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Abstract

The ~~theory~~ probability of internal pair production by radiationless transition of the radioactive nucleus was calculated and ~~its ratio the ratio~~ its ratio to the probability of emission of electrons from K levels by the same nuclear transition was compared with the experiment in the case of RaC'.

On the Theory of Internal Pair Production.[1]

By Hideki Yukawa and Shoichi Sakata.

§ 1. Introduction and Summary

The theory of the production of the pair of an electron and a positron by the internal conversion of the γ-ray with the energy larger than $2mc^2$, was developped recently by Jaeger and Hulme[2], as a natural extension of the theory of ordinary internal conversion of the γ-ray with the emission of the β-ray.

They compared their result with the experiment of Alichanow and Kosodaew[3], who measured the number of positrons emitted from a Ra(B + C) source. According to the theory the second and the third maxima of the experimental distribution curve, at about 0.7×10^6 eV and 10^6 eV, were to be due to the γ-rays of energies 1.7×10^6 eV and 2.2×10^6 eV, as the coefficient of internal pair production were maximum at the upper limit of the energy of the positron.

The exceptional case, where a pair of an electron and a positron is emitted by taking up the energy liberated by a nuclear transition between two S levels, was not treated by the former authors, because of the lack of the corresponding γ-ray.

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1) This paper was read before the meeting of Osaka branch on July 6, 1935.

2) J.C.Jaeger and H.R.Hulme, Proc.Roy.Soc. 148,708,1935.

3) A.I.Alichanow and M.S.Kosodaew, ZS.f.Phys. 90,249,1934.

4) See Fig. 1, below, which is the same with Fig. 15 of Alichanow and Kosodaew, loc.cit.

Now if the maximum of the number of positrons produced by such processes lies also at the upper limit of their energies, the first hump of Alichanow s curve at about 0.4×10^6 eV can be attributed to the missing γ-ray of RaC′ of energy 1.4×10^6 eV.

Hence it seems worth while to investigate this case in detail and to determine whether such aprocess is sufficient to account for the largest hump of the curve or not.

The nuclear transition between two levels of energy difference ΔW can be considered in general as a perturbing field on the electron varying with the frequency $\frac{\Delta W}{h}$. This perturbing field extends only over the region of nuclear dimension in the case of S-S transition, so that the probability of ~~the~~ transition of the electron is large only when the eigenfunctions of the initial and the final states are not small in the neighborhood of the nucleus. Accordingly, we have only to consider the case when the total angular momenta of both of the electron and the positron take the smallest values.

The distribution function of the positron calculated in this manner has the maximum in the neighborhood of the upper limit of the energy, which agrees with the first hump of the experimental curve.

To find the ~~experimental~~ absolute value of the probability of the pair production, we have to know the nuclear stucture in detail, ~~which is out of our present knowledge.~~ To avoid this difficulty, we considered only the ratio of the probability of the pair production to that of the emission of K electrons caused by the same nuclear transitions, as they have the common factor ~~relating~~ relating to the nuclear transition, if we neglect the vector potential of the perturbing field.

If we take the vector potential into account, the ratio depends on the relative magnitude of scalar and vector potentials and can in special forms be done fairly large enough to be in accord with the experiment.

The result obtained shows that the ratio is so small that only a small part of the hump in the experimental curve can be attributed to the positrons emitted by the above processes.

On the contrary, as it is possible that the application of the present theory of the electron up to the region of the nuclear dimension ends with entirely wrong result, we are not certain whether other processes such as the ~~direct~~ emission of single positrons from the nucleus or the internal conversion of the β-ray ~~is~~ are necessary for the explanation of the experiment or not.

A brief account of the calculation will be given in the following sections.

§ 2. Probability of Internal Pair Production

Now we consider the general case, where an electron moving in the field of the nucleus with the charge Ze is perturbed by a nuclear transition between two levels of energy difference ΔW.

The perturbation can be expressed by scalar and vector potentials of the form $A_0(\vec{r})\exp(-2\pi i\nu t)$ and $\vec{A}(\vec{r})\exp(-2\pi i\nu t)$, where $\nu = \frac{\Delta W}{h}$. If we assume that only one particle with the effective charge Qe in the nucleus and the reduced mass M falls from a state $\varphi(\vec{r})$ to a state $\varphi_0(\vec{r})$, the energy difference being ΔW, the potentials can be expressed in general by

$$A_0(\vec{r}) = Qe\int \frac{\exp(iq|\vec{r}-\vec{r}_1|)}{|\vec{r}-\vec{r}_1|}\varphi_0^*(\vec{r}_1)\varphi(\vec{r}_1)\,d\vec{r}_1 , \qquad (1)$$

$$\vec{A}(\vec{r}) = \frac{Qeh}{4\pi Mci}\int \frac{\exp(iq|\vec{r}-\vec{r}_1|)}{|\vec{r}-\vec{r}_1|}\{\varphi_0^*(\vec{r}_1)\,\mathrm{grad}_1\,\varphi(\vec{r}_1) - \varphi(\vec{r}_1)\,\mathrm{grad}_1\,\varphi_0^*(\vec{r}_1)\}\,d\vec{r}_1 , \qquad (2)$$

5) Anyhow, we can not have much confidence in the results of our calculation, as the application - - - - may happen to be entirely wrong and we are not certain whether - - - -

The ~~suffix~~ symbol → means always a vector quantity and * the conjugate complex quantity.

~~and → means always a vector quantity.~~

where $q = \frac{2\pi\nu}{c} = \frac{2\pi\Delta W}{hc}$.

