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NO. 1

Theory of Pair Production by ~~S-S-Transition~~ Radiationless of Radioactive Nucleus.

by H. Yukawa and S. Sakata

1. Jaeger and Hulme's[1] ~~have ca~~ calculated the probability of pair production by internal conversion of γ-rays and their result ~~agrees~~ is compared with the experiment of Alichanow and Kosodaew[2], ~~concerning using~~ who measured the number of positrons emitted from the Ra(B+C) source. The exceptional case of S-S-transition, corresponding to energy liberation of amount $1.4\cancel{26} \times 10^6$ e.V., was not ~~treated~~ calculated by the authors. If the for RaC′ maximum of the distribution of positron for this case ~~be also~~ lies also at the upper limit of the energy, which is 0.43×10^6 eV for RaC′, ~~a~~ it coincides with the first maximum of the curve of Alichanow and Kosodaew.

Hence it seems worth while to ~~cons treat this case~~ ~~and~~ investigate this case and to determine whether the S-S-transition of the nucleus ~~this~~ is sufficient or not to account for the ~~first maximum~~ large maximum of Alichanow's curve or not. The pair production by. ~~It is necessary or not to take into account the rather improbable internal conversion of~~ ~~natural~~ γ-rays, as ~~Alichanow and Kosodaew~~ the authors ~~assu~~ considered.

To avoid ~~the~~ special assumptions on nuclear ~~transition~~ structure we ~~are~~ considered only the ratio of the probabilities of the pair production and ~~the~~ emission of K-electron caused by the same nuclear transition.

1)

2) The result of our calculation shows that the ratio is ~~much~~ so small that ~~the~~ only a small part ~~to small~~ ~~to account for~~ of the large ~~maximum~~ hump can be attributed to the ~~positron~~ process.

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Dirac's equation

$$\left(E + \frac{e^2 Z}{r} + c(\vec{\alpha}\vec{p} + \beta m c^2)\right) u = 0$$

DATE............

NO. 2

by the nuclear transition of energy difference $h\nu$

2. ~~$u_{E,j,u}$~~ The probability of pair production, ~~emission~~ ~~i.e.~~ of ~~from~~ is given by

$$P_{pair} = \int p(E')dE'$$

$$= \frac{2\pi}{h} \int \sum_{j',u'} \sum_{j,u} \left| \int \tilde{u}_{E',j',u'}(\vec{r}) \{ e A_0(\vec{r}) + e \vec{\alpha} \vec{A}(\vec{r}) \} \right.$$

$$\left. \times u_{E,j,u}(\vec{r}) d\vec{r} \right|^2 dE', \qquad (1)$$

where $E\,(<mc^2)$, $E'\,(>mc^2)$, ~~$E' = E + h\nu$~~ denote the energies of the initial and the final states of the electron, satisfying the relation $E' - E = h\nu$. ~~$u_{E,j,u}$;~~ ~~E', ν~~ $u_{E,j,u}$ denotes the solution of ~~eigenfunction of the state~~ ~~in the Coulomb field~~ with energy E, ~~total angular~~ ~~Dirac~~ total angular momentum j (Dirac's j) and its z-component u. j ~~take~~ is ~~not zero~~ positive or negative integer excluding zero and u takes the value between $-|j|$ and $|j|-1$. ~~The~~ $\rightarrow$ means always a vector quantity.

The calculation ~~in~~ this case is formerly the same as ~~in the case~~ that of Beck and Sitte[3], ~~only the in~~ ~~The physical interpretation~~, ~~physical meaning~~ they concern, however, to ~~two~~ different processes, ~~the latter, the position of the electron emitted~~ either being reabsorbed by the nucleus. ~~Namely~~ ~~in~~ In the latter case, ~~however~~, either the positron or the electron emitted is reabsorbed by the nucleus,

3)

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$\frac{2\pi\nu}{c}$

DATE

NO. 3

so that the energy principle is violated, whereas in ~~the~~ former case both the electron and the positron escape the nucleus.

