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On the Theory of the β-Disintegration and the Allied Phenomenon[1)]

By Hideki Yukawa and Shoichi Sakata

(Read July 6. 1935)

Abstract

According to the theory of the β-disintegration, the occurrence of the following phenomenon is expected. Namely, if the difference ΔW of proper energies of two isobars with the atomic numbers Z and Z-1 respectively is larger than $-(mc^2 + \mu c^2)$, where m and μ are the masses of the electron and the neutrino respectively, the isobar Z will change into the isobar Z-1 by absorbing one of the orbital electrons. The probability of such a process is calculated, when the electron to be absorbed is initially in one of the K states and its bearing on the problems of the neutrino mass and the nuclear spin is discussed. The importance of this process relative to the ordinary disintegration process by positron emission, ~~when~~ is considered for ΔW larger than $mc^2 + \mu c^2$.

1) ~~This paper was read before the meeting of Osaka branch~~ on July 6, ~~1935~~.

§ 1. Introduction

The present quantum theory teaches us that almost all the phenomena occurring in nature can be resolved into a number of elementary processes, for each of which the laws of conservation of the energy and the momentum hold good.

The disintegration of natural or artificially produced radio-element by emitting electrons or positrons with the continuous energy spectrum was the only case ever known, ~~for~~ in which the conservation laws seemed to be violated.

In all other cases, new sorts of elementary particles, such as the photon and the neutron, have been introduced, whenever necessary for maintaining the conservation laws. So, in this last case also, it seems plausible to introduce a new sort of ~~the~~ elementary particle, which goes away unnoticed with the surplus energy and momentum.

The neutrino theory of ~~theory~~ β-disintegration, which was suggested by Pauli and developed by Fermi[2] on this point of view, was found to agree with the experiment on the whole, although there are still many points to be corrected and completed in future.

In this paper, we want to develope the general theory ~~deal with the problem~~ in a slightly modified form and to investigate especially the following phenomenon, which is closely related to the β-disintegration.

Consider two isobars with the atomic numbers Z and Z-1 respectively. The small difference ΔW of their proper energies will be, in general, positive or negative. If it is larger than the sum $mc^2+\mu c^2$ of the proper energies

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2) E.Fermi, Zeits.f.Phys. 88,161,1934.

of the electron and the neutrino, the isobar Z will be unstable, changing into Z-1 with the emission of the positron. On the contrary, if ΔW is smaller than $-(mc^2+\mu c^2)$, the isobar Z-1 will become unstable, changing into Z with the emission of the electron.

Finally, if ΔW lies between $mc^2+\mu c^2$ and $-(mc^2+\mu c^2)$, both types of β-disintegration can not occur. Then, are these two isobars both stable ? It is not always so, for if ΔW is larger than μc^2-E, where E is the total energy of one of the orbital electrons of the isobar Z, it can change into the isobar Z-1 with the absoption of this electron. For example, a K electron can be absorbed by the nucleus, if ΔW is larger than $\mu c^2-mc^2\sqrt{1-\alpha^2Z^2}$, where α is the fine structure constant.

Thus, in general , two isobars with atomic numbers differing by one are both stable, only if ΔW lies between $-(mc^2+\mu c^2)$ and $-mc^2+\mu c^2$, as the total energies of the electrons in outer orbits are very close to mc^2.

Consequently, if the mass μ of the neutrino is very small compared with that of the electron, as assumed by Fermi, it will be very rare that two isobars with consecutive atomic numbers happen to be both stable.[3)]

In the following sections, we want to deal with this problem in conformity with the general theory of β-disintegration and to show that the probability of occurrence of the above process becomes fairly large in certain cases.

In case, when ΔW is larger than $mc^2+\mu c^2$, such a process will occur side

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3) This point was suggested by Prof. Beck, to whom the authors are much obliged.

by side with the emission of the positron. The ratio of the occurrence of these two processes will also be calculated.

