On the Theory of β-Disintegration[1]

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§1. Introduction

The present quantum theory teaches us that almost all the phenomena occurring in nature can be resolved into a number of elementary processes, in which the laws of conservation of energy and momentum hold always good.

The disintegration of natural or artificially produced radio-element by emitting electrons or positrons with the continuous energy spectrum was the only case ever known, in which the conservation laws seemed to be violated. In all other cases, new sorts of elementary particles, such as the photon and the neutron, have been introduced, whenever necessary for the conservation of energy and momentum.

So, in this last case also, it seems more plausible to introduce a new sort of elementary particle, which ~~goes~~ goes away unnoticed with the surplus energy and momentum, than to consider that the conservation laws do not hold in principle.

~~Pauli's theory~~ The neutrino theory of β-disintegration, which was suggested by Pauli and developed by Fermi[2] on this point of view, was found to agree with ~~the~~ experiment on the whole, although there are still many points to be corrected and completed in future.

1) This paper was read before the meeting of Osaka branch on July 6, 1935.

2) E. Fermi,

In this paper, we want to ~~the~~ treat the problem in a little more definite form and to investigate especially the following phenomenon ~~, which~~ which is alone should occur ~~side by side with the emission of the position~~ ~~and~~ in certain case and occur ~~side by side with the emission of the position in another.~~ related closely to β-

Consider two isobars with the mass number M, and the atomic numbers Z and Z−1 respectively. The small difference ΔW of their proper energies ~~masses~~ will be, in general, either positive or negative.

~~Now~~, If ΔW is larger than the sum of the proper energies ~~masses~~ $mc^2+\mu c^2$ of the electron and the neutrino, the ~~element~~ isobar Z will be unstable and will ~~disintegrate~~ change ~~in~~ into ~~the element~~ in general Z−1 with the emission of the position.

On the contrary, if ~~ΔW is~~ ΔW is smaller than $-(mc^2+\mu c^2)$, the ~~element~~ isobar Z−1 will be unstable and will ~~disintegrate~~ change into ~~the element~~ the isobar Z with the emission of the electron.

Finally, if ΔW lies between $mc^2+\mu c^2$ and $-(mc^2+\mu c^2)$, both types of disintegration can not occur. Then, are these two isobars both stable? It is not always so, for if ΔW added by the total energy E of an orbital electron of ~~the element~~ the isobar Z is larger than the proper energy μc^2 of the neutrino, i.e.

$$\Delta W > \cancel{E}\ \mu c^2 - E,$$

~~This orbital electron can disappear this~~ the isobar Z will change into Z−1, with the absorption of this orbital electron.

For example, a K electron of ~~the~~ the isobar Z can ~~disappear~~ ~~be absorbed, in changing~~ can be absorbed, if $\Delta W > \mu c^2 - mc^2\sqrt{1-\alpha^2 Z^2}$, where ~~Z−1, ≤ 1~~ α is the fine structure constant.

Thus, in general, two isobars with atomic numbers differing by one are both stable, only if

$$-(mc^2+\mu c^2) < \Delta W < -mc^2+\mu c^2,$$

as the total energies of ~~the~~ electrons in outer orbits are very close to mc^2.

~~Where~~ Consequently, if the mass μ of the neutrino is zero or very small compared with m as assumed by Fermi, it will be ~~compared with~~ scarcely ~~impossible~~ very rare ~~very rare~~ that two isobars with atomic numbers differing by one ~~both~~ ~~can exist for a lo in nature,~~ ~~for a long time unless~~ happen to be both stable.)

In the following ~~sele~~ sections, we shall treat this problem quantitatively and show in what case the probability of occurrence of the above phenomenon will be appreciable.

In case, when $\Delta W > (m+\mu)c^2$, such a process will occur side by side with the emission of the position. Hence, the relative probability of occurrence of these two processes will also be calculated.

) This point was suggested by Prof. ~~Dr.~~ Beck, to whom the authors are much obliged.

§ 2. General Theory

According to Fermi's theory, β-disintegration can be considered as the emission of an electron and a neutrino at the same time with the transition of a heavy particle in the nucleus from a neutron state to a proton state.