Now, according to the perturbation theory, the probability per unit time per unit energy range of the transition of the electron from the continuous state of negative energy $-E_+(E_+>mc^2)$ to the continuous state of positive energy $E_-(>mc^2)$, satisfying the relation

$E_+ + E_- = \Delta W$, is given by

$$p(E_+) = \frac{4\pi^2}{h} \sum_{j,u} \sum_{j',u'} \left| \int \tilde{\psi}_{E_-,j',u'}(\vec{r}) \{ e A_0(\vec{r}) + e\vec{\alpha}\vec{A}(\vec{r}) \} \times \psi_{-E_+,j,u}(\vec{r})\, d\vec{r} \right|^2 \qquad (3)$$

where $\psi_{E,j,u}$ denotes the solution of Dirac's equation

$$\left(E + \frac{Ze^2}{r} + c\vec{\alpha}\vec{p} + \beta mc^2 \right) \psi = 0$$

with energy E, total angular momentum $|j|-\frac{1}{2}$ (Dirac's j), and its z-component u, $u+\frac{1}{2}$. j is a positive or a negative integer excluding zero and u takes a value between $-|j|$ and $|j|-1$. The eigenfunctions are normalized with respect to the energy scale, the definition of which is

$$\int \tilde{\psi}_E \psi_{E'} d\vec{r} = \delta(E'-E).$$

This corresponds to the process, in which a positron of energy E_+ and an electron of energy E_- are emitted simultaneously from the nucleus.

Hence, the total probability per unit time of the production of a pair is given by $\Delta W - mc^2$

$$P_{pair} = \int_{mc^2} p(E_+)\, dE_+ \quad .$$

(4) $\vec{\alpha}$, β are Dirac's matrices with four rows and four columns and ψ is to be considered as a matrix with one row and four columns, whereas $\tilde{\psi}$ is to be considered as a matrix with four rows and one column, the elements being complex conjugate to the corresponding elements of ψ.

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In the case of S-S transition, φ and φ_0 are functions of r only, so that the scalar potential becomes a function of r ~~only~~ and the vector potential reduces to ~~the form~~ a function of r multiplied by $\frac{\vec{r}}{r}$ another. Thus the perturbing potential $eA_0 + e\vec{\alpha}\vec{A}$ can be written in a form

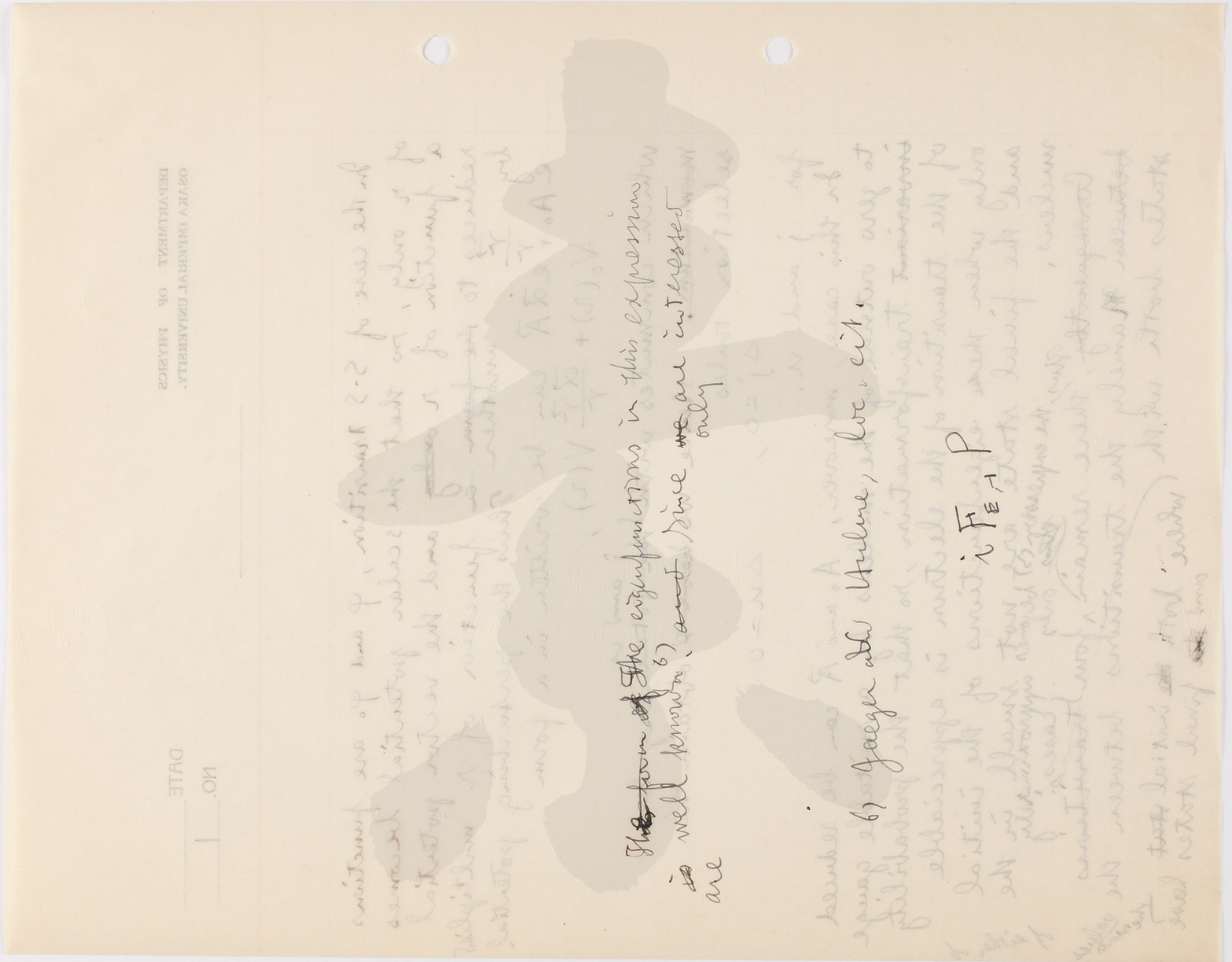
$$V_0(r) + \frac{\vec{\alpha}\vec{r}}{r} V(r),$$

which commutes with ~~the total angular momentum~~ j and κ. Hence we ~~have~~ obtain the selection rules

$$\Delta j = 0, \qquad \Delta \kappa = 0$$

for j and κ.