§ 2. General Theory

According to Fermi's theory, the β-disintegration can be considered as the emission of an electron and a neutrino at the same time with the transition of a heavy particle in the nucleus from a neutron state to a proton state. We can alternatively assume that, in this case, the light particle jumps from a neutrino state of negative energy to an electron state of positive energy at the same time with the nuclear transition, thus an electron and an anti-neutrino being emitted.[4)]

Similarly, in the case of the positron emission, we consider that the light particle jumps from an electron state of negative energy to a neutrino state of positive energy at the same time with the transition of a heavy particle in the nucleus from a proton state to a neutron state, thus a positron and a neutrino being emitted.

We want to assume further that the light particle satisfies Dirac's wave equation, in which the charge and the proper mass are, however, not constant, but take the values $-e$ or 0 and m or μ respectively, according as the particle is in an electron or in a neutrino state. Thus the wave equation takes the form

$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \vec{\alpha}(c\vec{p} + e'\vec{A}) + \beta m'c^2\right\}\psi = 0 \qquad (1)$$

where e' and m' are the matrices, which commute with each other and with all other quantities in (1). They can be written simultaneously in the diagonal forms

4) H.Yukawa, Proc.Phys.Math.Soc.Japan, **17**, 48, 1935. This paper will be referred to as I. Compare also E.J.Konopinski & G.E.Uhlenbeck, Phys.Rev. **48**, 7, 1935. and R.L.Dolecek, ibid. **48**, 13, 1935.

$$\begin{pmatrix} e\cdot 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} m\cdot 1 & 0 \\ 0 & \mu\cdot 1 \end{pmatrix}, \qquad (2)$$

where 1 and 0 denotes ~~the~~ unit and zero matrices respectively, with four rows and columns. Other notations are the same with those in the previous paper of the authors.[5] The wave function ψ has eight components, of which the first four refer to the electron state and the remaining four to the neutrino state in the representation, in which e' and m' are expressed by (2).

We want now to consider the case, when ΔW is larger than $-mc^2 + \mu c^2$. The change of the charge of the nucleus from Ze to $(Z-1)e$ with the liberation of energy of amount ΔW can be considered as a periodic perturbation with the frequency $\nu = \frac{\Delta W}{h}$, which induces the transition of the light particle from an electron state to a neutrino state.

If we assume that this perturbation can be described by a sort of field acting on the light particle and further that this field can be derived from scalar and vector potentials,[6] in analogy with the ordinary electromagnetic field, we have to add the term of the form

$$g'\tau'(U_0 + \vec{\alpha}\vec{U})\exp(-2\pi i\nu t)\psi \qquad (3)$$

to the left hand side of the wave equation (1), where τ' is an operator, which changes the electron state of the light particle into the neutrino state, having the form

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad (4)$$

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5) H.Yukawa & S.Sakata, Proc.Phys.Math.Soc.Japan, **17**,1935, in press. This paper will be referred to as II.

6) Compare I, in which, however, only the scalar potential was taken into account.

in the representation, in which e' and m' are expressed by (2). U_0 and $\vec{U}$ are scalar and vector functions of the coordinates and the momenta, in general, describing the potentials of the perturbing field due to the nuclear transition and g' is a ~~cn~~ constant analogous to the charge in the case of the ordinary electromagnetic interaction.

A possible theory of such a field, proposed recently by one of the authors[7], will not be discussed in this paper. Here we want to assume simply that the potentials U_0 and $\vec{U}$ vanish outside of the nucleus.

Since the total angular momentum of the system consisting of the light particle and the nucleus is considered to be constant throughout the process of disintegration, the change of the total angular momentum $\vec{M}$ of the light particle is connected closely with that of the nuclear spin. Especially, when both the absolute value and the direction of the nuclear spin do not change during the disintegration, the operator

$$\vec{M} = \vec{m} + \frac{h}{4\pi}\vec{\sigma} \qquad (5)$$

will be also constant, where $\vec{m} = \vec{r} \times \vec{p}$ and $\vec{\sigma}$ is the spin vector, so that $\vec{M}$ should commute with the perturbation operator (3).

