\right)\left(\frac{l+1+m \cdot l+1-m}{2l+3} \cdot D_{l+1.m}R_{l+1}^{m}\right)$
$+ \kappa A_{lm}^{z}F_{l}^{m} = 0.$

⑧ $\left(\frac{d}{dr} - \frac{l-1}{r}\right)X_{lm}^{(1)} + \left(\frac{d}{dr} + \frac{l+2}{r}\right)X_{lm}^{(2)} - (\kappa C_{lm}^{x})G_{l}^{m} = 0.$

⑨ $\left(\frac{d}{dr} - \frac{l-1}{r}\right)Y_{lm}^{(1)} + \left(\frac{d}{dr} + \frac{l+2}{r}\right)Y_{lm}^{(2)} - (\kappa C_{lm}^{y})G_{l}^{m} = 0$

⑩ $\left(\frac{d}{dr} - \frac{l-1}{r}\right)Z_{lm}^{(1)} + \left(\frac{d}{dr} + \frac{l+2}{r}\right)Z_{lm}^{(2)} - (\kappa C_{lm}^{z})G_{l}^{m} = 0$

⑦ $\left(\frac{d}{dr} - \frac{l-1}{r}\right)M_{lm}^{(1)} + \left(\frac{d}{dr} + \frac{l+2}{r}\right)M_{lm}^{(2)} + \kappa D_{lm}R_{lm} = 0.$