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100) DATE

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(van der Waerden, Gött. Nachr. 1929, 100)

O. Laporte and G. E. Uhlenbeck, Application of Spinor Analysis to the Maxwell and Dirac Equations (Phys. Rev. 37, 1931, 1380)

$$[\Gamma^k p_k + mc]\psi = 0 \qquad p_k = \frac{\hbar}{i}\frac{\partial}{\partial x^k} + \phi_k$$

$$\Gamma^1 = \begin{pmatrix} & & & -1 \\ & & -1 & \\ & 1 & & \\ 1 & & & \end{pmatrix} \quad \Gamma^2 = \begin{pmatrix} & & & i \\ & & -i & \\ & -i & & \\ i & & & \end{pmatrix} \quad \Gamma^3 = \begin{pmatrix} & & -1 & \\ & & & 1 \\ 1 & & & \\ & -1 & & \end{pmatrix} \quad \Gamma^4 = \begin{pmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{pmatrix}$$

$$x = x'L \qquad p = L^{-1}p'$$

(L) $\left\{ \begin{array}{l} \left.\begin{array}{l} \xi_1' = \alpha_{11}\xi_1 + \alpha_{12}\xi_2 \\ \xi_2' = \alpha_{21}\xi_1 + \alpha_{22}\xi_2 \end{array}\right\} \quad \left.\begin{array}{l} \bar{\xi}_1' = \bar{\alpha}_{11}\bar{\xi}_1 + \bar{\alpha}_{12}\bar{\xi}_2 \\ \bar{\xi}_2' = \bar{\alpha}_{21}\bar{\xi}_1 + \bar{\alpha}_{22}\bar{\xi}_2 \end{array}\right\} \\ \begin{vmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{vmatrix} = 1 \qquad 6 = 8 - 2 : \text{parameters} \end{array} \right.$

ξ_1, ξ_2 : spinor of 1st rank ; a_k ; $k = 1, 2$
ξ_1^*, ξ_2^* : 〃 〃 ; $b_{\dot r}$; $\dot r = \dot 1, \dot 2$.

$\xi_1\xi_1, \xi_1\xi_2, \xi_2\xi_1, \xi_2\xi_2$: 2nd rank ; a_{kl} ; $k, l = 1, 2$
$\bar\xi_1\bar\xi_1$, 〃 〃 〃 : 〃 ; $b_{\dot r\dot s}$; $\dot r, \dot s = \dot 1, \dot 2$
$\xi_1\bar\xi_1$, 〃 〃 〃 : mixed spinor 〃 ; $c_{\dot r k}$; $\dot r = \dot 1, \dot 2$; $k = 1, 2$

$\xi_1'\eta_2' - \xi_2'\eta_1' = \xi_1\eta_2 - \xi_2\eta_1$: invariant
$a^1 = a_2 \quad a^2 = -a_1$: contravariant spinor a^k
$b^{\dot 1} = b_{\dot 2} \quad b^{\dot 2} = -b_{\dot 1}$: 〃 〃 $b^{\dot r}$
$a_1c^1 + a_2c^2 = a_\lambda c^\lambda$; $b_{\dot 1}d^{\dot 1} + b_{\dot 2}d^{\dot 2} = b_{\dot\rho}d^{\dot\rho}$: invariant.

$$a^k = \epsilon^{k\lambda}a_\lambda$$

$$\epsilon^{kl} = \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix}, \quad \epsilon_{kl} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\epsilon^{k\lambda}\epsilon_{l\lambda} = \delta_l^k$$

$a_\lambda b^\lambda = -a^\lambda b_\lambda$

$\therefore\ a_\lambda a^\lambda = 0;\quad a^{\lambda\mu\nu} a_{\lambda\mu\nu} = 0$

$a^\lambda b_\lambda c_\mu + a_\lambda b_\mu c^\lambda + a_\mu b^\lambda c_\lambda = 0$

1°
2° $a_{\dot{r}\dot{l}t} = a_{l\dot{r}t} = a_{lt\dot{r}}$

$A^1 = A_1 = \frac{1}{2}(a_{\dot{2}1} + a_{\dot{1}2})$ $\quad \frac{1}{2}(a_{\dot{1}1} - a_{\dot{2}2}) = A^3 = A_3$