In such a case, if we assume U_0 and $\vec{U}$ to be the functions of coordinates $\vec{r}$ only, it follows that U_0 depends on r only, while $\vec{U}$ has the form

$$\frac{\vec{\alpha}\vec{r}}{r} U'$$

where U' is a certain function of r only. The proof of this theorem is the same as in the previous paper.[8]

In our problem, the light particle is initially in the electron state under the action of the central electrostatic field of the nucleus with the charge Ze, so that $\vec{A}$ can be reduced to zero everywhere and A_0 ~~can be~~ to a function of r only, which is equal to $\frac{Ze}{r}$ except in the neighborhood of the nucleus.

7) See I. 8) See II, §4.

Hence the perturbed wave ~~function~~ equation

$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \vec{\alpha}(c\vec{p} + e'\vec{A}) + \beta m'c^2 + g'\tau'(U_0 + \vec{\alpha}\vec{U})\exp(-2\pi i\nu t)\right\}\psi = 0 \qquad (6)$$

can be transformed into the form [9]

$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \varepsilon p_r c + i\varepsilon\rho_3 j\, hc/2\pi r + \rho_3 m'c^2 + g'\tau'(U_0 + \varepsilon U')\exp(-2\pi i\nu t)\right\}\psi = 0 \qquad (7)$$

under the above conditions, where

$$\left.\begin{aligned} \varepsilon &= \frac{\vec{\alpha}\vec{r}}{r} = \frac{\rho_1\vec{\sigma}\vec{r}}{r}, \qquad \frac{jh}{2\pi} = \rho_3\left(\vec{\sigma}\vec{m} + \frac{h}{2\pi}\right), \\ p_r &= \frac{1}{r}\left(\vec{r}\vec{p} - \frac{ih}{2\pi}\right) = -\frac{ih}{2\pi}\left(\frac{\partial}{\partial r} + \frac{1}{r}\right). \end{aligned}\right\} \qquad (8)$$

There appear in (7) only the operators ε, ρ_3, j, p_r and r, among which j commutes with all others, whereas ε and ρ_3, commuting with p_r and r, satisfy the relations

$$\varepsilon^2 = \rho_3^2 = 1, \qquad \varepsilon\rho_3 + \rho_3\varepsilon = 0$$

between themselves, so that they can be represented by the matrices

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad (9)$$

where the symbols 1 and 0 are, of course, different from those in the expressions (2) and (4).

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9) Compare, for example, P.A.M.Dirac, Quantum Mechanics, 2nd Ed. §73.

If the commuting quantities ρ_3, j, r and

$$u = M_z = m_z + \frac{h}{4\pi}\sigma_z$$

are taken as the arguments of the wave functions, instead of r, $\left\{\begin{matrix}\theta, \\ \phi\end{matrix}\right\}$, ρ_3 and σ_3, in addition to the quantity such as e', which determines whether the light particle is in the electron or in the neutrino state, the wave equation takes the form

$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \varepsilon p_r c + i\varepsilon\rho_3 j\, hc/2\pi r + \rho_3 m'c^2 + g'\tau'(U_0 + \varepsilon U')\exp(-2\pi i\nu t)\right\}\psi(\rho_3, j, r, u, e', t) = 0. \quad (10)$$

~~Since j and u commute with other operators in (10), they are constants of motion, even when the perturbation of the above type exists, and so that the components of the wave function~~

In this representation, the components of the wave functions can be written as

$$\psi^{(1)}_{j,u}(r,t),\quad \psi^{(2)}_{j,u}(r,t),\quad \varphi^{(1)}_{j,u}(r,t),\quad \varphi^{(2)}_{j,u}(r,t), \quad (11)$$

where ψ and φ correspond to $e' = e$ and $e' = 0$ respectively, while the suffixes 1 and 2 to $\rho_3 = 1$ and $\rho_3 = -1$ respectively. Thus, (10) takes the form