In this case, moreover, A_0 and $\vec{A}$ can be reduced to zero outside of the nucleus by a suitable gauge ~~invariant~~ transformation, so that the probability of the transition of the electron is appreciable only when the eigenfunctions of the initial and the final state are not small in the nucleus. ~~Consequently~~, thus the expression (5) becomes approximately ~~thus~~ only, there remain four cases; ~~transitions between~~ namely the transitions between the states both with (where both the initial ~~stat~~ and the final states have the same values of either of



1) $j=-1$, $u=-1$ 2) $j=-1$, $u=\pm 0$
3) $j=1$, $u=-1$ 4) $j=1$, $u=0$,

The eigenfunctions in these cases ~~have the~~ are

1)
$$\begin{pmatrix} i F_{E,-1}(r)\sin\theta\, e^{-i\phi} \\ -i F_{E,-1}(r)\cos\theta \\ 0 \\ -G_{E,-1}(r) \end{pmatrix}$$

2)
$$\begin{pmatrix} -i F_{E,-1}(r)\cos\theta \\ -i F_{E,-1}(r)\sin\theta\, e^{i\phi} \\ G_{E,-1}(r) \\ 0 \end{pmatrix}$$

3)
$$\begin{pmatrix} 0 \\ i F_{E,1}(r) \\ -G_{E,1}(r)\sin\theta\, e^{-i\phi} \\ G_{E,1}(r)\cos\theta \end{pmatrix}$$

4)
$$\begin{pmatrix} -i F_{E,1}(r) \\ 0 \\ G_{E,1}(r)\cos\theta \\ G_{E,1}(r)\sin\theta\, e^{i\phi} \end{pmatrix},$$

E being the energy of these states [6];
since only the values of the eigenfunctions in the neighborhood of the nucleus are important, their radial parts $F_{E,\mp 1}$, $G_{E,\mp 1}$ are expanded in powers of r and the first terms of the smaller power $r^{\gamma-1}$ are taken as $F_{E,\mp 1} r^{\gamma-1}$, $G_{E,\mp 1} r^{\gamma-1}$ in the first approximation.

6) Jaeger and Hulme, loc. cit.

are taken as the first approximation, where
$\gamma = \sqrt{1-\alpha^2 Z^2}$,

For $E = E_- > mc^2$

$$f_{E_-,\mp 1} = \pm N \sqrt{2(-\gamma \pm 1)(\gamma \pm \varepsilon)} \quad \pm N\sqrt{2(\gamma\pm1-\gamma)(\varepsilon\pm\gamma)} \quad \pm N\sqrt{2(1\mp\gamma)(\varepsilon\pm\gamma)}$$

$$g_{E_-,\mp 1} = N \sqrt{2(\gamma \pm 1)(\gamma \pm \varepsilon)} \quad \to N\sqrt{2(1\pm\gamma)(\varepsilon\pm\gamma)},$$

with $$N = \sqrt{\frac{2\pi m}{h^2}} (2k)^{\gamma - \frac{1}{2}} \exp\left(\frac{\pi b}{2}\right) \frac{|\Gamma(\gamma + ib)|}{\Gamma(2\gamma+1)},$$

where $\varepsilon_- = \frac{E_-}{mc^2} \geq 1$, $k = \frac{2\pi mc}{h}\sqrt{\varepsilon^2 - 1}$

$$b = \alpha Z \frac{\varepsilon}{\sqrt{\varepsilon^2 - 1}},$$

For $E = -E_+$, $E_+ > mc^2$

$$f_{-E_+,\mp 1} = \overline{g}_{E_+,\pm 1}$$

$$g_{-E_+,\mp 1} = \overline{f}_{E_+,\pm 1},$$

where the sign $-$ upon g and f means to change Z into $-Z$.

Now we neglect for simplicity the term $\frac{\alpha}{r}\frac{\partial r}{\partial r} V(r)$ ~~perturba~~ in the perturbing potential, which will be small compared with $V_0(r)$, owing to the fact that

$$\frac{4m}{\{\Gamma(2\gamma+1)\}^4 \hbar^3}\left(\frac{4\pi mc}{\hbar}\right)^{4\gamma}$$

$$\frac{16}{16}\quad \frac{4m}{\hbar^3}$$

$$\frac{9}{16}\quad 4$$

$$\frac{16}{256}$$

$$\frac{4\,(4\pi)^4}{\{\Gamma(2\gamma+1)\}^4}\,\frac{m^5c^4}{\hbar^7}\left(\frac{4\pi mc}{\hbar}\right)^{4\gamma-1}$$

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or p-disintegration

In this approximation the calculation is formally ~~the same as~~ similar to that of Beck and Sitte?! The physical interpretation however, differ in two cases. In the latter case, either the positron or the electron is assumed to be reabsorbed by the original nucleus, whereas in our case both the electron and the positron escape the nucleus.

The velocities of the ~~nuclear~~ heavy particles in the nucleus are far smaller than c. ~~Then, inserting the above the equations in (3), we have and performing~~ we have

$$p(E_+) = \frac{4\pi^2}{h} \sum_{j=\mp 1,\ u=0,-1} 1 \int \psi_{E,}$$

In this case

Then the equation (3) becomes

$$p(E_+) = \frac{4\pi^2}{h} \sum_{j=\mp 1,\ u=0,-1} 1 \int \tilde{\psi}_{E_-,j,u}(\vec{r})$$

$$\times V_0(r)\, \psi_{-E_+,j,u}(\vec{r})\, d\vec{r}\, |^2$$

Inserting the above expressions for the eigenfunctions and performing the integration with respect to θ, ϕ, we have

$$p(E_+) = \frac{8\pi^2}{h} \{ (f_{E_-,-1}\, \bar{g}_{E_+,+1} + g_{E_-,-1}\, \bar{f}_{E_+,+1})^2$$

$$+ (f_{E_-,1}\, \bar{g}_{E_+,-1} + g_{E_-,1}\, \bar{f}_{E_+,-1})^2 \} 1 \int_{R_p^2}^{2R} V_0(r)\, dr |^2$$

or

$$= 4\gamma^2 \frac{256\pi^4}{\{\Gamma(2\gamma+1)\}^4} \frac{m^5 c^4}{h^7} \left(\frac{4\pi m c}{h}\right)^{4(\gamma-1)}$$

$$\times R_p^2 \cdot F_p(\varepsilon_+) \, , \{ 1 + \frac{1-\gamma^2}{\gamma^2} \frac{\alpha^2 Z^2}{1-\alpha^2 Z^2} \left(\frac{S_p}{R_p}\right)^2 \}$$

R_p^2

ηb)

We have to take $k=0$ in (2β) for $j=-1$ and $k=1$ in (2β) for $j=1$.