$\frac{1}{2i}(a_{\dot{2}1} - a_{\dot{1}2}) = A^2 = A_2$ $\quad \frac{1}{2}(a_{\dot{1}1} + a_{\dot{2}2}) = A^4 = -A_4$

$\sim$ $a_{\dot{2}1} = -a^{\dot{1}2} = A^1 + iA^2 = A_1 + iA_2$

$a_{\dot{1}2} = -a^{\dot{2}1} = A^1 - iA^2 = A_1 - iA_2$

$a_{\dot{1}1} = a^{\dot{2}2} = A^3 + A^4 = A_3 - A_4$

$-a_{\dot{2}2} = -a^{\dot{1}1} = A^3 - A^4 = A_3 + A_4$

$-\frac{1}{2} a_{\dot{\sigma}\tau} a^{\dot{\sigma}\tau} = A_\gamma A^\gamma$

一つの(U) transf. に一つの(L) が対応.
逆に一つの(L) に対し二つの(U) (sign ±) が対応.

例へば.
$ct' = ct\cosh\theta + z\sinh\theta$
$z' = ct\sinh\theta + z\cosh\theta$
$\quad \alpha_{kl} = \begin{pmatrix} \pm e^{\frac{\theta}{2}} & 0 \\ 0 & \pm e^{-\theta/2} \end{pmatrix}$

$x' = x\cos\theta + y\sin\theta$
$y' = -x\sin\theta + y\cos\theta$
$\quad \alpha_{kl} = \begin{pmatrix} \pm e^{-i\theta/2} & 0 \\ 0 & \pm e^{i\theta/2} \end{pmatrix}$

$a_{\dot{2}\dot{2}11} = a^{\dot{1}\dot{1}22} = A^{11} - A^{22} + i(A^{12} + A^{21}) = \cdots$

a

$a_k \quad a_{\dot{m}}$

$a_{kl} \quad \underline{a_{\dot{m}l}} \quad a_{\dot{m}\dot{n}}$

$a_{klt} \quad a_{\dot{m}lt} \quad a_{\dot{m}\dot{n}t} \quad a_{\dot{m}\dot{n}\dot{r}}$

$a_{kltv} \quad a_{\dot{m}ltv} \quad \underline{a_{\dot{m}\dot{n}tv}} \quad a_{\dot{m}\dot{n}\dot{r}v} \quad a_{\dot{m}\dot{n}\dot{r}\dot{t}}$

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$$a_{kl} = \frac{1}{2}(a_{kl}+a_{lk}) + \frac{1}{2}(a_{kl}-a_{lk}) = \sigma_{kl} + \alpha_{kl}$$

$$\alpha_{12} = -\alpha_{21} = \frac{1}{2}(a_{12}-a_{21}) = \frac{1}{2}(a_1{}^1 + a_2{}^2) = \underbrace{\frac{1}{2}a_\rho{}^\rho}_{\text{invariant}}$$

F^{kl} : six vector

$$F^{*}_{kl} = \pm\frac{i}{2}\delta_{kl\alpha\beta}F^{\alpha\rho} \quad \text{~~self~~-dual tensor}$$

$$F^{**kl} = F^{kl}.$$

self-dual tensor : $G^{kl} = F^{kl} + F^{*kl}$

$$\bar{G}^{kl} = F^{kl} - F^{*kl}$$

$$G^{kl} = \begin{pmatrix} 0 & k_3 & -k_2 & -ik_1 \\ -k_3 & 0 & k_1 & -ik_2 \\ k_2 & -k_1 & 0 & -ik_3 \\ ik_1 & ik_2 & ik_3 & 0 \end{pmatrix}$$

$$\bar{G}^{kl} = \quad , \quad ,$$

$$\left.\begin{matrix} g_{11} = 2(\bar{k}_2 - i\bar{k}_1) \qquad g_{22} = 2(\bar{k}_2 + i\bar{k}_1) \\ g_{12} = g_{21} = 2ik_3 \end{matrix}\right\}$$

$$\left.\begin{matrix} g_{\dot{1}\dot{1}} = 2(k_2 + ik_1) \qquad g_{\dot{2}\dot{2}} = 2(k_2 - ik_1) \\ g_{\dot{1}\dot{2}} = g_{\dot{2}\dot{1}} = -2ik_3 \end{matrix}\right\}$$