$$\left.\begin{aligned}
&\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + eA_0 + mc^2\right\}\psi^{(1)}_{j,u}(r,t) - \frac{hc}{2\pi}\left\{\frac{\partial}{\partial r} + \frac{1+j}{r}\right\}\psi^{(2)}_{j,u}(r,t) = 0 \\
&\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + eA_0 - mc^2\right\}\psi^{(2)}_{j,u}(r,t) + \frac{hc}{2\pi}\left\{\frac{\partial}{\partial r} + \frac{1-j}{r}\right\}\psi^{(1)}_{j,u}(r,t) = 0 \\
&\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + \mu c^2\right\}\varphi^{(1)}_{j,u}(r,t) - \frac{hc}{2\pi}\left\{\frac{\partial}{\partial r} + \frac{1+j}{r}\right\}\varphi^{(2)}_{j,u}(r,t) \\
&\quad + g'U_0\exp(-2\pi i\nu t)\psi^{(1)}_{j,u}(r,t) - ig'U'\exp(-2\pi i\nu t)\psi^{(2)}_{j,u}(r,t) = 0 \\
&\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} - \mu c^2\right\}\varphi^{(2)}_{j,u}(r,t) + \frac{hc}{2\pi}\left\{\frac{\partial}{\partial r} + \frac{1-j}{r}\right\}\varphi^{(1)}_{j,u}(r,t) \\
&\quad + g'U_0\exp(-2\pi i\nu t)\psi^{(2)}_{j,u}(r,t) + ig'U'\exp(-2\pi i\nu t)\psi^{(1)}_{j,u}(r,t) = 0,
\end{aligned}\right\} \quad (12)$$

(9)

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Since j and u commute with all other operators in (10), they are constants of motion, even when the perturbation of the above type exists, so that they can be considered to have the same fixed values for the initial and the final states.

We consider a light particle initially in the electron state of energy E with the wave functions

$$\left.\begin{aligned} \psi^{(1)}_{j,u}(r,t) &= \psi^{(1)}_{E,j,u}(r)\exp(-2\pi i E t/h) \\ \psi^{(2)}_{j,u}(r,t) &= \psi^{(2)}_{E,j,u}(r)\exp(-2\pi i E t/h) \\ \varphi^{(1)}_{j,u}(r,t) &= \varphi^{(2)}_{j,u}(r,t) = 0 \end{aligned}\right\} \quad (13)$$

where $\psi^{(1)}_{E,j,u}(r)$ and $\psi^{(2)}_{E,j,u}(r)$ satisfy the unperturbed equations

$$\left.\begin{aligned} (E + eA_0 + mc^2)\psi^{(1)}_{j,u}(r) - \frac{hc}{2\pi}\left(\frac{d}{dr} + \frac{1+j}{r}\right)\psi^{(2)}_{j,u}(r) &= 0 \\ \frac{hc}{2\pi}\left(\frac{d}{dr} + \frac{1-j}{r}\right)\psi^{(1)}_{j,u}(r) + (E + eA_0 - mc^2)\psi^{(2)}_{j,u}(r) &= 0. \end{aligned}\right\} \quad (14)$$

On account of the perturbation expressed by the terms involving g′ in (12), the transition of the particle to the continuous neutrino state of positive energy E' with the same values of j and u as the initial state can take place, if the condition

$$E' = E + \Delta W > \mu c^2$$

is fulfilled. The wave functions of the final state can be written in the form

$$\left.\begin{aligned} \psi^{(1)}_{j,u}(r,t) &= \psi^{(2)}_{j,u}(r,t) = 0 \\ \varphi^{(1)}_{j,u}(r,t) &= \varphi^{(1)}_{E',j,u}(r)\exp(-2\pi i E' t/h) \\ \varphi^{(2)}_{j,u}(r,t) &= \varphi^{(2)}_{E',j,u}(r)\exp(-2\pi i E' t/h), \end{aligned}\right\} \quad (15)$$

where $\varphi^{(1)}_{E',j,u}(r)$ and $\varphi^{(2)}_{E',j,u}(r)$ satisfy the equations

$$\left.\begin{aligned}(E'+\mu c^2)\varphi^{(1)}_{j,u}(r) - \frac{hc}{2\pi}\left(\frac{d}{dr}+\frac{1+j}{r}\right)\varphi^{(2)}_{j,u}(r)=0\\ \frac{hc}{2\pi}\left(\frac{d}{dr}+\frac{1-j}{r}\right)\varphi^{(1)}_{j,u}(r) + (E'-\mu c^2)\varphi^{(2)}_{j,u}(r)=0\,.\end{aligned}\right\}\quad(16)$$