We can alternatively assume) that, in this case, the light particle jumps from a neutrino state of negative energy to an electron state of positive energy at the same time with the nuclear transition, thus an electron and an anti-neutrino being emitted. Similarly, in the case of positron emission, ~~the light particle~~ we consider that the light particle jumps from an electron state of negative energy to a neutrino state of positive energy, thus a positron and a neutrino being emitted.

Further ~~Furthermore, we~~ we want to ~~now~~ assume that the light particle satisfies Dirac's wave equation, in which the charge and the proper mass are, however, not constant and take the values $(-e, m)$ or $(0, \mu)$ according as the particle is in an electron state or in a neutrino state. Thus the wave equation has the form

$$\left\{ \frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \vec{\alpha}(c\vec{p} + e'\vec{A}) + \rho m'c^2 \right\}\psi = 0, \qquad (1)$$

where e', m' are the matrices, which commute with each other and ~~commute~~ with all other quantities. They can be written simultaneously in the diagonal ~~two~~ forms

$$\begin{pmatrix} e\cdot 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \begin{pmatrix} m\cdot 1 & 0 \\ 0 & \mu\cdot 1 \end{pmatrix}, \qquad (2)$$

where 1, 0 denote unit and zero matrices respectively, with four rows and columns. Other notations are the same as in the previous paper of the authors). The wave function ψ has eight components and in the representation, in which e', m' ~~take~~ the are expressed by (2), the first ~~four~~ four components refer to the electron state and the remaining four to the neutrino state.

Now the change of ~~atomic number~~ the charge of the nucleus from Ze to $(Z-1)e$ with the liberation of energy of amount ΔW can be considered as ~~a~~ periodic perturbation ~~varying~~ with the frequency $\nu = \frac{\Delta W}{h}$, which induces the transition of the light particle from an electron state to an neutrino state.

If we assume that this perturbation ~~act~~ is described by a new sort of field acting on the light particle) and that this field can be derived from scalar ~~potential~~ and vector potentials, in analogy with the ordinary electromagnetic ~~potential~~ field, we should add the term of the form

$$g'\tau'(U_0 + \vec{\alpha}\vec{U})\exp(-2\pi i\nu t) \qquad (3)$$

to the ~~opert~~ operator in the left hand side of the wave equation (1), where τ' is an operator, which changes the electron state of the light particle into the neutrino state, ~~and g' is a const~~ In the representation, in which e', m' are expressed by (2), τ' has the form

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \qquad (4)$$

) H. Yukawa, Proc. This paper will be referred to as I.

) ~~See I~~ Compare I, in which only the scalar potential of the field was taken into account

) H. Yukawa and S. Sakata, Proc. Phys. Math. Soc. Japan, 17 1935. This paper will be referred to as II.

$U_0, \vec{U}$ are the scalar and vector functions of the coordinates describing the potentials of the perturbing field due to the nuclear transformation and g' is a constant analogous to the charge in the case of electromagnetic interaction.

A possible theory of such a field, ~~was given~~ given ~~previous~~ recently by one of the authors;[1] Further ~~development of this theory~~ will be ~~formulated~~ discussed in detail in ~~the last section~~ §3 below. Here, we ~~assume~~ want to assume simply that the potentials $U_0, \vec{U}$ ~~are~~ vanish outside of the nucleus.

Since ~~in general~~, the total angular momentum of the system consisting of the light particle and the nucleus ~~ary~~ is assumed ~~does not change during~~ the process of disintegration is considered to be constant, the change of the total angular momentum $\vec{M}$ ~~with~~ of the light particle is closely connected with that of the nuclear spin $\vec{n}$ ~~with~~ by the relation

$$\Delta \vec{M} = -\Delta \vec{n}$$

both

Especially when the absolute value and the direction of the nuclear spin does not change by the ~~nucle~~ transition, the total angular momentum of light particle will ~~all~~ also be constant throughout the ~~transition~~ process, so that the perturbation ~~on the light~~ operator (3) should commute with the ~~the~~ operator

$$\vec{M} = \vec{m} + \frac{h}{4\pi}\vec{\sigma} \qquad (5)$$

where $\vec{m} = \vec{r} \times \vec{p}$ and $\vec{\sigma}$ ~~is~~ the spin ~~mat~~ vector. In this case, if we assume ~~that~~ ~~If we~~ U_0 and $\vec{U}$ to be the functions of coordinates $\vec{r}$ only, ~~they~~ ~~we can~~

[1] See I.