Next the probability of emission of K-electrons from the nucleus is given by

$$P_K = \frac{2\pi}{\hbar} \sum_{j'', u''} \sum_{u_0^*=0,-1} \left| \int \tilde{u}_{E'', j'', u''}(\vec{r}) \{ e A_0(\vec{r}) + e \vec{\alpha} \vec{A}(\vec{r}) \} \right.$$

$$\left. \times\, u_{n=1, j=-1, u_0}(\vec{r})\, d\vec{r} \right|^2, \qquad (2)$$

where $u_{n=1, j=-1, u_0}$ $\left(\text{or } u_{s, u_0}\right)$ denotes either of the two eigenfunction of 1S state of energy $E = mc^2\sqrt{1-\alpha^2 Z^2}$, $u_{E'', j'', u''}$ denotes that of energy $E''(> mc^2)$ ~~satisfying the relation~~ satisfying the condition $E'' = \cancel{E_{n=1}}\ mc^2\sqrt{1-\alpha^2 Z^2} + h\nu$.

~~$2mc^2\sqrt{1-\alpha^2 Z^2}$. $u_{E, j, u}$~~. The perturbing potentials $A_0(\vec{r})$, $\vec{A}(\vec{r})$ are expressed in general

$$A_0(\vec{r}) = Qe \int \frac{e^{iq|\vec{r}-\vec{r}_1|}}{|\vec{r}-\vec{r}_1|} \phi_a(\vec{r}_1) \phi_g^*(\vec{r}_1)\, d\vec{r}_1$$

$$\vec{A}(\vec{r}) = \frac{Qe\hbar}{4\pi mc\, i} \int \frac{e^{iq|\vec{r}-\vec{r}_1|}}{|\vec{r}-\vec{r}_1|} \{ \phi_g^*(\vec{r}_1)\, \mathrm{grad}_1 \phi_a(\vec{r}_1)$$

$$- \phi_a(\vec{r}_1)\, \mathrm{grad}_1 \phi_g^*(\vec{r}_1) \}\, d\vec{r}_1,$$

where $q = \frac{2\pi\nu}{c}$ and $\phi_g(\vec{r}_1)$ and $\phi_a(\vec{r}_1)$ are the wave functions of the normal and the excited states of the ~~the~~ one only, assuming that

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particle ~~out~~ with the charge Qe in the nucleus ~~wit~~ is concerned to the ~~tr~~ transition. q denotes $\frac{2\pi\nu}{c}$.
In the case of S-S transition, $A_0(\vec{r})$ is independent of θ, ϕ and $\vec{A}(\vec{r})$ can be made to zero by a suitable gauge invariant transformation, so that ~~we~~ $e A_0(\vec{r}) + e\vec{\alpha}\vec{A}(\vec{r})$ will be written simply $V(r)$ hereafter ~~we write~~

3. Now ~~we should write~~ the four components of the eigenfunctions $u_{E,j,u}(\vec{r})$ ~~and~~ or $u_{n,j,u}(\vec{r})$, which are the solutions of ~~the~~ Dirac's equations

$$\left(E + \frac{e^2 Z}{r} + c\,\vec{\alpha}\vec{p} + \beta mc^2\right)u = 0,$$

~~in detail~~ [2], are

$$u^{(1)}_{E\text{ or }n,\,j,\,u}(\vec{r}) = -i\,F_{E\text{ or }n,\,j}(r)\cdot w^{(1)}_{j,u}(\theta,\phi)$$

$$u^{(2)}_{E\text{ or }n,\,j,\,u}(\vec{r}) = -i\,F_{E\text{ or }n,\,j}(r)\cdot w^{(2)}_{j,u}(\theta,\phi)$$

$$u^{(3)}_{E\text{ or }n,\,j,\,u}(\vec{r}) = G_{E\text{ or }n,\,j}(r)\cdot w^{(1)}_{j,u}(\theta,\phi)$$

$$u^{(4)}_{E\text{ or }n,\,j,\,u}(\vec{r}) = G_{E\text{ or }n,\,j}(r)\cdot w^{(4)}_{j,u}(\theta,\phi),$$

1) Hulme,

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DATE

NO.

N_w

where $\quad N_{j,u}$

$$w^{(1)}_{j,u}(\theta,\phi) = \zeta(j,u)P^{u}_{-j} \quad \text{or} \quad \zeta(j,u)(j+u)P^{u}_{j-1}$$

$$w^{(2)}_{j,u}(\theta,\phi) = \zeta(j,u)P^{u+1}_{-j} \quad \text{or} \quad \zeta(j,u)(-j+u+1)P^{u}_{j-1}$$

$$w^{(3)}_{j,u}(\theta,\phi) = \zeta(j,u)(-j+u)P^{u}_{-j-1} \quad \text{or} \quad \zeta(j,u)P^{u}_{j}$$