According to the perturbation theory, the probability per unit time of the above transition is given by

$$P_{j,u} = \frac{4\pi^2 g'^2}{h}\left|\int_0^\infty\{(\varphi^{(1)*}_{E',j,u}\psi^{(1)}_{E,j,u}+\varphi^{(2)*}_{E',j,u}\psi^{(2)}_{E,j,u})U_0 + i(\varphi^{(2)*}_{E',j,u}\psi^{(1)}_{E,j,u}-\varphi^{(1)*}_{E',j,u}\psi^{(2)}_{E,j,u})U'\}r^2dr\right|^2, \quad (17)$$

where the wave functions are normalized with respect to the energy, i.e.

$$\int_0^\infty\{\psi^{(1)*}_{E_2,j,u}\psi^{(1)}_{E_1,j,u}+\psi^{(2)*}_{E_2,j,u}\psi^{(2)}_{E_1,j,u}+\varphi^{(1)*}_{E_2,j,u}\varphi^{(1)}_{E_1,j,u}+\varphi^{(2)*}_{E_2,j,u}\varphi^{(2)}_{E_1,j,u}\}r^2dr=1 \text{ (or } \delta(E_1-E_2)) \quad (18)$$

according as E_1 and E_2 belong to the discrete or the continuous energy spectrum. In this normalization, (17) expresses the probability per unit energy range, if the initial state is continuous, i.e. if $|E|>mc^2$.

Thus, the probability P per unit time of the transition from any electron state with the energy E to the neutrino state with the energy $E'=E+\Delta W$ is obtained by summing $P_{j,u}$ for all possible values of j and u. Namely,

$$P = \sum_{j,u} P_{j,u}\,. \quad (19)$$

The above process corresponds to the emission of a positron of energy $-E=E_+$ and a neutrino of energy $E'=\Delta W - E_+$, if E is smaller than $-mc^2$, whereas it corresponds to the disappearance of an orbital electron of energy E moving in the field of the nuclear charge Ze at the same time with the emission of a neutrino of energy $E'=E+\Delta W$, if E is one of the discrete energy values between mc^2 and $mc^2\sqrt{1-\alpha^2Z^2}$. In both cases, the atomic number is reduced by one during the process of disintegration.

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§ 3. Calculation of the Transition Probabilities

If the nuclear static field is expressed everywhere by the Coulomb potential $A_0 = \frac{Ze}{r}$, the normalized wave functions of the continuous electron state of negative energy $-E_+$ become approximately, for small value of r,

$$\left.\begin{aligned} \psi^{(1)}_{-E_+,j,u}(r) &\cong N_+\sqrt{2(j+\gamma')(j\varepsilon_+ +\gamma')}\cdot r^{\gamma'-1} \\ \psi^{(2)}_{-E_+,j,u}(r) &\cong + \text{ or } - N_+\sqrt{2(j-\gamma')(j\varepsilon_+ +\gamma')}\cdot r^{\gamma'-1} \end{aligned}\right\} \quad (20)$$

according as $j > 0$ or < 0, where

$$\left.\begin{aligned} N_+^2 &= \frac{1}{8\pi mc^2}\left(\frac{4\pi mc}{h}\right)^{2\gamma'+1}\frac{1}{\{\Gamma(2\gamma'+1)\}^2}\,\eta_+^{2\gamma'-1} \\ &\quad \times \exp\left(-\pi\alpha Z\frac{\varepsilon_+}{\eta_+}\right)\cdot\left|\Gamma\left(\gamma'+i\alpha Z\frac{\varepsilon_+}{\eta_+}\right)\right|^2, \\ \alpha &= \frac{2\pi e^2}{hc}, \qquad \gamma' = \sqrt{j^2-\alpha^2Z^2}, \\ \varepsilon_+ &= \frac{E_+}{mc^2}, \qquad \eta_+ = \sqrt{\varepsilon_+^2-1}. \end{aligned}\right\} \quad (21)$$

As the nuclear field is no more of Coulomb type in the neighborhood of the nucleus, the wave functions should be modified in this region. For simplicity, we take the values at $r = a_N$ of the expressions ~~(2)~~ (20) as the modified functions in the nucleus, a_N being the nuclear radius, whereas their values outside of the nucleus do not concern our problem.