~~show that~~ it follows that ~~they have the following forms~~ ~~U_0 is the function~~ ~~$U_0 =$~~

U_0 depends only on r and $\vec{U}$ has the form

$$\frac{\vec{\alpha}\vec{r}}{r} U'(r) \quad \text{~~(X) function of r~~},$$

where U' is a certain function of r only ~~multiplied by a function~~ as shown in the previous paper.[1]

We can ~~transform~~ transform the perturbed ~~Hamiltonian~~ wave equation

$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \vec{\alpha}(c\vec{p} + e'\vec{A}) + \rho_3 mc^2 + g'\tau'(U_0 + \vec{\alpha}\vec{U})\exp(-2\pi i\nu t)\right\}\psi = 0 \qquad (6)$$

into the form[2]

$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \varepsilon p_r c + e'\vec{\alpha}\vec{A} + i\varepsilon\rho_3 j\frac{hc}{2\pi r} + \rho_3 mc^2 + g'\tau'(U_0 + \varepsilon U')\exp(-2\pi i\nu t)\right\}\psi = 0, \qquad (7)$$

where
$$\varepsilon = \frac{\vec{\alpha}\vec{r}}{r} = \frac{\rho_1\vec{\sigma}\vec{r}}{r},$$
$$\frac{jh}{2\pi} = \rho_3\left\{\vec{\sigma}\vec{m} + \frac{h}{2\pi}\right\}$$
$$p_r = \frac{1}{r}\left(\vec{r}\vec{p} - \frac{ih}{2\pi}\right) = -\frac{ih}{2\pi}\left(\frac{\partial}{\partial r} + \frac{1}{r}\right) \qquad (8)$$

In particular, when the central electrostatic field of the nucleus acts on the electron,

[1] See II, §4.

[2] Dirac,

A_0 is a function of r only and $\vec{A}$ ~~varies~~ $=0$, ~~Thus that in ( )~~ so that there appears only the ~~op~~ operators $\varepsilon, j, \rho_3, i, \ p_r, r$ in ( ). ~~They commute with one another except that the~~ first two anticommute with ~~each other~~, which ~~can~~ satisfy the relations

$$\varepsilon^2 = \rho_3^2 = 1$$

Among these, j commutes with other variables, ~~and~~ while ε and ρ_3, ~~which~~ satisfying the relations (commute with j, p_r, r ~~and~~)

$$\varepsilon^2 = \rho_3^2 = 1 \qquad \varepsilon\rho_3 + \rho_3\varepsilon = 1 \text{ and}$$

~~In the representation, in which~~ can be represented as ~~a~~ matrices

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \qquad (9)$$

~~In this representation~~, ~~hence~~, if the commuting ~~variables~~ quantities ε, ρ_3, j, r and

$$u = M_z = m_z + \frac{h}{4\pi}\sigma_z$$

~~can be~~ are taken as the arguments of the wave functions, instead of ~~ε the coordinates and~~ r, θ ρ_3 and σ_3, the wave equation becomes ~~the form~~

$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \varepsilon p_r c + i\varepsilon\rho_3 j\frac{hc}{2\pi r} + \rho_3 m'c^2 + g'\tau'(U_0 + \varepsilon U')\exp(-2\pi i\nu t)\right\}\psi(\rho_3, j, r, u, e', t) = 0 \qquad (10)$$

~~besides the~~ in addition to the quantity ~~due~~, for example e', ~~characterizing the~~ determining ~~the~~ whether the light particle is in the electron state or in the neutrino state,

Since j and u ~~are~~ commute with other operators in (10), ~~and~~ they are the constants of ~~perturbed~~ motion, even when the perturbation of the above type exists, ~~they~~ and can be considered simply as numbers, so that the wave ~~equation~~ (10) can be ~~represented written as~~ represented ~~as follow~~ in the form

$$\psi^{(1)}_{j,u}(r,t) \qquad \psi^{(2)}_{j,u}(r,t), \quad \varphi^{(1)}_{j,u}(r,t), \quad \varphi^{(2)}_{j,u}(r,t) \qquad \cancel{i = 1, 2, 3}, 4$$

where the first two components refer to the electron state and the ~~&~~ remaining components to the neutrino states, ~~The wave which~~ satisfying the ~~following~~ equations

