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NO. 1

$k, u$ の代りに $j$ (Dirac), $u$ を 使ふために Jaeger and Hulme の (2A) 及 (2B) に於て $F_k$, $F_{-k-1}$ の代りに

$$\left.\begin{aligned} F_j &= F_k \quad \text{for (2A) 即 } j = -k-1 < 0 \\ &= F_{-k-1} \quad \text{for (2B) 即 } j = k > 0 \end{aligned}\right\} \quad (2')$$

とかく. (3) はそうすると

$$\left.\begin{aligned} \left(A^2 + \frac{\alpha Z}{r}\right)F_j + \frac{dG_j}{dr} + \frac{1+j}{r} G_j &= 0 \\ \left(B^2 - \frac{\alpha Z}{r}\right)G_j + \frac{dF_j}{dr} + \frac{1-j}{r} F_j &= 0 \end{aligned}\right\} \quad (3')$$

where

$$\left.\begin{aligned} A^2 &= \frac{1}{\Lambda}(1+\varepsilon) \\ B^2 &= \frac{1}{\Lambda}(1-\varepsilon) \\ \Lambda &= \frac{2\pi mc}{h}, \quad \varepsilon = \frac{E}{mc^2} \end{aligned}\right\} \quad (4')$$

(5)と同じく

$$\left.\begin{aligned} f_j &= AF_j - BG_j \\ g_j &= AF_j + BG_j \end{aligned}\right\} \quad (5')$$

を導入すると $\varepsilon^2 > 1$ に対する (3′) の solution は (6A) 及 (6B) により 次の様にかかれる. ($j$ を導入したため $S$, 及 $T$ は 同じ expression になるから $S$ のみを用ひる)

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NO. 2

$$F_j = C\{(-j-1-s)+i(b+c)\} r^s k^{2s+2} e^{-\pi b} \int_{-1}^{1} (1-u)^{s-ib} (1+u)^{s+ib+1} e^{ikru} du \quad \text{for } j<0$$

$$= \frac{C}{\cancel{D}} \{(b-c) - i(j-1-s)\} r^s k^{2s+2} e^{-\pi b} \int_{-1}^{1} (1-u)^{s-ib} (1+u)^{s+ib+1} e^{ikru} du \quad \text{for } j>0$$

$$G_j = C\{(-j-1-s) - i(b+c)\} r^s k^{2s+2} e^{-\pi b} \int_{-1}^{1} (1-u)^{s-ib+1} (1+u)^{s+ib} e^{ikru} du \quad \text{for } j<0$$

$$= \frac{C}{\cancel{D}} \{(b-c) + i(j-1-s)\} r^s k^{2s+2} e^{-\pi b} \int_{-1}^{1} (1-u)^{s-ib+1} (1+u)^{s+ib} e^{ikru} du \quad \text{for } j>0 \qquad (6')$$

where

$$\left.\begin{aligned} ik &= AB \qquad (k \text{ は Hulme の } \alpha) \quad k = \tfrac{1}{\lambda}\sqrt{\varepsilon^2-1} \\ -ib &= \frac{\alpha Z}{2}\left(\frac{A}{B} - \frac{B}{A}\right) \qquad \text{d.h.} \quad b = \alpha Z \frac{\varepsilon}{\sqrt{\varepsilon^2-1}} \\ -ic &= \frac{\alpha Z}{2}\left(\frac{A}{B} + \frac{B}{A}\right) \qquad \text{d.h.} \quad c = \alpha Z \frac{1}{\sqrt{\varepsilon^2-1}} \\ s &= \sqrt{j^2 - \alpha^2 Z^2} - 1 \end{aligned}\right\} \qquad (7')$$

$$\left.\begin{aligned} C &\doteq e^{\pi b} k^{-s-2} \qquad \text{for } \varepsilon > 1 \\ &= i e^{\pi b} k^{-s-2} \qquad \text{for } \varepsilon < -1 \end{aligned}\right\} \qquad (8')$$

( Hulme の C と D は 此處では 同一の expression になる)

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NO. 3

E-Skala で normalize しておくと normalization factor は

$$\left.\begin{aligned} &\xi(E, j) = \sqrt{\frac{2}{\hbar c}}\, k^{\frac{3}{2}} \Lambda^{-1} \\ &\Lambda = \sqrt{(\mp j - 1 - s)^2 + (b \pm c)^2}\, \left|\Gamma(s+1+ib)\right| e^{-\frac{\pi b}{2}} \cdot 2^{s+1} \\ &\text{according as } j \lessgtr 0 \end{aligned}\right\} \quad (10')$$

(Hulme は ν-Skala で normalize したから (10) と (10′) は $\frac{1}{\sqrt{\hbar}}$ だけ違ふ.)