Similarly, for small value of r, the wave functions of the neutrino state of positive energy E' can be expressed approximately by

$$\left.\begin{aligned} \varphi^{(1)}_{E',j,u}(r) &\cong N_0\cdot 2j\sqrt{\varepsilon'-\kappa}\cdot r^{j-1} \\ \varphi^{(2)}_{E',j,u}(r) &\cong 0 \end{aligned}\right\} \quad (22)$$

for $j > 0$ and

$$\left.\begin{aligned} \varphi^{(1)}_{E',j,u}(r) &\cong 0 \\ \varphi^{(2)}_{E',j,u}(r) &\cong N_0\cdot 2j\sqrt{\varepsilon'+\kappa}\; r^{-j-1} \end{aligned}\right\} \quad (23)$$

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for $j<0$, where

$$\left.\begin{aligned} N_0^2 &= \frac{1}{8\pi mc^2}\left(\frac{4\pi mc}{h}\right)^{2|j|+1}\frac{\{(|j|-1)!\}^2}{\{(2|j|)!\}^2}\,\eta'^{\,2|j|-1} \\ \varepsilon' &= \frac{E'}{mc^2}, \quad \eta' = \sqrt{\varepsilon'^2-\kappa^2}, \quad \kappa = \frac{\mu}{m} \end{aligned}\right\} \quad (24)$$

Inserting these expressions into (17) and summing $P_{j,u}dE_+$ up with respect to j and u, we obtain the probability $P_{E_+}dE_+$ per unit time of disintegration of the isobar Z with the emission of a positron of energy between E_+ and $E_+ + dE_+$ and a neutrino of energy between $\Delta W - E_+ - dE_+$ and $\Delta W - E_+$. In the summation, however, the terms other than those with $j = \pm 1$ are so small that they can be neglected without serious error. Hence the result is

$$P_{E_+}dE_+ = \frac{256\pi^4 m^5 c^4 g'^2}{h^7\{\Gamma(2\gamma+1)\}^2}\left(\frac{4\pi mc\,a_N}{h}\right)^{2(\gamma-1)}\left\{\frac{1+\gamma}{2}\left|\int U_0 d\vec{r}\right|^2+\frac{1-\gamma}{2}\left|\int U' d\vec{r}\right|^2\right\} \times F(\varepsilon_+,\kappa)\,d\varepsilon_+ \quad (25)$$

where

$$\left.\begin{aligned} \gamma &= \sqrt{1-\alpha^2 Z^2}, \quad \Delta w = \frac{\Delta W}{mc^2}, \quad d\varepsilon_+ = \frac{dE_+}{mc^2} \\ \text{and} \quad F(\varepsilon_+,\kappa) &= \eta_+^{2\gamma-1}\exp\left(-\pi\alpha Z\frac{\varepsilon_+}{\eta_+}\right)\left|\Gamma\left(\gamma+i\alpha Z\frac{\varepsilon_+}{\eta_+}\right)\right|^2 \\ &\times\{\varepsilon_+(\Delta w-\varepsilon_+)-\gamma\kappa\}\sqrt{(\Delta w-\varepsilon_+)^2-\kappa^2} \end{aligned}\right\} \quad (26)$$

which represents the energy distribution function for the positron.