$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + eA_0 \cancel{+} + \cancel{i\varepsilon\rho_3}\, mc^2\right\}\psi^{(1)}_{j,u}(r,t)$$
$$- \frac{hc}{2\pi}\left\{\frac{\partial}{\partial r} + \frac{1+j}{r} \cancel{+\frac{j}{r}}\right\}\psi^{(2)}_{j,u}(r,t) = 0$$

$$\cancel{+ g} \left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + eA_0 - mc^2\right\}\psi^{(2)}_{j,u}(r,t) = 0$$

$$\frac{hc}{2\pi}\left(\frac{\partial}{\partial r} + \frac{1-j}{r}\right)\psi^{(1)}_{j,u}(r,t) +$$

$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + \mu c^2\right\}\varphi^{(1)}_{j,u}(r,t) - \frac{hc}{2\pi}\left\{\frac{\partial}{\partial r} + \frac{1+j}{r}\right\}\varphi^{(2)}_{j,u}(r,t)$$
$$+ g'\{U_0 \cancel{+ \varepsilon U'}\}\exp(-2\pi i\nu t)\,\psi^{(1)}_{j,u}(r,t)$$
$$- ig'U'\exp(-2\pi i\nu t)\,\psi^{(2)}_{j,u}(r,t) = 0 \qquad (11)$$

$$\frac{hc}{2\pi}\left(\frac{\partial}{\partial r} + \frac{1-j}{r}\right)\varphi^{(1)}_{j,u}(r,t) + \left(\frac{ih}{2\pi}\frac{\partial}{\partial t} \cancel{+ eA_0 + mc^2} - \mu c^2\right)\varphi^{(2)}_{j,u}(r,t)$$
$$+ g'U_0\exp(-2\pi i\nu t)\,\psi^{(2)}_{j,u}(r,t)$$
$$+ ig'U'\exp(\ \text{''}\ )\,\psi^{(1)}_{j,u}(r,t) = 0.$$

Now ~~const~~ we consider ~~the~~ a light particle initially in the electron state ~~with~~ of ~~with an~~ negative energy $E_\ast$ ~~($E_\ast > mc^2$)~~ with ~~the~~ $\Psi'_{j,u}$ wave functions

$$\psi^{(1)}_{j,u}(r,t) = \psi^{(1)}_{E,j,u}(r)\exp\left(-\frac{2\pi i E_\ast t}{h}\right)$$
$$\psi^{(2)}_{j,u}(r,t) = \psi^{(2)}_{E,j,u}(r)\exp\left(-\frac{2\pi i E_\ast t}{h}\right)$$
$$\varphi^{(1)}_{j,u}(r,t) = 0$$
$$\varphi^{(2)}_{j,u}(r,t) = 0, \qquad (12)$$

satisfying the ~~equation~~ unperturbed equations

$$(E_\ast + eA_0 + mc^2)\psi^{(1)}_{j,u}(r) + \frac{hc}{2\pi}\left(\frac{d}{dr}+\frac{1+j}{r}\right)\psi^{(2)}_{j,u}(r)=0 \qquad (13)$$
$$\frac{hc}{2\pi}\left(\frac{d}{dr}+\frac{1-j}{r}\right)\psi^{(1)}_{j,u}(r) + (E_\ast + eA_0 - mc^2)\psi^{(2)}_{j,u}(r)=0.$$

On account of the perturbation expressed by the terms involving g' in ~~the eq~~ (11), ~~there are~~ ~~is the probability of~~ the transition may occur of the light particle to the ~~neutrino~~ continuous state of ~~energy~~ positive energy E' ~~($E' > mc^2$)~~ with the wave ~~eq~~ functions

$$\psi^{(1)}_{j,u}(r,t) = \psi^{(2)}_{j,u}(r,t) = 0$$
$$\varphi^{(1)}_{j,u}(r,t) = \varphi^{(1)}_{E',j,u}(r)\exp\left(-\frac{2\pi i E' t}{h}\right) \qquad (14)$$
$$\varphi^{(2)}_{j,u}(r,t) = \varphi^{(2)}_{E',j,u}(r)\exp\left(-\frac{2\pi i E' t}{h}\right),$$

