3

23

數物報告 拾月

数物記事原稿

校正送先

兵庫縣西宮市苦楽園
湯川秀樹

所要別刷 著者宛 百五十部
阪大理学部事務室宛
八百部

内部変換による
陰陽電子發生の理論.

① 24 数物報告 拾月

On the Theory of Internal Pair Production.[1]

By Hideki Yukawa and Shoichi Sakata

Abstract

The probability of internal pair production by radiationless transition of the radioactive nucleus was calculated and its ratio to the probability of emission of electrons from K levels by the same nuclear transition was compared with the experiment in the case of RaC′.

1) This paper was read before the meeting of Osaka branch on July 6, 1935.

25

數物報告 拾月

On the Theory of Internal Pair Production.[1]

~~By~~ Hideki Yukawa and Shoichi Sakata

§ 1. Introduction and Summary

The theory of the production of the pair of an electron and a positron by the internal conversion of the γ-ray with the energy larger than $2mc^2$, was developped recently by Jaeger and Hulme[2], as a natural extension of the theory of ordinary internal conversion of the γ-ray with the emission of the β-ray.

They compared their result with the experiment of Alichanow and Kosodaew[3], who measured the number of positrons emitted from a Ra(B+C) source. According to the theory the second and the third maxima of the experimental distribution curve,[4] at about 0.7×10^6 eV and 10^6 eV, were to be due to the γ-rays of energies 1.7×10^6 eV and 2.2×10^6 eV, respectively as the coefficient of internal pair production ~~were~~ was maximum at the upper limit of the energy of the positron.

The exceptional case, where a pair of an electron and a~~n~~ positron is emitted ~~by taking up~~ with the absorption of the energy liberated by a nuclear transition between two S levels, was not ~~treated~~ dealted with by the former authors, because of lack of the corresponding γ-ray.

~~1) This paper was read before the meeting of Osaka branch on July 6, 1935.~~

2) J.C.Jaeger & H.R.Hulme, Proc.Roy.Soc. **148**, 708, 1935.

3) A.I.Alichanow & M.S.Kosodaew, Zeits.f.Phys. **90**, 249, 1934.

4) See Fig. 1 below, which is the same with Fig. 15 of Alichanow & Kosodaew, loc.cit.

(3)

26

~~Now,~~ If the maximum of the distribution curve of the positron produced by such a process lies also at the upper limit of the energy, the first hump of the curve of Fig. 1, at about 0.4×10^6 eV, can be attributed to the missing γ-ray of energy 1.4×10^6 eV.

Hence it seems worth while to investigate this case in detail ~~and~~ to see ~~determine~~ whether such a process is sufficient to account for the large hump of the curve. ~~or not.~~

The nuclear transition between two levels of energy difference ΔW can be considered in general as a perturbing field on the electron varying with the frequency $\frac{\Delta W}{h}$. This perturbing field extends only over the region of nuclear dimension in the case of S-S transition, so that the probability of transition of the electron is large only when the eigenfunction of the initial and the final states are not small in the neighborhood of the nucleus. Accordingly, we have only to consider the case, where the total angular momenta of both of the electron and the positron take the smallest values.

[illegible]

The distribution function of the positron calculated in this manner has the maximum in the neighborhood of the upper limit of the energy, which agrees with the first maximum of the curve of Fig. 1.

To find the absolute value of the probability of the pair production, we must have ~~have to know~~ the detailed information of the nuclear structure which is too difficult as yet. ~~in detail.~~ ~~To avoid this difficulty,~~ For this reason we considered only the ratio of the probability of the pair production to that of the emission of K electrons caused by the same nuclear transition, as they have the common factor concerning ~~relating to~~ the nuclear transition, if the vector potential of the perturbing field

(4)

27 數物報告 拾月

is negligible compared with the scalar potential. The result obtained shows that the ratio is so small that only a small part of the hump in the expperimental curve can be attributed to the positrons emitted by the above process.

If the vector potential is also taken into account, the ratio depends on the special forms of scalar and vector potentials and in the favorable case it becomes large enough to be in accord with the experiment. It should be noticed, however, that ~~We can not, however, have much confidence of in our results anyhow, as~~ the application of the present theory of the electron up to the immediate neighborhood of the nucleus may happen to be totally wrong. It remains still uncertain ~~Hence we are not certain~~ therefore whether or not other processes such as the emission of the positron alone from the nucleus or the internal conversion of the β-ray may be considered ~~are~~ necessary for the complete explanation of the experimental result, ~~or not~~.

A brief account of the calculation will be given in the following sections.