The total probability per unit time of the positron emission is thus

$$P_+ = \int_{mc^2}^{\Delta W-\mu c^2} P_{E_+}dE_+$$
$$= \frac{256\pi^4 m^5 c^4 g'^2}{h^7\{\Gamma(2\gamma+1)\}^2}\left(\frac{4\pi mc\,a_N}{h}\right)^{2(\gamma-1)}\left\{\frac{1+\gamma}{2}\left|\int U_0 d\vec{r}\right|^2+\frac{1-\gamma}{2}\left|\int U' d\vec{r}\right|^2\right\}\int_1^{\Delta w-\kappa} F(\varepsilon_+,\kappa)\,d\varepsilon_+. \quad (27)$$

If we put the neutrino mass equal to zero, i.e. $\kappa=0$, in the above expressions, they become identical with the corresponding expressions in the theory of Fermi and Wick,[10] except that the factor

10) Fermi, loc.cit. G.C.Wick, Atti Accad.Lincei **19**,319,1934.

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$$g^2 \left| \int v_m^* u_n d\tau \right|^2 \tag{28}$$

is replaced by

$$g'\left\{ \frac{1+\gamma}{2} \left| \int U_0(r)\, d\vec{r} \right|^2 + \frac{1-\gamma}{2} \left| \int U'(r)\, d\vec{r} \right|^2 \right\}. \tag{29}$$

Similarly, we can calculate the probability per unit time of the absorption of any one of the orbital electrons by the nucleus, provided that ΔW added by the total energy of the electron is larger than μc^2. Since it can be easily seen that the probability of such a process is large, when the electron is initially in one of the K states, compared with other cases, if $\Delta W + \gamma mc^2$ is larger than μc^2, it will be enough for us to consider this case only.

The wave functions of either of two K states with j = -1, u = -1 and j = -1, u = 0 respectively are given approximately by

$$\left.\begin{aligned} \psi_K^{(1)} &= -N_K \sqrt{1-\gamma}\, a_N^{\gamma-1} \\ \psi_K^{(2)} &= N_K \sqrt{1-\gamma}\, a_N^{\gamma-1} \end{aligned}\right\} \tag{30}$$

in the nucleus, where

$$N_K^2 = \frac{1}{2\Gamma(2\gamma+1)} \left(\frac{2\pi^2 m e^2 Z}{h^2} \right)^{2\gamma+1}. \tag{31}$$

Inserting these expressions and (22), (23) into (17) and summing the results for u = -1 and 0, the required probability per unit time of absorption of any one of the K electrons by the nucleus becomes finally

$$P_K = (\alpha Z)^{2\gamma+1} \frac{256\pi^5 m^5 c^4 g'^2}{h^7\, \Gamma(2\gamma+1)} \left(\frac{4\pi m c\, a_N}{h} \right)^{2(\gamma-1)}$$
$$\times \left\{ \frac{1+\gamma}{2} \left| \int U_0 d\vec{r} \right|^2 + \frac{1-\gamma}{2} \left| \int U' d\vec{r} \right|^2 \right\} (\Delta W + \gamma + \kappa) \sqrt{(\Delta W + \gamma)^2 - \kappa^2} \tag{32}$$

Similar result will be obtained by using the original theory of Fermi, except that the factor (29) in (32) should be replaced by (28), as in the previous case.

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In case, when ΔW is larger than $mc^2 + \mu c^2$, both of the above processes can take place and the ratio of the probabilities is given by

$$\sigma = \frac{P_+}{P_K} = \frac{\int_1^{\Delta W - \kappa} F(\varepsilon_+, \kappa)\, d\varepsilon_+}{\pi (\alpha Z)^{2\gamma+1} \Gamma(2\gamma+1)(\Delta W + \gamma + \kappa)\sqrt{(\Delta W + \gamma)^2 - \kappa^2}} \tag{33}$$

which does not depend on U_0 and $\vec{U}$, so that it can be determined in any special case without detailed assumptions on the nuclear transition.

The mean life time of the isobar Z becomes thus

$$\tau_K = \frac{1}{P_K}, \tag{34}$$

if ΔW lies between $-\gamma mc^2 + \mu c^2$ and $mc^2 + \mu c^2$, whereas it becomes

$$\tau = \frac{1}{P_+ + P_K} \tag{35}$$

if ΔW is larger than $mc^2 + \mu c^2$.

It should be noticed that the normal states of two isobars was assumed to have the same spin value throughout the above calculation. If it is not the case, the probabilities P_{K*} and P_+ will be both much smaller.