satisfying the equations

$$(E' + \mu c^2)\varphi^{(1)}_{j,u}(r) - \frac{hc}{2\pi}\left(\frac{d}{dr}+\frac{1+j}{r}\right)\varphi^{(2)}_{j,u}(r)=0 \qquad (15)$$
$$\frac{hc}{2\pi}\left(\frac{d}{dr}+\frac{1-j}{r}\right)\varphi^{(1)}_{j,u}(r) + (E' + \cancel{eA_0} - mc^2)\varphi^{(2)}_{j,u}(r)=0$$

can ~~may~~ take place, ~~provided that~~ ~~the~~ if the condition

$$E' = E + \Delta W > \mu c^2$$

is ~~sati~~ fullfiled.

The probability of the transition per unit time per unit energy range is given by

$$P_{\ast j,u} = \frac{4\pi^2 g'^2}{h}\sum_{j,u}\left|\int_0^\infty \left\{(\varphi^{(1)*}_{E',j,u}\psi^{(1)}_{E,j,u} + \varphi^{(2)*}_{E',j,u}\psi^{(2)}_{E,j,u})U_0 + hi(\varphi^{(2)*}_{E',j,u}\psi^{(1)}_{E,j,u} - \varphi^{(1)*}_{E',j,u}\psi^{(2)}_{E,j,u})U'\right\} 4\pi r^2 dr\right|^2 \qquad (16)$$

~~where~~ ~~where the wave~~ functions are normalized ~~with respect to energy~~, if the ~~energy~~ initial ~~electron state belongs to~~ continuous energy state ~~energy E of the~~ initial state is continuous, ~~what On the~~ i.e. $|E| > mc^2$, the wave ~~On the other~~ functions are normalized ~~to~~ with respect to the energy, i.e.

$$\int_0^\infty \left\{\psi^{*(1)}_{j,u}(E_1)\psi^{(1)}_{j,u}(E_2) + \psi^{*(2)}_{j,u}(E_1)\psi^{(2)}_{j,u}(E_2) + \varphi^{(1)}_{j,u}(E_1)\varphi^{(1)}_{j,u}(E_2) + \varphi^{(2)}_{j,u}(E_1)\varphi^{(2)}_{j,u}(E_2)\right\} 4\pi r^2 dr = \delta(E_1 - E_2).$$

~~On the other hand~~, The probability per unit time of the transition from the discrete initial state is given by the same expression (16), ~~where~~

~~$P = \frac{4\pi^2}{}$~~ if the wave functions of the initial state ~~is~~ are normalized ~~unit~~ to 1.

$E' = E + \Delta W > mc^2$

If $E \gtrless < -mc^2$, ~~this~~ the above process corresponds to the emission of a neutrino of energy $E' = E + \Delta W$ and a positron of energy $-E = E_+$, while if ~~the initial state~~ E ~~is one of~~ belongs to the discrete energy level $(mc^2 > E > 0)$ of the electron moving in the static field of the charge Ze, this corresponds to the disappearance of an orbital electron with the energy E ~~with~~ at ~~wi~~ the same time with the ~~de~~ decrease of the nuclear charge by one.

and the emission of an neutrino of energy $E' = E + \Delta W$

~~In the neighborhood of the~~ If the nuclear static field is ~~given by~~ expressed by the Coulomb potential

$$A_0 = \frac{Ze^2}{r},$$

the normalized wave functions of the continuous electron state can be taken in the neighborhood of the nucleus ~~become~~ approximately as ~~equal to~~

$$\psi^{(1)}_{E_+,j,u}(r) \cong \pm N_+ \sqrt{2(j+\gamma')(j\varepsilon_+ + \gamma')} \cdot r^{\gamma'-1}$$

$$\psi^{(2)}_{E_+,j,u}(r) \cong \pm N_+ \sqrt{2(j-\gamma')(j\varepsilon_+ + \gamma')} \cdot r^{\gamma'-1} \quad (17)$$