§ 2. Probability of Internal Pair Production

~~Now~~ We consider the general case, where an electron moving in the field of the nucleus with the charge Ze is perturbed by a nuclear transition between two levels of energy difference ΔW. 20

The perturbation can be expressed by scalar and vector potentials of the form $A_0(\vec{r})\exp(-2\pi i\nu t)$ and $\vec{A}(\vec{r})\exp(-2\pi i\nu t)$, where $\nu = \frac{\Delta W}{h}$.
If we assume ~~that~~ only one particle with the effective charge Qe and the reduced mass M in the nucleus to fall from a state $\varphi(\vec{r})$ to a

(5)

28 数物報告 拾月

state $\varphi_0(\vec{r})$, the energy difference being ΔW, the potentials can be expressed in general by

$$A_0(\vec{r}) = Qe \int \frac{\exp(iq|\vec{r}-\vec{r}_1|)}{|\vec{r}-\vec{r}_1|} \varphi_0^*(\vec{r}_1)\varphi(\vec{r}_1)\,d\vec{r}_1, \quad (1)$$

$$\vec{A}(\vec{r}) = \frac{Qeh}{4\pi Mci} \int \frac{\exp(iq|\vec{r}-\vec{r}_1|)}{|\vec{r}-\vec{r}_1|} \{\varphi_0^*(\vec{r}_1)\,\mathrm{grad}_1\varphi(\vec{r}_1) - \varphi(\vec{r}_1)\,\mathrm{grad}_1\varphi_0^*(\vec{r}_1)\}\,d\vec{r}_1, \quad (2)$$

5)

where $q = \frac{2\pi\nu}{c} = \frac{2\pi\Delta W}{hc}$. The symbol → means always a vector quantity and $*$ a conjugate complex quantity.

Now, according to the perturbation theory, the probability per unit time per unit energy range of the transition of the electron from the continuous state of negative energy $-E_+(E_+ > mc^2)$ to the continuous state of positive energy $E_-(> mc^2)$, satisfying the relation $E_+ + E_- = \Delta W$, is given by

$$p(E_+) = \frac{4\pi^2}{h} \sum_{j',u'} \sum_{j,u} \left| \int \tilde{\psi}_{E_-,j',u'}(\vec{r})\{eA_0(\vec{r}) + e\vec{\alpha}\vec{A}(\vec{r})\} \times \psi_{-E_+,j,u}(\vec{r})\,d\vec{r}\right|^2, \quad (3)$$

where $\psi_{E,j,u}$ denotes the solution of Dirac's equation

$$\left(E + \frac{Ze^2}{r} + c\vec{\alpha}\vec{p} + \beta mc^2\right)\psi = 0$$

with energy E, inner quantum number $|j| - \frac{1}{2}$, and its z-component ~~u~~ $u + \frac{1}{2}$. j is a positive or a negative integer excluding zero and u takes a value between $-|j|$ and $|j|-1$. $\vec{\alpha}, \beta$ are Dirac's matrices with four rows and four columns and ψ can be considered as a matrix with one row and four columns, while $\tilde{\psi}$ as that with four rows and one column, the corresponding elements being complex conjugate to each other.

---

5) H.M.Taylor & N.F.Mott, Proc.Roy.Soc. **138**, 665, 1932.

29 數物報告 拾月

The eigenfunctions are normalized with respect to energy, the definition of which is

$$\int \tilde{\psi}_{E'} \psi_E \, d\vec{r} = \delta(E'-E).$$

This corresponds to the process, in which a positron of energy E and an electron of energy E are emitted simultaneously from the nucleus.

Hence, the total probability per unit time of the production of the pair is given by

$$P_{pair} = \int_{mc^2}^{\Delta W - mc^2} p(E_+)\, dE_+ \qquad (4)$$

In the case of S-S transition, φ_0 and φ are functions of r only, so that the scalar potential becomes a function of r and the vector potential is reduced to the form $\frac{\vec{r}}{r}$ multiplied by a function of r. Thus the perturbing potential $eA_0 + e\vec{\alpha}\vec{A}$ can be written in a form

$$V_0(r) + \frac{\vec{\alpha}\vec{r}}{r} V(r)$$

which commutes with j and u.

Hence we obtain the selection rules

$$\Delta j = 0 \quad , \quad \Delta u = 0$$

for the transition of the electron.

In this case, moreover, A_0 and $\vec{A}$ can be reduced to zero outside of the nucleus by a suitable gauge transformation, which does not affect the transition probability in general. Consequently, the probability of the transition of the electron is appreciable only when the eigenfunctions of the initial and the final states are not small in the

(7)

neighborhood of ~~r/~~ r=0, i.e. only when $j=\mp 1$.