$j=-1$ $\quad -k-1$

$j=+1$ $\quad k$

$k=1$

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where $R_p = \int r^{2(\gamma-1)} V_0(r)\, d\vec{r}$,

$F_p(\varepsilon_+)$ is the only factor, which depends on the energy of the positron and has the form

$$F_p(\varepsilon_+) = \eta_+^{2\gamma-1} e^{-\pi b_+} |\Gamma(\gamma+ib_+)|^2 \times \eta_-^{2\gamma-1} e^{\pi b_-} |\Gamma(\gamma-ib_-)|^2 (\varepsilon_+\varepsilon_- - \gamma^2),$$

where $\varepsilon_\pm = \frac{E_\pm}{mc^2}$ $\quad \eta_\pm = \sqrt{\varepsilon_\pm^2 - 1}$

$$b_\pm = \alpha Z \frac{\varepsilon_\pm}{\eta_\pm}.$$

The forms of $F_p(\varepsilon_+)$ in special cases are shown below.

Now the total probability of pair production is given by

$$P_{pair} = 4\gamma^2 \frac{256\pi^4}{\Gamma(2\gamma+1)} \frac{m^5c^4}{h^7} \left(\frac{4\pi mc}{h}\right)^{4(\gamma-1)} \times R_p^2 \int_1^{\omega-1} F_p(\varepsilon_+)\, d\varepsilon_+,$$

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If we neglect the vector potential, the calculation is formally similar to that of Beck and Sitte on β-disintegration. The physical interpretations, however, differ in two cases.

where $\Delta w = \frac{\Delta W}{mc^2}$.

In these formulae the factor R_p^2 depends on ~~the~~ the detailed structure of the nucleus and can not be calculated without special assumptions. To eliminate this factor we want to consider the emission of K-electrons by the same the prob. of nuclear transitions in the next section.

§ 3. Probability of Electron Emission from K ~~levels, states~~ levels.

The ~~calculation in this case~~ probability per unit time of the transition of the electrons ~~from the K levels~~ from K ~~states~~ to the continuous states of energy $E'(>mc^2)$, ~~of energy $mc^2\sqrt{1-\alpha^2 Z^2}$~~ satisfying the condition $E' = mc^2\sqrt{1-\alpha^2 Z^2} + \Delta W$, is given ~~by~~ in general by

$$P_K = \frac{4\pi^2}{h} \sum_{j,u'} \sum_{u=0,-1} \left| \int \tilde{\psi}_{E';j;u'}(\vec{r}) \{ eA_0(\vec{r}) + e\vec{\alpha}\vec{A}(\vec{r}) \} \psi_{K,u}(\vec{r})\, d\vec{r} \right|^2,$$

($V_0(r)+e$)

where ~~$A_0(\vec{r})$ and $\vec{A}(\vec{r})$~~ ~~ΔW~~ ΔW, $A_0(\vec{r})$, $\vec{A}(\vec{r})$ and $\psi_{E';j;u'}$

~~have the same meaning~~ as in the previous case.
~~multiplied respective~~ by the factor $\exp\left(-\frac{2\pi i z w_0}{a}\right)$
~~are~~ have the same meaning
~~mean the perturbing potentials due to the nuclear~~
~~transition~~ as in the previous section.

$\psi_{K,u}$ is ~~either of two~~ the eigenfunction of either of two K states, ~~and has the form~~ The components of ψ_{Ku} are

$$+i N_0 \sqrt{1-\gamma}\, r^{\gamma-1} \exp\left(-\frac{r}{a_0}\right) \sin\theta\, e^{-i\phi}$$
$$-i N_0 \sqrt{1-\gamma}\, r^{\gamma-1} \exp\left(-\frac{r}{a_0}\right) \cos\theta$$
$$0$$
$$-N_0 \sqrt{1+\gamma}\, r^{\gamma-1} \exp\left(-\frac{r}{a_0}\right)$$