according as $j \gtrless 0$, where

$j \gtrless 0$

$$N_+^2 = \frac{1}{8\pi mc^2}\frac{1}{\{\Gamma(2\gamma'+1)\}^2}\left(\frac{4\pi mc}{h}\right)^{2\gamma'+1}\eta_+^{2\gamma'-1}e^{-\pi\alpha Z\frac{\varepsilon_+}{\eta_+}} \times \left|\Gamma\left(\gamma' + i\alpha Z\frac{\varepsilon_+}{\eta_+}\right)\right|^2$$

$$\alpha = \frac{2\pi e^2}{hc}, \quad \gamma' = \sqrt{j^2 - \alpha^2 Z^2},$$

$$\varepsilon_+ = \frac{E_+}{mc^2}, \quad \eta_+ = \sqrt{\varepsilon_+^2 - 1}.$$

$$\alpha Z C^{(1)}_{\gamma'-1} - (\gamma' - 1 + 1 + j)C^{(2)}_{\gamma'-1} = 0$$

$$(\gamma' - 1 + 1 - j)C^{(1)}_{\gamma'-1} + \alpha Z C^{(2)}_{\gamma'-1} = 0$$

$$C^{(1)}_{\gamma'-1} : C^{(2)}_{\gamma'-1} = \cancel{\alpha Z :}\ \gamma' + j : \alpha Z \ \cancel{= \gamma' + j}$$

$$= \gamma' + j : \sqrt{j^2 - \gamma'^2} = \sqrt{\gamma' + j} : \sqrt{j - \gamma'} \quad \text{for } j > 0$$

$$= \sqrt{-j-\gamma'} : \sqrt{-j+\gamma'} \quad \text{for } j < 0.$$

$$\psi^{(1)}_{-E_+,j,u}(r) \cong N'_+ \sqrt{j+\gamma'}\, r^{\gamma'-1}$$

$$\psi^{(2)}_{-E_+,j,u}(r) \cong N'_+ \alpha Z\, r^{\gamma'-1}.$$

$$N_+'^2 = \frac{2(j\varepsilon_+ + \gamma')}{(j+\gamma')} N_+^2$$

~~for small value of r~~ ~~in the nucleus~~ Similarly the wave functions of the neutrino state of positive energy E' ~~are given by for small value of r~~ can be expressed approximately by

$$\varphi^{(1)}_{E',j,u}(r) \cong \pm N_0 \sqrt{2(j+|j|)(j\varepsilon' + |j|)}\, r^{|j|-1}$$

$$\varphi^{(2)}_{E',j,u}(r) \cong$$

$$+ \frac{1-\gamma}{1+\gamma} \} \}]^2$$

per unit energy range per unit time

Inserting (17) into (16), ~~we obtain for~~ the probability of disintegration of the nucleus with the emission of ~~an neutrino of ener~~ a positron of energy between $(E_+, E_+ + dE_+)$ and a neutrino of energy between $(\Delta W - E_+, \Delta W - E_+ - dE_+)$ is ~~go~~ becomes finally

$$P_{\beta+} = \frac{\gamma+1}{2} \frac{256\pi^4 m^5 c^4}{h^7 \{\Gamma(2\gamma+1)\}^2} \left(\frac{4\pi mc}{h}\right)^{2(\gamma-1)}$$

$$\left\{ \int r^{\gamma-1} g' U_0(r) d\vec{r} \right\}^2 F(\varepsilon_+, \kappa)\, d\varepsilon_+ \qquad (18)$$

where $\Delta w = \frac{\Delta W}{mc^2}$, $\kappa = \frac{\mu}{m}$, $d\varepsilon_+ = \frac{dE_+}{mc^2}$,

and

$$F(\varepsilon_+, \kappa) = \eta_+^{2\gamma-1} e^{-\pi\alpha Z \frac{\varepsilon_+}{\eta_+}} \left|\Gamma\left(\gamma + i\alpha Z \frac{\varepsilon_+}{\eta_+}\right)\right|^2 \times \{\varepsilon_+(\Delta w - \varepsilon_+)^2 - \gamma \cdot \kappa (\Delta w - \varepsilon_+)\} \qquad (19)$$

, which represents the energy distribution of the positron.