Thus, the expression (3) becomes approximately

$$p(E_+) = \frac{4\pi^2}{h}\sum_{j=\mp1,\ u=0,-1}\left|\int \tilde{\psi}_{E_-,j,u}(\vec{r})\left\{V_0(r)+\frac{\alpha\vec{r}}{r}V(r)\right\} \times \psi_{-E_+,j,u}(\vec{r})\,d\vec{r}\right|^2, \qquad (5)$$

The eigenfunctions in this expression are well known.[6] Since only the values of the eigenfunctions in the neighborhood of r=0 are important, their radial parts are expanded in powers of r and the first terms with the power $r^{\gamma-1}$ are taken as the first approximation, where $\gamma=\sqrt{1-\alpha^2Z^2}$, ($\alpha$ being the fine structure constant.) Inserting these expressions for the eigenfunction in (5), ~~and performing the integrations with respect to θ,ϕ,~~ we ~~have finally~~ (obtain at length)

$$p(E_+) = \frac{4m}{\{\Gamma(2\gamma+1)\}^4 h^3}\left(\frac{4\pi mc}{h}\right)^{4\gamma}\{(1-\alpha^2Z^2)R_p^2+\alpha^2Z^2S_p^2\} \times F_p(\varepsilon_+), \qquad (6)$$

where

$$R_p = \int r^{2(\gamma-1)}V_0(r)\,d\vec{r},$$

$$S_p = \int r^{2(\gamma-1)}V(r)\,d\vec{r}.$$

$F_p(\varepsilon_+)$ is the only factor, which depends on the energy of the positron emitted, ~~and has~~ (having) the form

$$F_p(\varepsilon_+) = \eta_+^{2\gamma-1}e^{-\pi b_+}|\Gamma(\gamma+ib_+)|^2 \times \eta_-^{2\gamma-1}e^{\pi b_-}|\Gamma(\gamma-ib_-)|^2(\varepsilon_+\varepsilon_- - \gamma^2), \qquad (7)$$

where

$$\varepsilon_\pm = \frac{E_\pm}{mc^2}, \qquad \eta_\pm = \sqrt{\varepsilon_\pm^2-1}$$

$$b_\pm = \alpha Z\frac{\varepsilon_\pm}{\eta_\pm}.$$

---

6) Jaeger & Hulme, loc.cit. We have to take k=0 in (2_A) for j=-1 and k=1 in (2_B) for j=1, u being the same as in our case.

The forms of the distribution function $F_p(\varepsilon_+)$ in special cases are shown in Fig. 2 and Fig. 3.

The total probability of pair production is now becomes

$$P_{pair} = \frac{4m}{\{\Gamma(2\gamma+1)\}^4 \hbar^3}\left(\frac{4\pi mc}{\hbar}\right)^{4\gamma}\{(1-\alpha^2Z^2)R_p^2 + \alpha^2Z^2S_p^2\}$$

$$\times \int_1^{\Delta w-1} F_p(\varepsilon_+)\,d\varepsilon_+ , \qquad (8)$$

where $\Delta w = \frac{\Delta W}{mc^2}$.

In these formulae, ~~R_p and S_p depend on~~ being dependent on the detailed structure of the nucleus, Rp and Sp, ~~and~~ can not be estimated easily.

If we neglect the vector potential, the calculation ~~is~~ becomes formally similar to that of Beck and Sitte[7] on β-disintegration. The physical interpretations, however, differ ~~in two cases~~ from each other. In the latter case, either the electron or the positron is reabsorbed by the original nucleus, whereas, in ~~our~~ the former case, both of them escape from the nucleus.

§ 3. Probability of Electron Emission from K Levels

Next we consider the emission of K electrons by the same nuclear transitions.

In general, the probability per unit time of the transition of the the electrons from K states of energy $mc^2\sqrt{1-\alpha^2Z^2}$ to continuous states of energy $E'(>mc^2)$, satisfying the condition $E'=mc^2\sqrt{1-\alpha^2Z^2}+\Delta W$, is given by

$$P_K = \frac{4\pi^2}{\hbar}\sum_{j',u'}\sum_{u=0,-1}\left|\int \tilde{\Psi}_{E';j';u'}(\vec{r})\{eA_0(\vec{r}) + e\vec{\alpha}\vec{A}(\vec{r})\}\right.$$

$$\left.\times\, \Psi_{K,u}(\vec{r})\,d\vec{r}\right|^2 , \qquad (9)$$

7) G.Beck und K.Sitte, Zeits.f.Phys. **86**, 105, 1933.