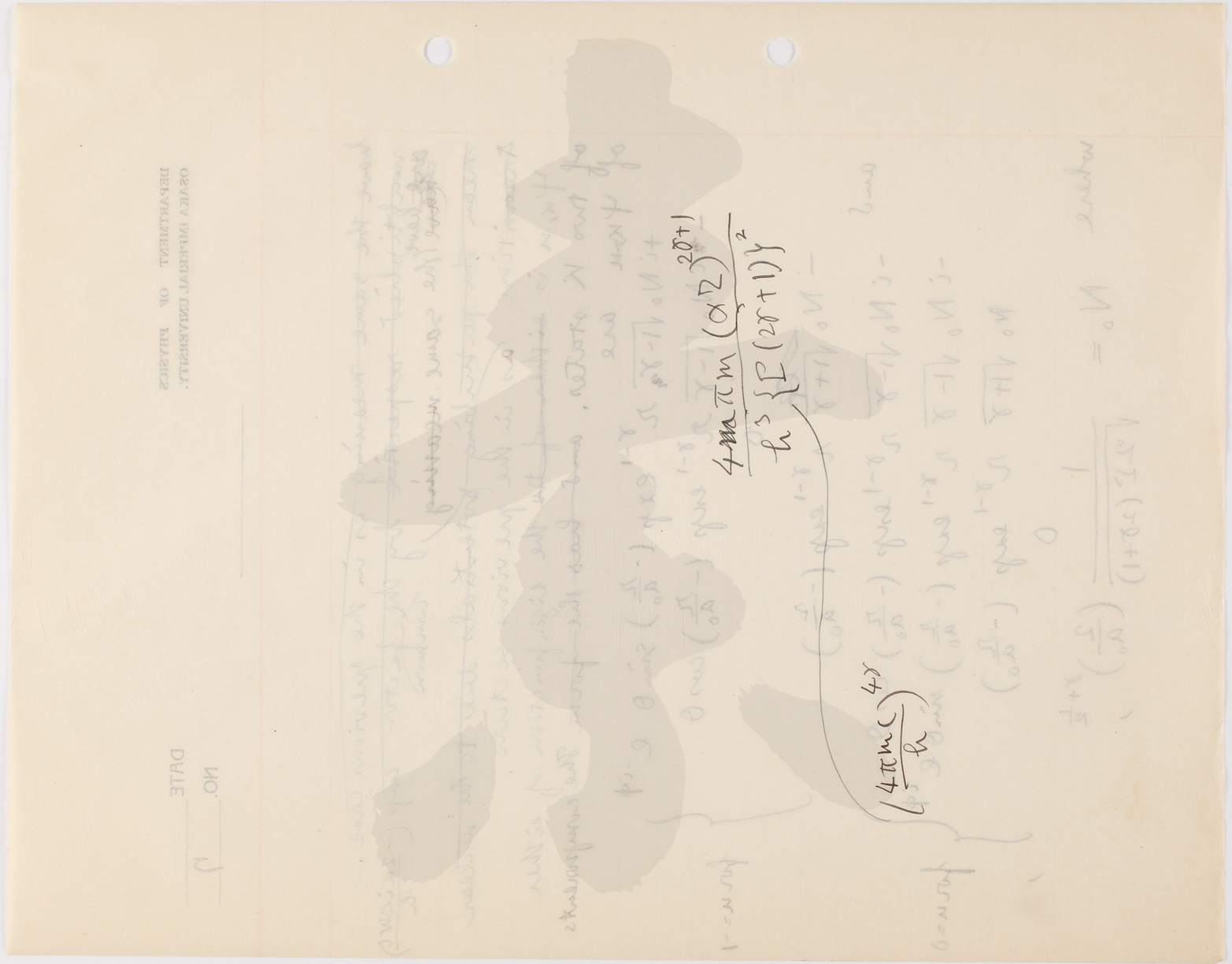
for $u=-1$

and

$$-i N_0 \sqrt{1-\gamma}\, r^{\gamma-1} \exp\left(-\frac{r}{a_0}\right) \cos\theta$$
$$-i N_0 \sqrt{1-\gamma}\, r^{\gamma-1} \exp\left(-\frac{r}{a_0}\right) \sin\theta\, e^{i\phi}$$
$$N_0 \sqrt{1+\gamma}\, r^{\gamma-1} \exp\left(-\frac{r}{a_0}\right)$$
$$0$$

for $u=0$,

where $N_0 = \dfrac{1}{\sqrt{2\Sigma(2\gamma+1)}} \left(\dfrac{2}{a_0}\right)^{\gamma+\frac{1}{2}}$,



$a_0 = \frac{h^2}{4\pi^2 m e^2 Z}$, the ~~normalization~~ eigenfunctions being normalized to 1.

In the case of S-S transition of the nucleus, ~~j, u' should be the same with~~ j, u' of the final states should the same with that of the inital states, ~~as above~~ previous section, so that $j' = -1$, $u' = 0$ or -1. ~~the transitions are confined to~~ only the terms with remain in the summation of P_K. Further neglecting the vector potential and omitting the factor $\exp(-\frac{r}{a_0})$ ~~in th~~ in ~~the~~ $\psi_{K,u}$, The expression in the neighborhood of the nucleus ~~we have the expres~~ P_K is transformed to the form

$$P_K = 4(\alpha Z)^{2\gamma+1} \frac{256\pi^5}{\{\Gamma(2\gamma+1)\}^2} \frac{m^5 c^4}{h^7}$$

$$\times \left(\frac{4\pi mc}{h}\right)^{4(\gamma-1)} R_p^2 \eta'^{2\gamma-1} e^{\pi b'} |\Gamma(\gamma+ib')|^2$$

$$\times (\varepsilon'+\gamma),$$

Taking $\psi_{K,u}$ for $\psi_{K,u}$ and $\psi_{K,u'}$ the approximal first terms of the ~~expression~~ expansion in powers of r ~~and we~~ have finally

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where $\varepsilon' = \frac{E'}{mc^2}$ $\eta' = \sqrt{\varepsilon'^2 - 1}$

$b' = \alpha Z \frac{\varepsilon'}{\eta'}$

and R_p is the same integral as in the previous selection, if the nuclear transition is the same in both cases.

Now the ratio of the pair production to ~~by~~ probability of the ~~that the~~ S transition of the nucleus to that of the emission of electrons from K levels by the same nuclear transition is given by

$$\rho = \frac{P_{pair}}{P_{K\ emiss.}} = \frac{(1-\alpha^2 Z^2)\gamma^2 \int F_p(\varepsilon_+) d\varepsilon_+}{\pi (\alpha Z)^{2\gamma+1} \Gamma(2\gamma+1) \eta'^{2\gamma-1}}$$

$$\frac{}{\times |\Gamma(\gamma + ib')|^2 e^{\pi b'} (\varepsilon' + \gamma)} \left\{1 + \frac{\alpha^2 Z^2}{(1-\alpha^2 Z^2)} \left(\frac{S_p}{R_p}\right)^2\right\}.$$

This expression is independent of the detailed structure of the nucleus and can be compared with the experiment in the special case.

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~~Results~~

§4. General discussions ~~and Numerical~~

In the above calculation there are two points ~~which are to be discussed before entering into~~ calculations (results) numerical values of $F_p(\xi_+)$ and ρ obtaining in special cases, ~~two~~ Two points should be noticed in detail in the above formulation.