$$w^{(4)}_{j,u}(\theta,\phi) = \zeta(j,u)(j+u+1)P^{u+1}_{-j-1} \quad \text{or} \quad \zeta(j,u)P^{u}_{j}$$

according as $j \lessgtr 0$ or $\gtrless 0$ and

$$P^{u}_{j}(\theta,\phi) = \frac{(j-u)!}{2^j\, j!}\{\sin^2\theta\}^{u}\frac{d^{j+u}}{(d\cos\theta)^{j+u}}(\cos^2\theta-1)^{j}\, e^{iu\phi}$$

$$\zeta(j,u) = \frac{1}{2\sqrt{\pi}}\frac{1}{\sqrt{(|j|-u-1)!\,(|j|+u)!}},$$

The first terms of $F(r)$ and $G(r)$ in powers of r are expension of $F_{E\sigma n;j}$ $E\sigma n;j$

$$f_{E\sigma n;j}\, r^{\gamma-1} \quad \text{and} \quad g_{E\sigma n;j}\, r^{\gamma-1}$$

respectively, where $\gamma = \sqrt{j^2-\alpha^2 z^2}$ $*1$

$\dagger \int \bar{u}_E dv \int_{E-\Delta E}^{E+\Delta E} u_{E'} dE' = 1$,

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DATE
NO.

For E > mc^2

$f_{E,j} = N_E \sqrt{2(j+\gamma)(j\varepsilon-\gamma)}$ for $j \gtrless 0$

$g_{E,j} = N_E \sqrt{2(j-\gamma)(j\varepsilon+\gamma)}$ according as

~~and~~ For $E = E_+ < mc^2$

$f_{-E_+,j} = \bar{g}_{E_+,-j}$ ~~according as $j \gtrless 0$~~

$g_{-E_+,j} = \bar{f}_{E_+,-j}$,

where the signe — upon g or f means to change Z into $-Z$.

where $N_E = \sqrt{\frac{2\pi m}{h^2}}(2k)^{\gamma-\frac{1}{2}} e^{\frac{\pi b}{2}} \frac{|\Gamma(\gamma+ib)|}{\Gamma(2\gamma+1)}$

$\varepsilon = \frac{E}{mc^2}$

$k = \frac{2\pi mc}{h}\sqrt{\left(\frac{E}{mc^2}\right)^2-1}$

$b = \alpha Z \cdot \frac{E}{mc^2} \Big/ \sqrt{\left(\frac{E}{mc^2}\right)^2-1}$,

normalization being made with respect to energy†.

Next for K-electrons

$f_{n=1,j=-1} = N_0\sqrt{1-\gamma}$

DEPARTMENT OF PHYSICS
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DATE..........
NO..........

$$g_{u=1,j=-1} = N_0\sqrt{1+\gamma}\,,$$

where

$$N_0 = \frac{1}{\sqrt{2}\sqrt{\Gamma(2\gamma+1)}}\left(\frac{2}{a_0}\right)^{\gamma+\frac{1}{2}}$$

$$a_0 = \frac{h^2}{4\pi^2 m e^2 Z}\,.$$

4. Inserting the above expressions ~~for~~ in the equations (1), (2), we have

$$p(E') = \frac{2\pi}{h}\sum_{j',u'}\sum_{j,u}\{f_{E',j'}f_{E,j}A_{j',u';j,u} + g_{E',j'}g_{E,j}B_{j',u';j,u}\}^2 R^2$$

$$P_K = \frac{2\pi}{h}\sum_{j'',u''}\sum_{u_0=0,-1}\{f_{E'',j''}f_{1,-1}A_{j'',u'';-1,u_0} + g_{E'',j''}g_{1,-1}B_{j'',u'';-1,u_0}\}^2 R^2,$$

where

$$R = \int r^{2\gamma}V(r)dr$$

$$A_{j',u';j,u} = \int d\varphi\int \sin\theta d\theta\,(w^{(1)}_{j,u}w^{*(1)}_{j',u'} + w^{(2)}_{j,u}w^{*(2)}_{j',u'})$$

DEPARTMENT OF PHYSICS
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DATE ....
NO. ....

$$B_{j'u';j,u} = \int d\varphi \int \sin\theta\, d\theta \, (w^{(3)}_{j,u} w^{*(3)}_{j'u'} + w^{(4)}_{j,u} w^{*(4)}_{j'u'}) .$$

Performing the integration, we have

$$A_{j'u';j,u} = B_{j'u';j,u} = \delta_{j'j}\,\delta_{u'u} ,$$

from which the selection rules ~~for j and u~~

$$\Delta j = 0 \qquad \Delta u = 0$$

for ~~j and u~~ the transition of the electron are obtained.