(9) (8)

32 數物報告 拾月

the meaning of ΔW, $A_0(\vec{r})$, $\vec{A}(\vec{r})$ and $\tilde{\psi}_{E',j',u'}$ being the same as in the previous section. ~~$\psi_{K,u=0,-1}$ are orthogonal~~ $\psi_{K,u=0,-1}$ are normarized eigenfunctions of two K states.[8)]

Now we consider the same S-S transition of the nucleus as in the previous section. The expression (9) becomes simply

$$P_K = \frac{4\pi^2}{h^2}\sum_{u=0,-1}\left|\int \tilde{\psi}_{E',-1,u}(\vec{r})\left\{V_0(r)+\frac{\alpha Z}{r}V(r)\right\}\psi_{K,u}(\vec{r})\,d\vec{r}\right|^2. \quad (10)$$

By taking for $\psi_{K,u}$ and $\tilde{\psi}_{E',-1,u}$ the first terms of the expansion in powers of r, we ~~have~~ obtain finally

$$P_K = \frac{4\pi m(\alpha Z)^{2\gamma+1}}{\{\Gamma(2\gamma+1)\}^3 h^3}\left(\frac{4\pi mc}{h}\right)^{4\gamma} R_p^2\, \eta'^{2\gamma-1} e^{\pi b'}|\Gamma(\gamma+ib')|^2 \times(\varepsilon'+\gamma) \quad (11)$$

where

$$\varepsilon' = \frac{E'}{mc^2}, \quad \eta' = \sqrt{\varepsilon'^2-1},$$

$$b' = \alpha Z\frac{\varepsilon'}{\eta'}.$$

In this case, the probability does not depend on the magnitude of the vector potential.

By comparing (11) with (8), ~~Now~~ the ratio of the probability of pair production by the S-S transition of the nucleus to that of the emission of electrons from K levels by the same transition is given by

$$\rho = \frac{P_{pair}}{P_K} = \frac{(1-\alpha^2Z^2)\int F_p(\varepsilon_+)\,d\varepsilon_+\cdot\left\{1+\frac{\alpha^2Z^2}{1-\alpha^2Z^2}\left(\frac{S_p}{R_p}\right)^2\right\}}{\pi(\alpha Z)^{2\gamma+1}\Gamma(2\gamma+1)\,\eta'^{2\gamma-1}|\Gamma(\gamma+ib')|^2 e^{\pi b'}(\varepsilon'+\gamma)}. \quad (12)$$

8) See, for example, H.R.Hulme, Proc.Roy.Soc. **138**, 643, 1932.

(10)
33

數物報告 拾月

§ 4. General Discussions

Before entering into numerical calculations in special cases, it should be noticed that the above formulation is incomplete in two points.

The first point is that the perturbing potentials ~~are~~ were assumed to have the special forms (1) and (2). Now, we can show more generally that the perturbing potentials are expressed in the forms

$$F(r)\exp(-2\pi i\nu t),$$

$$\frac{\vec{\alpha}\vec{r}}{r}G(r)\exp(-2\pi i\nu t),$$

where F and G are ~~F, G being~~ certain functions of r only, whenever both the initial and the final states of the nucleus have zero spin or, still more generally, when they have the same spin in the same direction.

To prove this, we consider the fact that the total angular momentum of the system consisting of the nucleus and the electron should be constant throughout the above processes. So, if the spin of the nucleus does not change, the total angular momentum of the electron

$$\vec{M} = \vec{m} + \frac{h}{4\pi}\vec{\sigma}$$

should become constant by itself ~~also~~, where $\vec{m} = \vec{r}\times\vec{p}$ and $\vec{\sigma}$ is the spin vector. Hence, the perturbed Hamiltonian of the electron should commute with $\vec{M}$, so that we have

$$\vec{M}(eA_0 + e\vec{\alpha}\vec{A}) - (eA_0 + e\vec{\alpha}\vec{A})\vec{M} = 0,$$

where $A_0(\vec{r})\exp(-2\pi i\nu t)$, $\vec{A}(\vec{r})\exp(-2\pi i\nu t)$ are perturbing potentials due to the nuclear transition. As the coefficients of 1 (the unit matrix), $\alpha_x = \rho_1\sigma_x$, $\alpha_y = \rho_1\sigma_y$, $\alpha_z = \rho_1\sigma_z$ in the left hand sides of these equations should vanish separately, we have

$$\vec{m}A_0 - A_0\vec{m} = 0 \qquad (13)$$

$$\left.\begin{aligned} m_x A_x - A_x m_x &= 0 \\ m_y A_x - A_x m_y &= -\frac{ih}{2\pi} A_z \\ m_z A_x - A_x m_z &= \frac{ih}{2\pi} A_y \end{aligned}\right\} \qquad (14)$$

and similar equations for A_y, A_z.