§ 4. Numerical Results

We want now to obtain the numerical values of the mean life time of the isobar Z for special values of Z and ΔW.

By comparing the theoretical result with the experiment in the case of natural β-disintegration, Fermi took μ equal to zero and the constant g equal to

$$4 \times 10^{-50}\ \text{cm}^3\,\text{erg}, \tag{36}$$

assuming $|\int v_m^* u_n d\tau|^2$ to be of the order of 1, when the nuclear spin does not change during the disintegration. Accordingly, in the formulae (25), (27) and (32), we put κ equal to zero and the factor given by (29) equal to

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the above value of g.

In this case, the mean life time is given by the general form

$$\tau_K = \frac{\text{const.}}{(\Delta w + \gamma)^2}$$

for the value of Δw between $-\gamma$ and 1, the numerical values for several cases being summarized in the following table.

αZ	Z	τ_K
1/137	1	$1370(\Delta w+1)^{-2}$ years
0.1	14	$200(\Delta w+1)^{-2}$ days
0.2	27	$25(\Delta w+1)^{-2}$ days
0.5	69	$14(\Delta w+1)^{-2}$ hours

Thus, the life time of the isobar Z becomes shorter and shorter as Z increases for a given value of $\Delta w+\gamma$. It is only several hours for Z as large as 70, unless $\Delta w+\gamma$ is very close to zero. These results seem to be in conflict with the fact that several pairs of stable isobars with consecutive atomic numbers are found in nature, especially for heavy elements.

Hence, we are obliged to assume either that such a pair has different nuclear spins in every case, or that the neutrino mass is not small compared with the electron mass. If the first alternative happen to be excluded by future experiments determining the nuclear spins, the second alternative will have to be adopted, necessitating an essential modification of Fermi's theory. The recent proposal of Uhlenbeck and Konopinski[11] is not sufficient for this purpose, as the distribution curve of the β-ray does not agree with the experiment near the upper limit of the energy in this case also, if the mass of the neutrino is not small compared with that of the electron.

On the other hand, the special case Z=1 is a little interesting, as

11) Konopinski & Uhlenbeck, loc.cit.

it shows that the proton is not stable and can disintegate into the neutron and the neutrino by uniting itself with the orbital electron. The mean life time of the proton is thus of the order of thousand years, if the difference of the masses of the proton and the neutron is not smaller than $-mc^2$. On the contary, if the difference of their masses is smaller than $-mc^2$, the neutron in its turn becomes unstable and can disintegrate into the proton and the electron by absorbing a neutrino in the negative energy state. Thus, whatever be the difference of the masses of the neutron and the proton, one of them will be unstable, if the neutrino mass is zero.

It should be noticed, however, that the above general theory of the β-disintegration can not be safely extended up to the case Z=1, so that the above results need not be taken too seriously.

Next, for the case $\Delta w > 1$, the ratio σ of the probabilities of the disintegration by positron emission and by K electron absorption are calculated for several values of Z and Δw, the results being summarized in the following table.

αZ	Δw	σ	αZ	Δw	σ	αZ	Δw	σ
0.1	2	2.9	0.2	2	0.2	0.6	2	0.8×10^{-3}
0.1	3	31.3	0.2	3	2.6	0.6	5	0.1
0.1	5	250	0.2	5	20.7			

From these results we see that, for artificially produced radioelements of the positron type, for which αZ is small and Δw is larger than 2 always, the ratio σ is so large that the disintegration by the absorption of the orbital electrons does not seriously affect the mean life time. On the contrary, for large values of Z, such a process becomes much more frequent than the ordinary process of the positron emission, as long as Δw is not too large compared with 1.

In conclusion, it should be remarked that the whole theory of the β-disintegration is based on the assumption that the equation of Dirac's type hold for the light particle up to the region of the nuclear radius ~~immediate neighborhood of the nucleus~~, at least approximately, which is so doubtfull that the radical ~~change~~ revision of the above results may well ~~happen~~ ~~to~~ be needed in future.

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mc + and c of the electron and the neutrino, the isobar Z will be thestable and will change into Z-1 with the emission of the positron. On the contary