~~Thus~~ Hence, the total probability per unit time of the positron emission is given by

$$P_{\beta+} = \left\{ \frac{\gamma+1}{2} \frac{256\pi^4 m^5 c^4}{h^7 \{\Gamma(2\gamma+1)\}^2} \times \left(\frac{4\pi mc}{h}\right)^{2(\gamma-1)} \left\{ \int r^{\gamma-1} g' U_0(r) d\vec{r} \right\}^2 \int_1^{\Delta w - \kappa} F(\varepsilon_+, \kappa)\, d\varepsilon_+ \right\} \qquad (20)$$

These expressions ~~are~~ (18) (20) become identical with those of Fermi and Wick[11], if

11)

$$\sigma_K = \frac{\mathrm{const}}{(\Delta w + \gamma)^2}$$

On the other hand, the probability of absorption of ~~the~~ orbital electron by the nucleus is largest when the electron is in ~~the normal state~~ one of the K states, ~~so that~~ in which case the prob. per unit time ~~of absorption~~ is given by

$$P_{K,abs} = (\alpha Z)^{2\gamma+1} \cancel{\gamma+1} \frac{256\pi^5 m^5 c^4}{h^7 \Gamma(2\gamma+1)} \left(\frac{4\pi mc}{h}\right)^{2(\gamma-1)}$$

$$\times \eta'(\varepsilon' + \kappa) \left\{ \frac{\gamma+1}{2} \left|\int r^{\gamma-1} g' U_0(r) d\vec{r}\right|^2 + \frac{1-\gamma}{2} \left|\int r^{\gamma-1} g' U(r) \ldots\right|^2 \right\} \qquad (21)$$

The ratio of (21) and (20) is given by

$$\sigma = \frac{P_{\beta+}}{P_{K,abs}} = \frac{\int_1^{\Delta w - \kappa} F(\varepsilon_+, \kappa)\, d\varepsilon_+}{\pi (\alpha Z)^{2\gamma+1} \Gamma(2\gamma+1) \{\Delta w + \gamma + \kappa\}}, \qquad (22)$$

(1)

$$\sqrt{(\Delta w - \varepsilon_+)^2 - \kappa^2}$$

The mean life time of the isobar Z, ~~when ΔW lies between −γmc² + μc²~~ and $mc^2 + \mu c^2$, is given ~~by~~ becomes

$$\tau = \frac{1}{P_K} \qquad \eta' = \sqrt{(\Delta w + \gamma + \kappa)^2 - \kappa^2}$$

~~and~~ whereas, if ΔW lies between ~~−γmc² + μc²~~ $-\gamma + \kappa$ and ~~mc² + μc²~~ $1 + \kappa$, whereas it becomes

$$\tau = \frac{1}{P_+ + P_K} \qquad dN = -N(P_+ + P_K)dt$$

~~when~~ if ΔW ~~lies between~~ is larger than $mc^2 + \mu c^2$ $1 + \kappa$.

(3)

§3. Numerical Results

We want now to obtain the numerical values ~~in~~ for ~~several cases special cases~~. special values of Z and ΔW. for of the mean life time of the isobar Z

~~a~~ According to Fermi, the factor (26)

By comparing with the experimental ~~value~~, ~~Fermi~~ in the case of ~~ordinary~~ natural β-disintegration,

Fermi ~~took~~ for ~~the factor (26)~~ a value the constant g a value

$$4 \cdot 10^{-50}\ \mathrm{cm}^3\ \mathrm{erg}. \qquad (33)$$

and ~~assumed~~ assumed ~~for~~

$$\left|\int v_m^* u_n d\tau\right|^2$$

~~a value of~~ to be of order 1, so ~~we~~ in the above ~~result~~ formulae we should take for the factor (27) (23), (29), ~~(30), (31)~~, ~~to~~ the ~~value~~ above value (25), (33).

~~Th~~ Further putting the ~~numerical values~~ mass of the neutrino equal to zero, i.e. $\kappa = 0$, we have the following results for ~~$\Delta W < 1$~~ any the value of ΔW between $-\gamma$ and 1.

$$\tau_K = \frac{200}{(\Delta W + 1)^2}\ \text{days}$$

for $\alpha Z = 0.1$, i.e. $Z \cong 14$

and

(2) In the above calculations it was always assumed that the normal states of ~~the~~ ~~both the~~ two isobars ~~have~~ had the same ~~The normal states~~ spin value. If this ~~was~~ is not the case, the probabilities P_K, P_+ will be, in general, much smaller.