The equations (13) show that A_0 is a function of r only, while the ~~remaining~~ equations (14) show that the relations of commutation between $\vec{m}$ and $\vec{A}$ are the ~~sm~~ same as those between $\vec{m}$ and $\vec{r}$.

Now, by ~~using~~ the help of (14) and the relations of commutation between $\vec{m}$ and $\vec{r}$, we have

$$m_y(xA_y - yA_x) - (xA_y - yA_x)m_y = \frac{ih}{2\pi}(yA_z - zA_y) \qquad (15)$$

On the other hand, by using (14) and the relations of commutation between $\vec{p}$ and $\vec{r}$, we ~~have~~ obtain

$$p_x(xA_y - yA_x) - (xA_y - yA_x)p_x = x(p_xA_y - A_yp_x) - \frac{ih}{2\pi}A_y - (yp_xA_x - A_xyp_x). \qquad \cancel{(15)}\ (16)$$

From the third equation

$$(xp_y - yp_x)A_x - A_x(xp_y - yp_x) = \frac{ih}{2\pi}A_y$$

of (14), the last two terms ~~together~~ of the right hand side of (15) ~~the above equation~~ become $-xp_yA_x + A_xxp_y$, so that we have

$$p_x(xA_y - yA_x) - (xA_y - yA_x)p_x = x(p_xA_y - A_yp_x - p_yA_x + A_xp_y). \qquad \cancel{(16)}\ \cancel{(16)}\ (17)$$

Further, by using the relations

$$(zp_x - xp_z)A_y - A_y(zp_x - xp_z) = 0, \text{ and } (yp_z - zp_y)A_x - A_x(yp_z - zp_y) = 0,$$

we can deduce the equation as follows:

$$p_z(xA_y - yA_x) - (xA_y - yA_x)p_z = z(p_xA_y - A_yp_x - p_yA_x + A_xp_y). \qquad (18)$$

Combining ~~(16)~~ (17) and (18), we have

$$(zp_x - xp_z)(xA_y - yA_x) - (xA_y - yA_x)(zp_x - xp_z) = 0.$$

Comparing this with (15), we ~~have~~ obtain finally

$$yA_z - zA_y = 0.$$

Similarly, we can further deduce the relations

$$zA_x - xA_z = 0, \qquad xA_y - yA_x = 0,$$

and also $\qquad \vec{m}(\vec{r}\vec{A}) - (\vec{r}\vec{A})\vec{m} = 0.$

These equations together show clearly that $\vec{A}$ has the form $\frac{\vec{r}}{r}G(r)$.

Thus the proof is ended.

We can show, moreover, that the perturbing field is zero outside of the nucleus in this general case. Firstly, the magnetic field $\vec{H}$ vanishes everywhere, since

間を詰める

$$\vec{H} = \text{curl } \vec{A} \, \exp(-2\pi i \nu t) = 0.$$

Secondly, from the field equations

$$\frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \text{curl } \vec{H} - \frac{4\pi}{c}\vec{I},$$

in which $\vec{H} = 0$ and both the electric field $\vec{E}$ and the current density $\vec{I}$ have the time factor $\exp(-2\pi i \nu t)$, we ~~have~~ obtain the relation

$$\vec{E} = \left(\frac{2}{i\nu}\right)\vec{I},$$

~~which~~ showing that the electric field vanishes outside of the nucleus, where the current density due to the nuclear transition vanishes.

Hence, in this case, we can reduce the potentials themselves to zero outside of the nucleus by a suitable gauge transformation.

The second point is that the solutions of Dirac's equation in Coulomb field of the nucleus were taken as the eigenfunctions of the unperturbed states of the electron. ~~As~~ Since the field is ~~no more~~ not of Coulomb type any more in the neighborhood of the nucleus, we should modify the forms of the eigenfunctions in this region. Thus, each eigenfunction in the previous sections should be multiplyed by a certain function, which deviates appreciably from 1 for ~~small~~ the value of r comparable with the nuclear radius a_N.

As the exact determination of such functions is very complicated, ~~and~~ necessitating special assumptions on the nuclear structure, we should be ~~satisfied~~ contented with taking for the values of the eigenfunctions in the nucleus simply the values of the solutions in Coulomb field at $r = a_N$, while their values outside of the nucleus do not ~~concern us here. in this problem~~ matter, as far as the results of our calculation are concerned.

Then, in the formulae of the preceeding sections, we have only to change the definitions of R_p, S_p, other points being unaltered. Namely, we have to put

$$R_p = a_N^{2(\gamma-1)} \int V_0 \, d\vec{r},$$

$$S_p = a_N^{2(\gamma-1)} \int V \, d\vec{r},$$

instead of the corresponding expressions in § 2.