---

(6′) を r に就き展開した第一項を考へる.

$$e^{ikru} = 1 \qquad \text{(i)}$$

とおき

$$\left.\begin{aligned} &\int_{-1}^{1} (1-u)^{p-1} (1+u)^{q-1}\, du = 2^{p+q-1} \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \\ &(\text{real part of } p \text{ and } q > 0) \end{aligned}\right\} \quad \text{(ii)}$$

及

$$\Gamma(p+1) = p\,\Gamma(p)$$

の関係を用ひると

$$\left.\begin{aligned} &\int_{-1}^{1} (1-u)^{s-ib} (1+u)^{s+ib+1}\, du = 2^{2s+2} (s+1+ib) \frac{\left|\Gamma(s+ib)\right|^2}{\Gamma(2s+3)} \\ &\int_{-1}^{1} (1-u)^{s-ib+1} (1+u)^{s+ib}\, du = 2^{2s+2} (s+1-ib) \frac{\left|\Gamma(s+ib)\right|^2}{\Gamma(2s+3)} \end{aligned}\right\} \text{(iii)}$$

$f_j$, $g_j$ は従て

$$f_j = \delta\cdot K\cdot\left[\{(s+1)(-j-s-1)-b(b+c)\}+i\{(s+1)(b+c)+b(-j-s-1)\}\right]\cdot r^s,\quad j<0$$

$$= \delta\cdot K\cdot\left[\{(s+1)(b-c)+b(j-s-1)\}-i\{(s+1)(j-s-1)-b(b-c)\}\right]\cdot r^s,\quad j>0$$

$$g_j = \delta\cdot K\cdot\left[\{(s+1)(-j-s-1)-b(b+c)\}-i\{(s+1)(b+c)+b(-j-s-1)\}\right]\cdot r^s,\quad j<0$$

$$= \delta\cdot K\cdot\left[\{(s+1)(b-c)+b(j-s-1)\}+i\{(s+1)(j-s-1)-b(b-c)\}\right]\cdot r^s,\quad j>0 \qquad \text{(iv)}$$

where

$$\left.\begin{aligned} K &= 2^{2s+2}\, k^s\, \frac{|\Gamma(s+1+ib)|^2}{\Gamma(2s+3)} \\ \delta &= \begin{cases} 1 & \text{for } \varepsilon>1 \\ i & \text{for } \varepsilon<-1 \end{cases} \end{aligned}\right\} \qquad \text{(v)}$$

(5') より

$$\left.\begin{aligned} F_j &= \frac{g_j+f_j}{2A} = \delta\cdot\frac{K}{A}\cdot\{(s+1)(-j-s-1)-b(b+c)\}\cdot r^s \\ &= \delta\cdot\frac{K}{A}\cdot\{(s+1)(b-c)+b(j-s-1)\}\cdot r^s \\ G_j &= \frac{g_j-f_j}{2B} = -i\cdot\delta\cdot\frac{K}{B}\{(s+1)(b+c)+b(-j-s-1)\}\cdot r^s \\ &= i\cdot\delta\cdot\frac{K}{B}\{(s+1)(j-s-1)-b(b-c)\}\cdot r^s \end{aligned}\right\} \qquad \text{(vi)}$$

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(4)から

$$\left.\begin{aligned} A &= \begin{cases} \left(\frac{1}{\Lambda}\right)^{\frac{1}{2}} (\varepsilon+1)^{\frac{1}{2}}, & \varepsilon > 1 \\ i\left(\frac{1}{\Lambda}\right)^{\frac{1}{2}} (-\varepsilon-1)^{\frac{1}{2}}, & \varepsilon < -1 \end{cases} \\ B &= \begin{cases} i\left(\frac{1}{\Lambda}\right)^{\frac{1}{2}} (\varepsilon-1)^{\frac{1}{2}}, & \varepsilon > 1 \\ \left(\frac{1}{\Lambda}\right)^{\frac{1}{2}} (-\varepsilon+1)^{\frac{1}{2}}. & \varepsilon > -1 \end{cases} \end{aligned}\right\} \quad \text{(vii)}$$