~~τ_K~~ αZ	Z	τ_K
$\frac{1}{137}$	1	$\frac{1370}{(\Delta W+1)^2}$ years
0.1	~14	$\frac{200}{(\Delta W+1)^2}$ days
0.2	~27	$\frac{25}{(\Delta W+1)^2}$ days
0.5	~69	$\frac{14}{(\Delta W+0.87)^2}$ hours

Thus, ~~These results shows that~~ the life time of the isobar Z becomes shorter and shorter as ~~the~~ ~~atomic~~ Z increases, being of order of several hours for $Z \cong 70$, ~~except when~~ unless ΔW is very near to $\gamma = \sqrt{1-\alpha^2 Z^2}$.

~~It Such a result~~

~~The fact that sever~~ On the other hand, we find several pairs of isobars with consecutive atomic numbers in the table of ~~too~~ known isotopes, especially in the region of higher atomic number.

This shows that either the two isobars have ~~the~~ different spin values or the theory ~~of the light particle~~ ~~can not be~~ are not above applicable to the immediate neighborhood

~~of the nucleus. If the latter be the case,~~
~~the whole theory of the β-disintegration~~
the neutrino

~~The mass~~

This ~~shows that~~ either seems to be in conflict
with the fact that ~~the~~ several pairs of
stable isobars with consequtive atomic
number are found in nature, especially
for ~~large values of atomic number~~ heavier elements.

~~The reason~~ Hence, we have to assume either
that ~~all~~ ~~these~~ such a pair ~~pairs~~ ~~have~~ has ~~always~~ ~~all~~ always different
spins values ~~every each of~~ in every case, ~~or~~ which
~~that the mass of the neutrino~~ or that the
mass of the neutrino is not small compared
with that of the electron. If The first alternative
~~can be~~ will ~~be able to~~ verified by future ~~measu determination~~ experiment, and
~~of the nuclear spins not~~
if it ~~would~~ will ~~not~~ happen to be ~~not~~ not always the case,
~~the theory of Fermi~~ the ~~neutrino mass~~ second
alternative should be adopted ~~as~~ the theory
were not confirmed by future experiment
of Fermi should be essentially modified.

The recent proposal of Uhlenbeck and
Konopinski is not sufficient for this purpose,
as the distribution curve of the β ray do not agree
theoretic near the upper limit of the electron energy
in this case also, if the mass of the neutrino
is ~~comparable~~ not small compared with that
of the electron.
In any case ~~In the~~ ~~as~~ the whole theory of the β-disintegration
~~Anyhow,~~ is based on the assumption, that ~~the~~
~~present theory~~ ~~Dirac's equation of the~~ the equation
of Dirac's type hold for the light particle
up to the region of the nuclear radius,
at least approximately, which is ~~very~~ so doubtfull, ~~we~~ the radical experiments
happen to be excluded by ~~the~~ future, will have to be adopted,
the second alternative ~~which~~ will necessitating the essential modification
of Fermi's theory.

~~it may happ~~
The revision ~~change~~ of the above results
may happen to be needed
in future.

~~so~~ that the above results ~~should~~ can not be accepted
~~only~~ without much reserve.

On the Theory of β-Disintegration ~~and~~
and of the a phenomenon related
~~On the phenomenon~~ related with it.
~~to the β-disintegration~~

~~the Possibility of~~

Abstract

The possibility of

The probability of such a process ~~the following~~ ~~process~~ phenomenon, which is ~~closely~~ connected with β-disintegration, ~~are di~~ is ~~di~~ considered and its bearing on the general theory of β-disintegration is discussed.

(2) Namely, ~~If~~ the ~~mass~~ difference of proper energies of two isobars with atomic numbers Z and Z−1 respectively is larger than $-(mc^2+\mu c^2)$, where m and μ are the masses of the electron and the neutrino respectively, the isobar Z will ~~disin~~ change into the isobar Z−1 by absorbing one of the orbital electron.

(1) According to ~~the~~ ~~present~~ general theory of β-disintegration, ~~the following~~ phenomenon is expected.
the occurrence of

(3) The probability of such a process is calculated and its ~~relation~~ ~~to~~ on the problem of the neutrino mass and the nuclear spin is discussed.