§ 5. Numerical Results ~~and Discussions~~

We want now to obtain the numerical values of $F_p(\varepsilon_+)$ and ρ in the case of RaC′. ~~Inserting~~ By taking the values

$$Z=84,\quad \alpha=\frac{1}{137},\quad \Delta W=1.4\times10^6\,\mathrm{eV}=2.8mc^2,$$

we ~~have~~ obtain

$$\alpha Z=0.6,\quad \gamma=0.8,\quad \frac{\gamma^2}{\pi(\alpha Z)^{2\gamma+1}\Gamma(2\gamma+1)}=0.5,\quad \varepsilon_++\varepsilon_-=2.8$$

$$\varepsilon'=3.6,\quad \eta'=3.5,$$

$$|\Gamma(\gamma+ib')|^2e^{\pi b'}(\varepsilon'+\gamma)\cong(4.5+1.6\eta')\,e^{-1.2\pi\frac{\varepsilon'}{\eta'}}(\varepsilon'+\gamma)=44.4,^{9)}$$

so that

$$F_p(\varepsilon_+)\cong(4.5+1.6\eta_-)(4.5+1.6\eta_+)(\varepsilon-\varepsilon_+-0.6)\,e^{-1.2\pi\frac{\varepsilon_+}{\eta_+}}$$

and

$$\rho=1.1\times10^{-2}\cdot\int_1^{1.8}F_p(\varepsilon_+)\,d\varepsilon_+\cdot\left\{1+0.56\left(\frac{S_p}{R_p}\right)^2\right\}.$$

The distribution function $F_p(\varepsilon_+)$ in this case is given in Fig. 2. Its maximum ~~is~~ lies near the upper limit of the energy of the positron, ~~and~~ coinciding approximately with the first maximum of Alichanow's curve, as ~~is~~ shown in Fig. 1, which represents the energy distribution of positrons emitted from a ~~Ra~~ Ra(B+C) source.[10)]

From Fig. 2, we ~~have~~ obtain

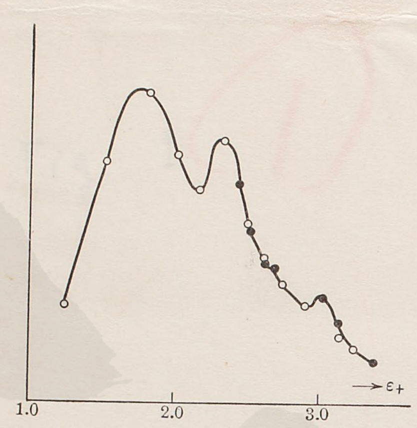
$$\int_1^{1.8}F_p(\varepsilon_+)\,d\varepsilon_+\cong0.18,$$

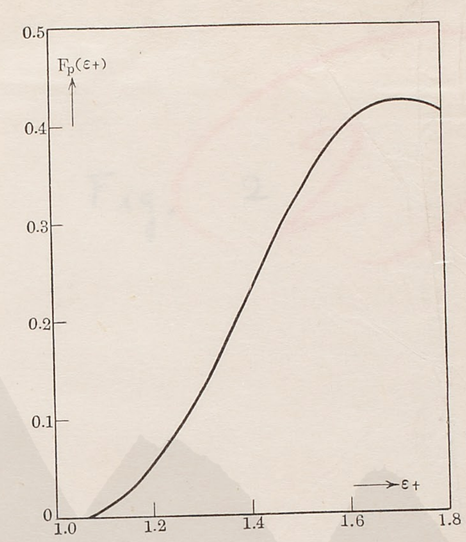
so that

$$\rho\cong2\times10^{-3}\left\{1+0.56\left(\frac{S_p}{R_p}\right)^2\right\}. \qquad (19)$$

10) The energy is written in unit of $mc^2=0.5\times10^6$ eV, the proper energy 1 being inclusive.

9) E.Fermi, Zeits.f.Phys. **88**, 161, 1934.





~~Now~~ The number of ~~the electron~~ electrons emitted from the K levels is known to be about 0.003 per disintegration in this case, ~~and~~ while on the other hand, according to Alichanow, total number of ~~the~~ positrons is also about 0.003 per disintegration.

Hence, if we assume all of the first hump in Fig. 1 to be due to the process above considered, ρ should be at least about 0.2, ~~from~~ from which we ~~have~~ obtain

$$1 + 0.56\left(\frac{S_p}{R_p}\right)^2 \geq 100$$

$$\text{or} \quad \int r^{2\gamma-1} V_0(r)\,d\vec{r} \ll \int r^{2\sigma'-1} V(r)\,d\vec{r}. \qquad (20)$$

Otherwise, ρ will be of order 10^{-2}.