従て

$$\left.\begin{aligned} F_j &= K\cdot\Lambda^{\frac{1}{2}} \cdot \frac{(s+1)(-j-s-1)-b(b+c)}{(\pm\varepsilon\pm1)^{\frac{1}{2}}} r^s, & j<0 \\ &= K\cdot\Lambda^{\frac{1}{2}} \frac{(s+1)(b-c)+b(j-s-1)}{(\pm\varepsilon\pm1)^{\frac{1}{2}}} r^s, & j>0 \\ G_j &= \mp K\cdot\Lambda^{\frac{1}{2}} \frac{(s+1)(b+c)+b(-j-s-1)}{(\pm\varepsilon\mp1)^{\frac{1}{2}}} \cdot r^s, & j<0 \\ &= \pm K\cdot\Lambda^{\frac{1}{2}} \cdot \frac{(s+1)(j-s-1)-b(b-c)}{(\pm\varepsilon\mp1)^{\frac{1}{2}}} \cdot r^s. & j>0 \end{aligned}\right\} \quad \text{(viii)}$$

（複号は 上/下 $\varepsilon > 1$ / $\varepsilon < -1$）

所が

$$(s+1)(-j-s-1)-b(b+c) = -(j-s-1)(-j\varepsilon+s+1)/(\varepsilon-1)$$

$$(s+1)(b-c)+b(j-s-1) = \alpha z\cdot(j\varepsilon-s-1)/(\varepsilon^2-1)^{\frac{1}{2}}$$

$$(s+1)(b+c)+b(-j-s-1) = \alpha z\,(-j\varepsilon+s+1)/(\varepsilon^2-1)^{\frac{1}{2}}$$

$$(s+1)(j-s-1)-b(b-c) = -(j-s-1)(j\varepsilon-s-1)/(\varepsilon+1)$$

$$K\cdot\Lambda^{\frac{1}{2}}\cdot\xi(E,j) = N\cdot\frac{2\sqrt{\varepsilon^2-1}}{\sqrt{(\mp j-1-s)^2+(b\pm c)^2}},$$ （複號は $j \lessgtr 0$）

$$N = \sqrt{\frac{2\pi m}{\hbar^2}}\,(2k)^{s+\frac{1}{2}}\, e^{\frac{\pi b}{2}} \frac{|\Gamma(s+1+ib)|}{\Gamma(2s+3)} \quad \text{(ix)}$$

$$(\mp j-1-s)^2+(b\pm c)^2 = \frac{2(-j-s-1)(-j\varepsilon+s+1)}{\varepsilon-1}, \quad j<0$$

$$= \frac{2(j-s-1)(j\varepsilon-s-1)}{\varepsilon+1}, \quad j>0$$

だから今

$$\left.\begin{aligned} \xi(E,j)F_j &= f_j\cdot r^s \\ \xi(E,j)G_j &= g_j\cdot r^s \end{aligned}\right\} \quad (x)$$

とかくと

$$\begin{cases} f_{j<0} = -N\cdot\sqrt{\pm2(-j\varepsilon+s+1)(-j-s-1)} \\ g_{j<0} = \mp N\cdot\sqrt{\pm2(-j\varepsilon+s+1)(-j+s+1)} \end{cases} \quad (xi)$$

$$\begin{cases} f_{j>0} = N\cdot\sqrt{\pm2(j\varepsilon-s-1)(j+s+1)} \\ g_{j>0} = \mp N\sqrt{\pm2(j\varepsilon-s-1)(j-s-1)} \end{cases} \quad (xii)$$

(複号は $\pm$, $\mp$ $\quad \varepsilon \begin{smallmatrix}>1\\<-1\end{smallmatrix}$)

$$N=\sqrt{\frac{2\pi m}{\hbar^2}}\,(2k)^{s+\frac12}\,e^{\frac{\pi\alpha Z}{2}\frac{\varepsilon}{\sqrt{\varepsilon^2-1}}}\,\frac{\left|\Gamma\left(s+1+i\alpha Z\frac{\varepsilon}{\sqrt{\varepsilon^2-1}}\right)\right|}{\Gamma(2s+3)}$$