The first point is that ~~the for the eigen~~ the solutions of Dirac's equation in Coulomb field of charge Ze ~~are~~ were taken as the eigenfunctions of ~~the initial and final~~ the electron. As in the immediate neighborhood of the nucleus the field is not of Coulomb type, we should have ~~modified~~ multiplied ~~each~~ eigenfunctions by ~~an unknown factor~~ an unknown function of r, which ~~is take~~ be seen to the value 1 ~~for r large~~ compared with the nuclear radius ~~and~~ the value of deviates from 1 ~~only~~ in the immediate neighborhood of the nucleus. ~~By~~ This modification can be ~~considered at length~~ reduced in the end to the change of the definitions of R_{p_1}, S_p, other factors in the ~~expressions~~ above being unaltered. (For $p(\xi_+)$ and P_k and ρ)

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By this modification. In this case, R_p and S_p are depend more or less on the initial and final states of the electron, ~~approx.~~

Instead of this, as a first approx. for the eigenvalues, we can simply take the constant values for the eigenfunction of ~~the eigenfunctions~~ at the boundary of the nucleus, $r = a_N$ being the radius of the nucleus. Then, in the formulae of the preceding sections, we have only to change the definition of R_p, S_p, in the following way. Namely,

$$R_p = a_N^{2(\gamma-1)} \int V_0 \, d\vec{r}$$

$$S_p = a_N^{2(\gamma-1)} \int V \, d\vec{r} \quad ,$$

so that other things being unaltered.

The second point is that we have assumed for the perturbing potentials the special forms (1) and (2). We can show, however, that the nucleus perturbing potential $eA_0 + e\vec{\alpha}\vec{A}$ has always included in effect, a form

$$V_0(r) + \frac{\alpha \cdot r}{r} V(r) \quad ,$$

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whenever the nucleus falls from a state of zero spin to another of zero spin.

These two points will be discussed in detail in a paper of the authors,† which will soon appear concerning a treat in which similar calculations in the case of β-disintegration will be performed.

† This paper will soon appear in this journal.

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§4. General Discussions

Before entering into numerical calculation in special cases, ~~we should notice~~ it should be noticed that the above formulation is incomplete in two points.

The first of them is that we ~~have~~ assumed for the perturbing potentials special form (1) and (2). We can show more generally that the perturbing potentials take the form

$$A_0(r,\theta,\varphi)\exp(-2\pi i\nu t) = F(r)\exp(-2\pi i\nu t)$$

$$A_1(r,\theta,\varphi)\exp(-2\pi i\nu t) = \frac{\vec{z}\vec{r}}{r}G(r)\exp(-2\pi i\nu t), \qquad (\ )$$

F and G ~~being~~ the functions of r only, wherever ~~the nucleus fall~~ the initial and the final states both of the nucleus have ~~the~~ zero spin or, more generally have the same ~~value~~ spin in the same and the same direction.

~~The proof is simple. For~~ To proof this, we consider the total angular momentum of the system consisting of the ~~nucleus~~ and the electron, which should ~~be constant~~ constant throughout the motion of the system. ~~Since the initial and the final states of the nucleus have both zero spin, the~~

Since the ~~total~~ angular momentum of the nucleus does not change ~~by~~ the above transition, being zero in any direction, ~~throughout~~ ~~throughout~~, the ~~total~~ angular momentum of the electron should not change by the transition. Hence, the perturbed Hamiltonian of the electron should commute with the total angular momentum also

$$\vec{M} = \vec{m} + \frac{\hbar}{4\pi}\vec{\sigma},$$

defined to be

$\vec{m}$ being $\vec{r}\times\vec{p}$, orbital and spin angular momenta respectively,

so that we have ~~in this case~~

$$\vec{M}(eA_0 + e\vec{\alpha}\vec{A}) - (eA_0 + e\vec{\alpha}\vec{A})\vec{M} = 0, \quad (\ )$$

where $A_0(x,0)\exp(-2\pi i\nu t)$, $\vec{A}(x,0)\exp(-2\pi i\nu t)$ are the potentials of the perturbing ~~perturbation~~ and $\vec{\alpha} = \vec{p}$, on the electron due to the nuclear transition.

As ~~Now~~ the ~~equ~~ coefficient ~~left hand sides~~ of the equations are ~~the~~ linear functions of p_1, the coefficients of 1 (the unit matrix), p_1, $p_1\sigma_x$, ~~$p_1\sigma_y$, $p_1\sigma_z$~~, $p_1\sigma_x$, $p_1\sigma_y$, $p_1\sigma_z$ in the left hand sides of the equations should vanish separately, we have

$$\vec{m}A_0 - A_0\vec{m} = 0, \quad (\ ,1)$$

$$\left.\begin{aligned} m_x A_x - A_x m_x &= 0 \\ m_y A_x - A_x m_y &= -\frac{i\hbar}{2\pi}A_z \\ m_z A_x - A_x m_z &= \frac{i\hbar}{2\pi}A_y \end{aligned}\right\} \quad (\ ,2)$$

and similar equations for A_y, A_z.

From (,1) the equations (,1) shows that the A_0 is a function of r only. From of Remaing equations shows that the relations of commutation between $\vec{m}$ and $\vec{A_x}, \vec{A_y}, \vec{A_z}$ are the same as those between $\vec{m}$ and $\vec{r}$. Further, by using the above equations repeatedly, we have the relation of commutation between $\vec{m}$ and $\vec{r}$, we have

$$m_y(xA_y - yA_x) - (xA_y - yA_x)m_y = \frac{ih}{2\pi}(yA_z - zA_y) \quad (\ ,2)$$

On the other hand, by using the above equations and the commutation relations between $\vec{p}$ and $\vec{r}$, we have