Since ~~as~~ the occurence of such an extreme case ~~satisfying~~ as to satisfy the condition (20) seems very improbable, we ~~should conclude~~ draw a conclusion that the the calculated value of the pair production is probably too small to account for the experiment

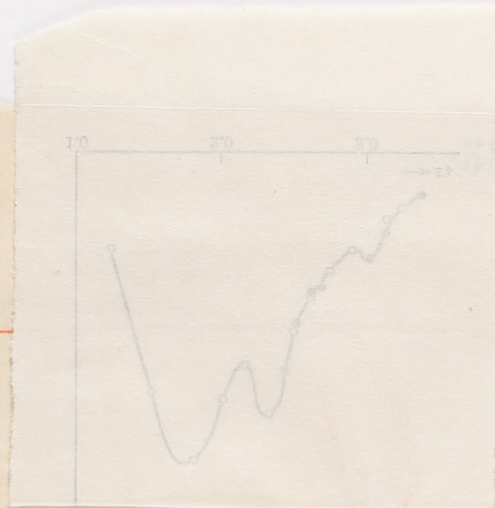


Fig. 1 (1)

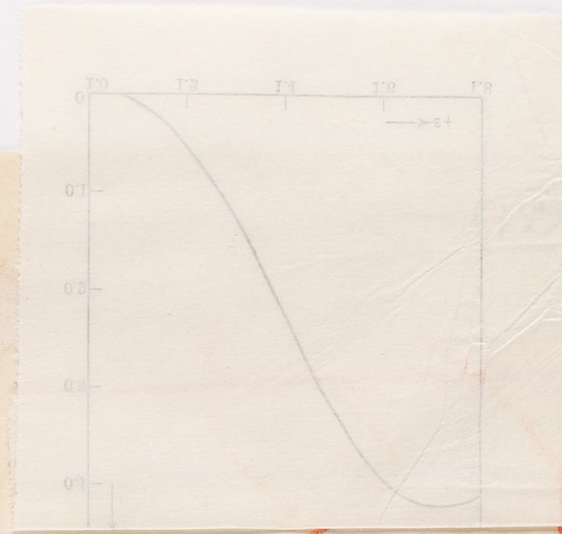


Fig. 2 (2)

~~Now~~ The number of ~~the electron~~ electrons emitted from the K levels is known to be about 0.003 per disintegration in this case, ~~and~~ while on the other hand, according to Alichanow, total number of ~~the~~ positrons is also about 0.003 per disintegration.

Hence, if we assume all of the first hump in Fig. 1 to be due to the process above considered, ρ should be at least about 0.2, ~~from~~ from which we ~~have~~ obtain

$$1 + 0{,}56\left(\frac{S_p}{R_p}\right)^2 \geqq 100$$

or

$$\int r^{2\gamma-1} V_0(r)\, d\vec{r} \ll \int r^{2\gamma-1} V(r)\, d\vec{r}. \qquad (20)$$

Otherwise, ρ will be of order 10^{-2} as to satisfy ~~satisfying~~ the condition (20)

Since ~~as~~ the occurence of such an extreme case seems very improbable, we draw a conclusion ~~should conclude~~ that the the calculated value of the pair production is probably too small to account for the experiment

(16) (11)

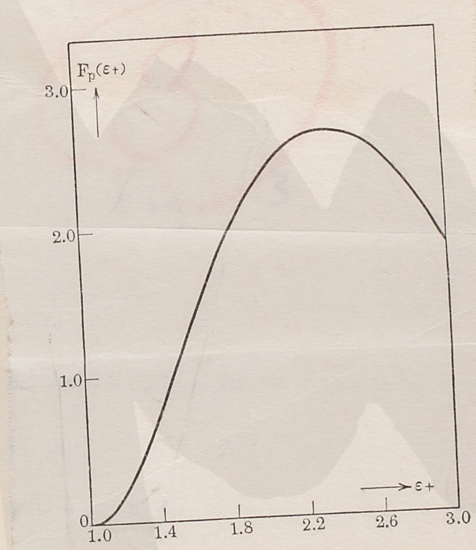
39

數物報告 拾月

exhaustively.

As shown by the formula (12), ρ depends, in general, on ΔW and Z. It increases rapidly with the increase of ΔW ~~as ΔW increases~~ and the maximum of $F_p(\varepsilon_+)$ displaces towards $\varepsilon_+ = \frac{\Delta W}{2} = \frac{\Delta W}{2mc^2}$ at the same time.