$$\left.\begin{matrix} \partial^{\dot{1}}{}_1 = \partial_{\dot{2}1} = \frac{\partial}{\partial x^1} + i\frac{\partial}{\partial x^2} \\ -\partial^{\dot{2}}{}_2 = \partial_{\dot{1}2} = \frac{\partial}{\partial x^1} - i\frac{\partial}{\partial x^2} \end{matrix}\right\} \quad \left.\begin{matrix} -\partial^{\dot{2}}{}_1 = \partial_{\dot{1}1} = \frac{\partial}{\partial x^3} - \frac{\partial}{\partial x^4} \\ -\partial^{\dot{1}}{}_2 = -\partial_{\dot{2}2} = \frac{\partial}{\partial x^3} + \frac{\partial}{\partial x^4} \end{matrix}\right\}$$

$$(x^1, x^2, x^3, x^4) \rightarrow (x, y, z, ct)$$

$$\left.\begin{matrix} \frac{\partial \phi^\alpha}{\partial x^\alpha} = -\frac{1}{2}\partial_{\dot{\sigma}\tau}\phi^{\dot{\sigma}\tau} \\ \frac{\partial^2 S}{\partial x_\alpha \partial x^\alpha} = -\frac{1}{2}\partial_{\dot{\sigma}\tau}\partial^{\dot{\sigma}\tau}S. \end{matrix}\right\}$$

$$F^{kl} = \begin{pmatrix} 0 & H_z & -H_y & -F_x \\ & 0 & H_x & -E_y \\ & & 0 & -E_z \\ & & & 0 \end{pmatrix} \qquad \overset{*}{F}_{kl} = -i \begin{pmatrix} 0 & E_z & -E_y & H_x \\ & 0 & E_x & H_y \\ & & 0 & H_z \\ & & & 0 \end{pmatrix}$$

$$\frac{\partial F^{k\lambda}}{\partial x^\lambda} = S^k, \qquad \frac{\partial F^{*k\lambda}}{\partial x^\lambda} = 0 \qquad S^k\left(\frac{\rho v_x}{c}, \ldots, \rho\right)$$

$$\frac{\partial S^\lambda}{\partial x^\lambda} = 0 \qquad \searrow \quad \frac{\partial G^{k\lambda}}{\partial x^\lambda} = S^k \qquad G^{kl} = F^{kl} + F^{*kl}$$

$$(\vec{F}_k = \vec{H} - i\vec{E}) \qquad \phi^k = (A_x, \ldots \phi) \qquad G^{kl} = \frac{\partial \phi^l}{\partial x_k} - \frac{\partial \phi^k}{\partial x_l}$$

$$\frac{\partial \phi^\lambda}{\partial x^\lambda} = 0. \qquad \frac{\partial^2 \phi^k}{\partial x_\lambda \partial x^\lambda} = S^k$$

$$\phi^k = \frac{\partial Z^{k\lambda}}{\partial x^\lambda} \qquad \frac{\partial}{\partial x^\lambda}\frac{\partial^2 Z^{k\lambda}}{\partial x_\alpha \partial x^\alpha} = S^k$$

$$Z^{kl} = i\begin{pmatrix} 0 & Z_z & -Z_y & iZ_x \\ & 0 & Z_x & iZ_y \\ & & 0 & iZ_z \\ & & & 0 \end{pmatrix} \qquad A = i\,\mathrm{curl}\,Z - \frac{1}{c}\frac{\partial Z}{\partial t}$$

$$\phi = \mathrm{div}\,Z$$

$$S^k = \frac{\partial Q^{k\lambda}}{\partial x^\lambda}. \qquad \underset{\sim\sim\sim\sim\sim\sim\sim\sim}{\frac{\partial^2 Z^{kl}}{\partial x_\alpha \partial x^\alpha} = Q^{kl}}.$$

$$T_k{}^l = \frac{1}{4}\left(G_{k\alpha} \overline{G}^{l\alpha} + \overline{G}_{k\alpha} G^{l\alpha}\right)$$