We have finally

$$P(E') = \frac{4\pi}{\hbar} \sum_j \{ f_{E',j} f_{E,j} + g_{E',j} g_{E,j} \}^2 R^2$$

$$\equiv \frac{4\pi}{\hbar} \sum_j M_j^2 \cdot R^2$$

$$P_K = \frac{4\pi}{\hbar} \{ f_{E'',-1} f_{n=1,-1} + g_{E'',-1} g_{n=1,-1} \}^2 R^2$$

$$\equiv \frac{4\pi}{\hbar} L^2 R^2 ,$$

so that the required ratio becomes

$$\rho = \frac{P_{paar}}{P_K} = \frac{\int \sum_j M_j^2 dE'}{L^2} ,$$

DEPARTMENT OF PHYSICS
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DATE..........
NO..........

where $M_j^2 = (f_{E',j} f_{E,j} + g_{E',j} g_{E,j})^2$

$$= (f_{E',j} \bar{g}_{E+,-j} + g_{E',j} \bar{f}_{E+,-j})^2$$

$$L^2 = (f_{E',-1} f_{1,-1} + g_{E',-1} g_{1,-1})^2,$$

~~The~~ In the ~~den~~ numerator of ρ, M_j ~~is small~~ is small except for $j = \pm 1$, so that

$$\rho = \frac{\int (M_{-1}^2 + M_1^2) dE'}{L^2}$$

approximately, where

$$M_{-1}^2 = 16 N'^2 N^2 \gamma_1^2 (\varepsilon_+ + \gamma_+)(\varepsilon' - \gamma_+)$$

$$M_1^2 = 16 N'^2 N_+^2 \gamma^2 (\varepsilon_+ - \gamma_+)(\varepsilon' + \gamma_+)$$

$$M_1^2 + M_{-1}^2 = 32 N'^2 N_+^2 \gamma^2 (\varepsilon_+ \varepsilon' - \gamma_+^2)$$

$$L^2 = 8 N^2 N_0^2 (\varepsilon + \gamma_+)^{\dagger}.$$

~~and $\gamma_1 = \sqrt{1-\alpha^2 Z^2}$~~ Now N, N_0 can be rewritten in the following manner:

$$N^2 = \frac{1}{8\pi mc^2}\left(\frac{2}{\Lambda}\right)^{2\gamma+1} \eta^{2\gamma-1} \frac{e^{\pi b} |\Gamma(\gamma+ib)|^2}{\{\Gamma(2\gamma+1)\}^2}$$

$$N_0^2 = \frac{1}{2}(\alpha Z)^{2\gamma+1} \left(\frac{2}{\Lambda}\right)^{2\gamma+1} \frac{1}{\Gamma(2\gamma+1)},$$

† Hereafter $\gamma = \sqrt{1-\alpha^2 Z^2}$.

DEPARTMENT OF PHYSICS
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DATE
NO.

where $\eta = \sqrt{\varepsilon^2 - 1}$, $\Lambda = \frac{h}{2\pi mc}$.

If we ~~We can~~ write further

$$\Phi(\varepsilon) = e^{\pi b} |\Gamma(\gamma + ib)|^2 \eta^{2\gamma-1}$$

we have

$$\rho = \frac{\gamma^2}{\pi(\alpha Z)^{2\gamma+1}\Gamma(2\gamma+1)} \int \frac{\Phi(\varepsilon')\overline{\Phi}(\varepsilon_+)(\varepsilon_+\varepsilon' - \gamma^2)}{\Phi(\varepsilon'')(\varepsilon''+\gamma)} d\varepsilon'$$

$$\left(\frac{\int F(\varepsilon')\,d\varepsilon'}{\Phi(\varepsilon'')(\varepsilon''+\gamma)} \right),$$

where $F(\varepsilon') \equiv \Phi(\varepsilon')\,\overline{\Phi}(\varepsilon_+)\,(\varepsilon_+\varepsilon' - \gamma^2)$

indicates the energy distribution of the emitted positive and negative electrons.

5. We want to obtain the numerical values of $F(\varepsilon')$ and ρ in the case of RaC', in which

$Z = 84$ $\quad \alpha = \frac{1}{137}$

$h\nu = 1.426 \times 10^6$ e.V. $= 2.8\,mc^2$

$\alpha Z = 0.6$

$\gamma = 0.8$.