$$p_x(xA_y - yA_x) - (xA_y - yA_x)p_x$$

$$= x(p_xA_y - A_yp_x) - \frac{ih}{2\pi}A_y - y(p_xA_x - A_xp_x)$$

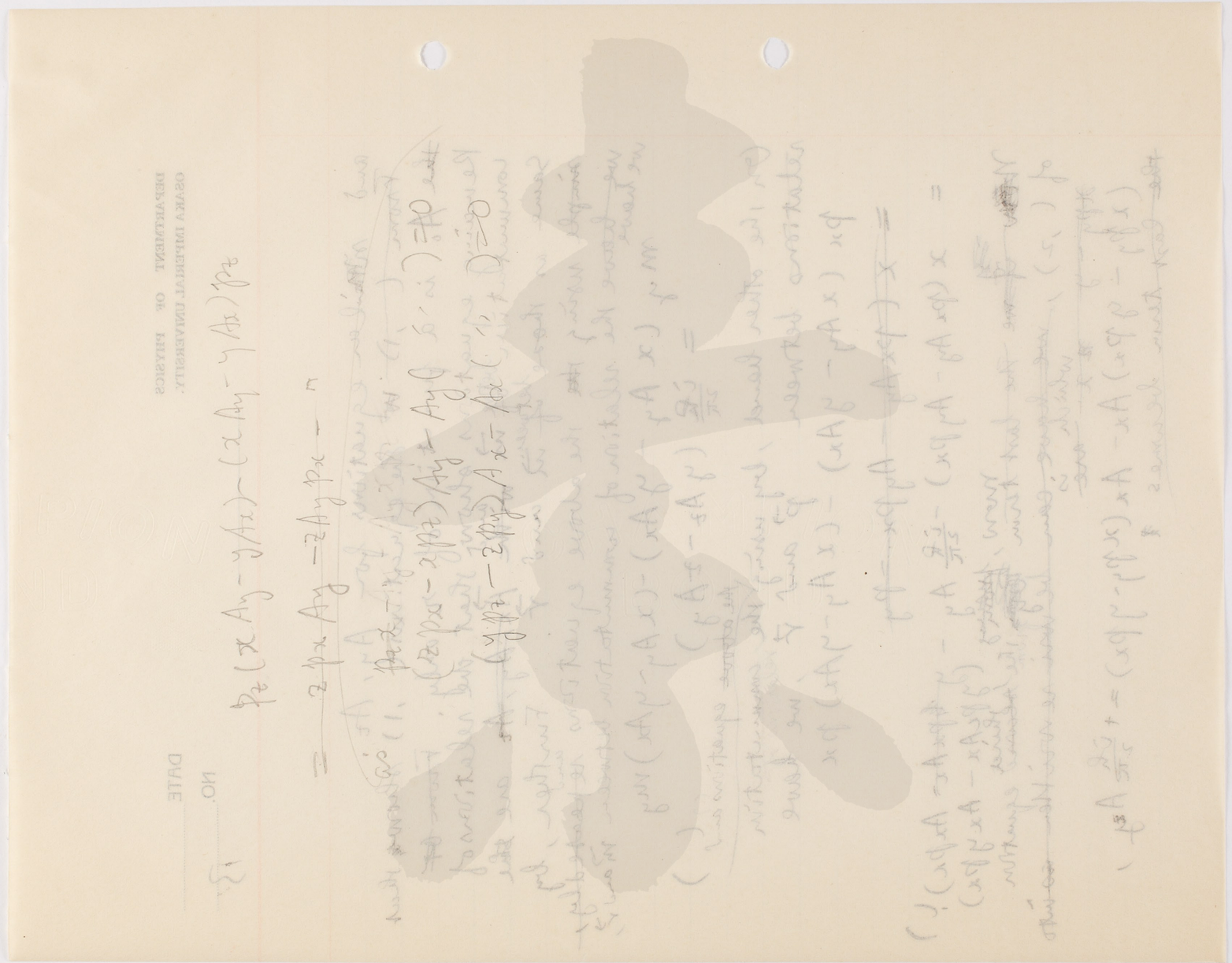
$$(yp_xA_x - A_xyp_x)$$

Now, if we the last term from the second equation of (,2), we have can be written which is into

$$xp_y - yp_x \text{ and } x \text{ are}$$

$$(xp_y - yp_x)A_x - A_x(xp_y - yp_x) = +\frac{ih}{2\pi}A_z,$$

the last term becomes



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we have

$$y p_x A_x - A_x y p_x = x p_y A_x - A_x x p_y - \frac{ih}{2\pi} A_y .$$

Hence the right hand side of the ( ) becomes

$$x (p_x A_y - A_y p_x - p_y A_x + A_x p_y) .$$

Similarly we can deduce the relation

$$p_z (x A_y - y A_x) - (x A_y - y A_x) p_z$$
$$= z (p_x A_y - A_y p_x - p_y A_x - A_x p_y) .$$

Combining these two equations, we have finally

$$M_y (x A_y - y A_x) - (x A_y - y A_x) M_y$$
$$= (z p_x - x p_z)(x A_y - y A_x) - (x A_y - y A_x)(z p_x - x p_z)$$
$$= 0 .$$

Comparing this with ( ), we have finally

$$y A_z - z A_y = 0 .$$

Similarly, we have further the relations

$$z A_x - x A_z = 0$$
$$x A_y - y A_x = 0 .$$

Hence, $\vec{A}$ should have the form $\frac{\vec{r}}{r} \times$ multiplied by a function of r, θ, ϕ.

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~~Now~~ By using the equations ( ) once more, ~~it~~ we have the equations

$$\vec{m}\,(\vec{r}\vec{A}) - (\vec{r}\vec{A})\vec{m} = 0,$$

which shows that $\vec{r}\vec{A}$ is a function of r only, ~~Hence~~, These relations combined, ~~we~~ $\vec{A}$ should have the form $\frac{\vec{r}}{r}$ multiplied by a function of r only. Thus the above ~~formulae~~ ( ) is ~~proved~~. the proof is ended.

Next we can show that in this general case, the perturbing field is zero outside of the nucleus. ~~In the first place~~ For ~~it~~ the magnetic field $\vec{H}$ vanishes everywhere, since, ~~mostly~~,

$$\vec{H} = \mathrm{curl}\,\vec{A} = 0.$$

from the field equations.

Secondly, ~~by the field equations~~

$$\frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \mathrm{curl}\,\vec{H} - \frac{4\pi}{c}\vec{I}$$

in which $\vec{H} = 0$ and both the electric field $\vec{E}$ and the current density $\vec{I}$ have the same factor $\exp(-2\pi i\nu t)$ in this case, we have the relations

$$\vec{E} = \left(\frac{2}{i\nu}\right)\vec{I},$$

The exact treatment

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and necessitates special assumptions ~~of~~ the nuclear structure, we should be satisfied with

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which shows that the electric field vanishes ~~where~~ outside of the nucleus, where the current density due to the nuclear transition vanishes. Hence, by a suitable gauge transition, we can reduce the potentials ~~itself~~ themselves ~~zero~~ to zero outside of the nucleus.