$$T_\alpha{}^\alpha = 0$$

$$\left.\begin{aligned} &\partial^{j}{}_{l}\, g_{ji n} = 2 S_{inl} \\ &\partial^{il} s_{i\lambda} = 0 \end{aligned}\right\} \qquad g_{rs} = \partial_{r\lambda}\phi_s{}^\lambda + \partial_{s\lambda}\phi_r{}^\lambda$$

$$\partial^{i\lambda}\phi_{i\lambda} = 0$$

$$\partial_{\rho\alpha}\partial^{\rho\alpha}\phi_{inl} = 2 S_{inl}$$

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$$\left.\begin{aligned} mc\chi_l - \left(\frac{\hbar}{i}\partial^{\dot\sigma}{}_l + \phi^{\dot\sigma}{}_l\right)\psi_{\dot\sigma} &= 0 \\ mc\psi_{\dot\mu} + \left(\frac{\hbar}{i}\partial_{\dot\mu}{}^\lambda + \phi_{\dot\mu}{}^\lambda\right)\chi_\lambda &= 0 \end{aligned}\right\}$$

$$\left.\begin{aligned} mc\chi_{\dot\mu} - \left(-\frac{\hbar}{i}\partial_{\dot\mu}{}^\lambda + \phi_{\dot\mu}{}^\lambda\right)\psi_\lambda &= 0 \\ mc\psi_l + \left(-\frac{\hbar}{i}\partial^{\dot\sigma}{}_l + \phi^{\dot\sigma}{}_l\right)\chi_{\dot\sigma} &= 0 \end{aligned}\right\}$$

$$p^{\dot s}{}_l = \frac{\hbar}{i}\partial^{\dot s}{}_l + \phi^{\dot s}{}_l$$

$$m^2c^2\psi_{\dot\mu} + p_{\dot\mu}{}^\lambda p^{\dot\sigma}{}_\lambda \psi_{\dot\sigma} = 0$$

$$\partial^{\dot\mu\lambda} j_{\dot\mu\lambda} = 0 \qquad j_{\dot\mu l} = \psi_{\dot\mu}\psi_l + \chi_{\dot\mu}\chi_l\,.$$

$$\Delta = \psi_\lambda\chi^\lambda\,, \quad \bar\Delta = \psi_{\dot\sigma}\chi^{\dot\sigma} \quad : \text{invariant}$$

$$S_{\dot r l} = \frac{\hbar}{4imc}\left(\psi_\alpha\partial_{\dot r l}\chi^\alpha + \chi_\alpha\partial_{\dot r l}\psi^\alpha + \psi^{\dot\sigma}\partial_{\dot r l}\chi_{\dot\sigma} + \chi^{\dot\sigma}\partial_{\dot r l}\psi_{\dot\sigma}\right) + \frac{1}{2mc}\phi_{\dot r l}\left(\psi_\alpha\chi^\alpha + \psi_{\dot\sigma}\chi^{\dot\sigma}\right)$$

$$j_{\dot r l} = S_{\dot r l} + \frac{1}{4}\left(\partial^{\dot\sigma}{}_l m_{\dot\sigma\dot r} + \partial_{\dot r}{}^\alpha m_{\alpha l}\right)$$

$$\left.\begin{aligned} m_{\dot r\dot s} &= \frac{\hbar}{imc}\left(\psi_{\dot r}\chi_{\dot s} + \psi_{\dot s}\chi_{\dot r}\right) \\ m_{kl} &= \frac{-\hbar}{imc}\left(\psi_k\chi_l + \psi_l\chi_k\right) \end{aligned}\right\}$$

$$L = \frac{\hbar}{i}\left\{\psi^{\dot\mu}\partial_{\dot\mu}{}^\lambda\psi_\lambda - \chi^{\dot\mu}\partial_{\dot\mu}{}^\lambda\chi_\lambda - \psi^\lambda\partial^{\dot\mu}{}_\lambda\psi_{\dot\mu} + \chi^\lambda\partial^{\dot\mu}{}_\lambda\chi_{\dot\mu}\right\} + 2mc(\Delta + \bar\Delta) + \phi^{\dot\mu\lambda} j_{\dot\mu\lambda} - \frac{1}{2}g^{\dot\mu\dot\varsigma}g_{\dot\mu\dot\varsigma}$$

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G. E. Uhlenbeck and Otto Laporte,
New Covariant Relations Following from the Dirac
Equations (R Phys. Rev. 37, 1552, 1931)