For example, assuming Z=84 and $\Delta W = 4mc^2$, we have for $F_p(\varepsilon_+)$ a curve as shown in Fig. 3



and for ρ a value

$$4.0\times 10^{-2}\left\{1+0.56\left(\frac{S_p}{R_p}\right)^2\right\}.$$

ρ decreases rapidly as Z increases, owing to the factor $(\alpha Z)^{2\gamma+1}$ in the denominator.

In conclusion the authors wish to express their cordial thanks to Prof. G. Beck for valuable discussions on the occasion of his visit to Osaka.

Department of Physics,
Osaka Imperial University.

(Received ~~July~~ August 22, 1935)

exhaustively.

As shown by the formula (12), ρ dep[illegible] on ΔW and Z. It increases rapidly with the increase ~~as ΔW increas~~ [illegible] +) displaces towards $\varepsilon_+ = \frac{\Delta W}{2} = \frac{\Delta W}{2mc^2}$ at the sam[illegible]

For example, assuming Z=84 and ΔW=[illegible] curve as shown in Fig. 3 and for ρ a value

$$4.0\times10^{-2}\left\{1+0.56\left(\frac{S_p}{R_p}\right)^2\right\}.$$

ρ decreases rapidly as Z increases, owing to the factor $(\alpha Z)^{2\gamma+1}$ in the denominater.

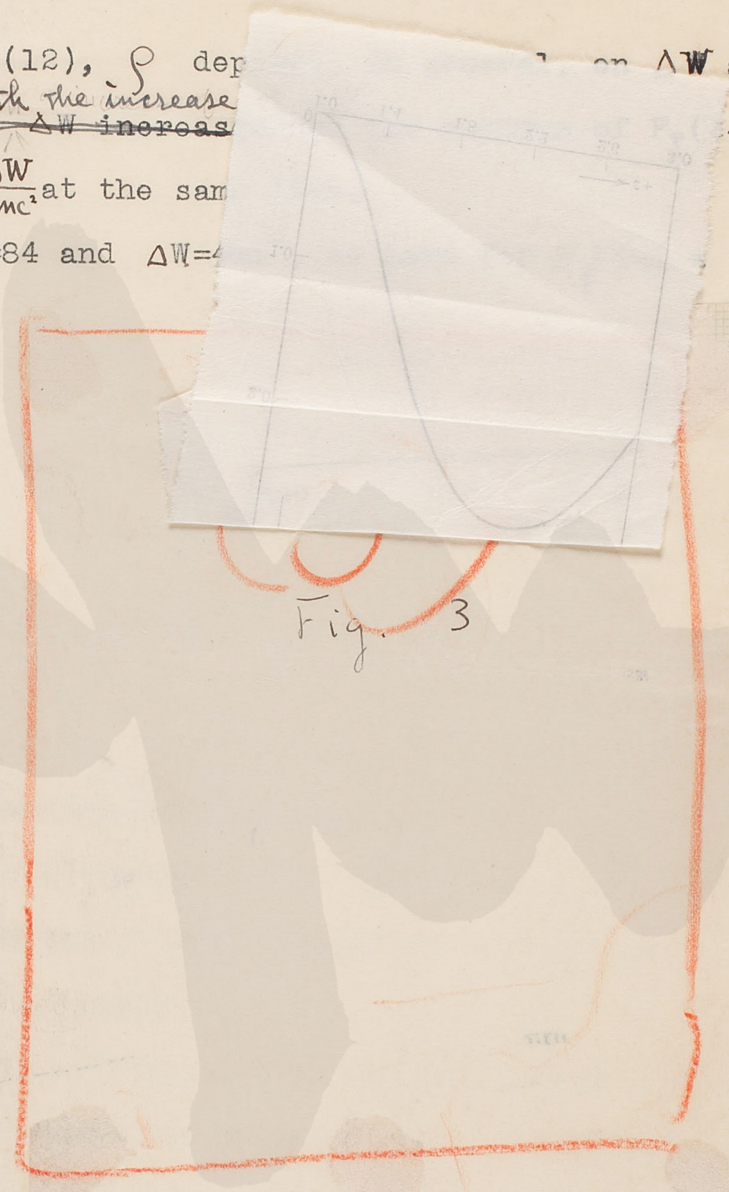


Fig. 3

In conclusion the authors wish to express their cordial thanks to Prof. G. Beck for valuable discussions on the occasion of his visit to Osaka.

Department of Physics,
Osaka Imperial University.

(Received ~~July~~ August 22, 1935)

It will easily be seen that

Note added in Proof. — ~~The ratio ρ~~ magnitude of the depends much on the ~~electrostatic~~ static ~~field~~ potential acting on the electron in the nuclear region. Especially, if we assume rather large ~~repulsion pos~~ repulsive ~~that the electron~~, potential, ~~to act near the nucleus~~ that exists in this region, the