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DATE..........
NO..........

Insert these values, we have
~~so that~~

$$\frac{\gamma^2}{\pi(\alpha 2)^{2\gamma+1}\Gamma(2\gamma+1)} = 0.5,$$

$$\Phi(\varepsilon) = \eta^{0.6} e^{0.6\pi\frac{\varepsilon}{\eta}} \left|\Gamma\left(0.8+0.6i\frac{\varepsilon}{\eta}\right)\right|^2$$

$$\cong 4.5 + 1.6\eta,$$

$$\overline{\Phi}(\varepsilon) \cong (4.5+1.6\eta)\, e^{-1.2\pi\frac{\varepsilon}{\eta}}$$

$$\varepsilon'' = 2.8 + 0.8 = 3.6$$

$$\eta'' = 3.5$$

$$\Phi(\varepsilon'')(\varepsilon''+\delta) = 44.4$$

so that

$$\rho = 1.1 \times 10^{-2} \int F(\varepsilon')\, d\varepsilon'$$

$$F(\varepsilon') = (4.5+1.6\eta')(4.5+1.6\eta_+)(\varepsilon_+\varepsilon' - 0.6) \times e^{-1.2\pi\frac{\varepsilon_+}{\eta_+}}$$

$$\varepsilon' + \varepsilon_+ = 2.8.$$

The distribution function $F(\varepsilon')$ is ~~tabulated;~~ given by ~~the following~~ figure~~, from which we have~~ ~~figure~~ Fig. 1.

) E. Fermi,

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DATE
NO.

~~It has a~~ The maximum of the curve lies at the upper limit of the energy of positron, ~~so that~~ ~~the~~ and it agrees with the first maximum of Alichanow's curve.

$\varepsilon \quad \eta' \quad \varepsilon + \eta + \frac{4.5}{9} 405 \quad F(\varepsilon')$

From the curve we have
~~From which~~ $\int F(\varepsilon')\,d\varepsilon' \cong 0.4\frac{\cancel{87}}{18}$, $\frac{\frac{4.5}{9}}{18}$
~~and~~ ~~from which~~ so that $\rho \cong 1.1 \times 10^{-2} \times 0.\cancel{4}18$
the number of $= \cancel{4.4 \times 10^{-3}}$ 2×10^{-3}

Now ~~the~~ the K-electrons in question ~~are~~ is about ~~emitted~~ 0.003 ~~for every~~ per ~~one~~ disintegration of the nucleus *), so that the ~~tota~~ number of positrons in question ~~per one disintegration~~ ~~are~~ is about

$$0.003 \times \cancel{4.4}\,2 \times 10^{-3}$$
$$\cong \cancel{1.3}\,0.6 \times 10^{-5} \quad \text{per disintegration},$$

On the other hand,
According to Alichanow and Kosodaew, total number of positron emitted ~~at~~ is
$$0.003$$
per disintegration., ~~so that~~ Hence, if the above result of the calculation is correct, ~~the contribution of~~ the number of the positrons emitted by the SS-transition ~~of the~~ in question ~~only~~ ~~contribute only~~ amounts only $0{,}2$ ~~0.44~~ % ~~4.4×10^{-3}~~ of the total number of positrons emitted,

DEPARTMENT OF PHYSICS
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DATE..................
NO..................

The above assumption is insufficient to explain
~~Thus the first large~~ maximum of the ~~to~~ distribution curve of ~~Alichanow~~ can not be explained by the above assumption only.
Thus only very small part of the first maximum of the distribution curve of Alichanow can be interpreted ~~by~~ as the ~~above~~ process above considered.

Abstract
The ratio ρ increases rapidly
§6. As the ~~nuclear~~ energy ~~difference of~~ liberated by the nuclear transition increases, and ~~For example~~, the maximum of the distribution curve ~~of~~ displaces by ~~to~~ and by ~~to~~ from the upper limit of the energy of the positron to the middle.
For example, assuming $Z = 84$, $h\nu = 4mc^2$, we have for $F(\varepsilon')$ a curve shown by Fig 2. and

$$\rho = \frac{\cancel{4.7\times10^{-2}}}{4.7\times10^{-2}} \quad 0.047.$$

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Theory of Pair Production by Radiationless
Transition of Radioactive Nucleus

By H. Yukawa and S. Sakata

1. Jaeger and Hulme recently calculated the probability of

p
pi