The second point is that the solutions of Dirac's equation in Coulomb field of the nucleus were taken as the eigenfunctions of initial and final states of the electron. As, ~~in the neighborhood of the nucleus~~, the field is no more of Coulomb type in the neighborhood of the nucleus, we should modify the ~~above form~~ forms of the eigenfunctions in this region. ~~In general~~ Thus, each eigenfunction in the previous ~~sets~~ sections should be multiplyed by the function of r which deviates appreciably from 1 ~~in the~~ for ~~small~~ the value of r comparable with the nuclear radius. In general, as the exact treatment of such a case ~~as a first approximation~~, instead of this ~~modification~~ is too complicated, we ~~take~~ ~~as a first~~ for the values of the eigenfunctions ~~in~~ ~~for~~ in the nucleus simply ~~those values~~ Those

at $r = a_N$ of the solutions in Coulomb field, where a_N is the radius of the nucleus.

Then, in the formulae of the preceding sections, we have only to change the definitions of R_p, S_p, other points being unaltered; We have, namely, to put

$$R_p = a_N^{2(\gamma-1)} \int V_0 \, d\vec{\tau},$$

$$S_p = a_N^{2(\gamma-1)} \int V \, d\vec{\tau}.$$

~~Those~~ instead of the corresponding expressions in ~~the~~ §2.

* We, while, ~~whereas~~ their values outside of the nucleus do not concern us

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$\frac{0.36}{0.64} = \frac{9}{16}$, 0.56

§4. The Numerical Results

We want now to obtain the numerical values of $F(\varepsilon_+)$ and ρ in the case of RaC'. Inserting the values

$$Z = 84, \quad \alpha = \frac{1}{137}, \quad \Delta W = 1.4 \times 10^6 eV = 2.8 mc^2,$$

we have

$$\alpha Z = 0.6, \quad \gamma = 0.8,$$

$$\frac{\gamma^2}{\pi (\alpha Z)^{2\gamma+1} \Gamma(2\gamma+1)} = 0.5,$$

$$\varepsilon' = 3.6 \qquad \eta' = 3.5$$

$$|\Gamma(\gamma + ib')|^2 e^{\pi b'} (\varepsilon' + \gamma) \cong (4.5 + 1.6\eta') e^{-1.2\pi \frac{\varepsilon'}{\eta'}} (\varepsilon' + \gamma)$$ [8]

$$= 44.4,$$

so that

$$F_p(\varepsilon_+) \cong (4.5 + 1.6\eta_-)(4.5 + 1.6\eta_+)(\varepsilon_- \varepsilon_+ - 0.6) e^{-1.2\pi \frac{\varepsilon_+}{\eta_+}}$$

$$\varepsilon_+ + \varepsilon_- = 2.8$$

and

$$\rho = 1.1 \times 10^{-2} \int F_p(\varepsilon_+) d\varepsilon_+ .$$

The distribution function $F_p(\varepsilon_+)$ in this case is given by Fig. 2. The maximum of the curve lies near the upper limit of the energy of the positron and it coincides with the first maximum of Alichanow's curve (approximately) (Fig. 1).

From Fig. 2 we have

8) E. Fermi, Z. f. Phys. 88, 161, 1934.

9) ~~The~~ Energy, is written in ~~the~~ unit of $mc^2 = 0.5\times10^6$ eV, ~~and included~~ the proper energy ~~being~~ is ~~included~~ included in it.

$\frac{S_p}{R_p} \approx 13 R^2 \sim 8$ $\frac{13}{164}$ $(1+0.56)$ $\frac{1.8}{1.56}$ $\frac{10^2 \cdot 56}{440}$ $\simeq 10^0$

~~On~~ On the contrary, if we assume the large hump in Fig. 1 is all due to the ~~above~~ above process, ~~The ratio~~ S_p/R_p should be about ~~20~~ twenty times as large as R_p, which means that the contribution of ~~the~~ due to the vector potential is far larger than that due to the scalar potential.

~~As~~ The occurrence of such an extreme case seems very improbable, we should conclude that the ~~ratio~~ calculated value of the pair production ~~seems~~ seems to be too small to account for the experiment exhaustively.

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On the contrary, if we assume the contribution due to the vector potential to be large compared with that of the scalar potential, the vector becomes fairly large. For example,

$$\int F(\varepsilon_+)\,d\varepsilon_+ \cong 0.18$$

and

$$\rho \cong 1.1\times10^{-2}\times0.18 = 2\times10^{-3}$$

On the other hand the number of the electron emitted from the K levels is known to be about 0.003 per disintegration in this case, so that the number of positrons in question is about

$$0.003\times2\times10^{-3} \cong 0.6\times10^{-5}$$

per disintegration.

On the other hand according to Alichanow and Kosodaew, total number of positrons emitted is 0.003 per disintegration. Hence, if we neglect the vector potential, we have $\frac{s_p}{k_p}=0$ and $\cdots=1$, the above results of the calculation is correct, and the number of positrons emitted by the S-S transition in question amounts only to 0.002 of total number of positrons emitted. Thus only very small part of the first hump of the experimental curve can be interpreted as due to the process above considered.

Even if we take into account the term

$$\frac{\vec{\alpha}\vec{\nu}}{\nu}V(r)$$

in the perturbation potential, the ratio changes it can be easily shown

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only slightly.
~~hardly larger than the above value~~, ~~as long as the~~ in any case ~~except~~ so. R_p is not small compared with S_{zp}.

$S_p =$

that the change of the ratio is too small to ~~be account for~~ compensate for the above discrepancy.

As seen from Fig. the formula f/p depends on ΔW and Z. In general, the ratio ingenuity f/p increases rapidly as ΔW increases, and the maximum of $F(\varepsilon_+)$ displaces by and by to $\varepsilon_+ = \frac{\Delta W}{2}$ at the same time. For example, assuming, assuming $Z = 84$ cm^{-1} $\Delta W = 4mc^2$, we have for $F(\varepsilon_+)$ a curve as shown by Fig. 3 and for f/p a value 4.0×10^{-2}. ~~On the other $f/p = 4.0$ hand~~ f/p increases rapidly as Z decreases, owing to the factor $(\alpha Z)^{2s+1}$ in the denometer.

In conclusion the