On the Theory of the β-Disintegration and the Allied Phenomenon[1]

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Abstract

According to general theory of the β-disintegration, the occurrence of the following phenomenon is expected. Namely, if the difference ΔW of proper energies of two isobars with atomic numbers Z and Z-1 respectively is larger than $-(mc^2+\mu c^2)$, where m and μ are the masses of the electron and the neutrino respectively, the isobar Z will change into the isobar Z-1 by absorbing one of the orbital electrons. The probability of such a process is calculated, when the electron is initially in one of the K states. The ~~relative~~ importance of this process, when $\Delta W > mc^2+\mu c^2$, relative to the ordinary disintegration by positron emission is also considered, and its bearing on the problems of the neutrino mass and the nuclear spin is discussed.

1) This paper was read before the meeting of Osaka branch on July 6, 1935.

§ 1. Introduction

The present quantum theory teaches us that almost all the phenomena occurring in nature can be resolved into a number of elementary processes, in which the laws of conservation of energy and momentum hold always good.

The disintegration of natural or artificially produced radio-element by by emitting electrons or positrons with continuous energy spectrum was the only case ever known, in which the conservation laws seemed to be violated.

In all other cases, new sorts of elementary particles, such as the photon and the neutron, have been introduced, whenever necessary for the conservation of energy and momentum. So, in this last case also, it seems more plausible to introduce a new sort of elementary particle, which goes away unnoticed with the surplus energy and momentum, than to ~~consider that~~ renounce the applicability of the conservation laws ~~do not hold in principle~~.

The neutrino theory of β-disintegration, which was suggested by Pauli and developed by Fermi[2] on this point of view, was found to agree with the experiment on the whole, although there are still many points to be corrected and completed in future.

In this paper, we want to ~~treat~~ deal with the problem in a ~~more definite~~ little modified form and to investigate especially the following phenomenon, which is related closely to the β-disintegration.

Consider two isobars with ~~the mass number M and~~ the atomic numbers Z and Z-1 respectively. The small difference ΔW of their proper energies will be, in general, either positive or negative. If ΔW is larger than the sum of the proper energies mc^2 and μc^2 of the electron and the neutrino,

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2) E. Fermi, Zeits. f. Phys. **88**, 161, 1934.

the isobar Z will be unstable and will change into Z-1 with the emission of the positron.

On the contrary, if ΔW is smaller than $-(mc^2+\mu c^2)$, the isobar Z-1 will be unstable, changing into Z with the emission of the electron.

Finally, if ΔW lies between $mc^2+\mu c^2$ and $-(mc^2+\mu c^2)$, both types of β-disintegration can not ~~occur~~ occur. Then, are these ~~both of two~~ two isobars both stable ? It is not always so, for if ΔW is larger than $\mu c^2 - E$, where E is the total energy of one of the orbital electrons of the isobar Z, it will change into Z-1 with the absorption of this electron. For example, a K electron of the isobar Z can be absorbed by the nucleus, if ΔW is larger than $\mu c^2 - mc^2\sqrt{1-\alpha^2 Z^2}$, where α is the fine structure constant.

Thus, in general, two isobars with atomic numbers differing by one are both stable, only if ΔW lies between $-(mc^2+\mu c^2)$ and $-mc^2+\mu c^2$, as the total energies of the electrons in outer orbits are very close to mc^2.

Consequently, if the mass μ of the neutrino is very small compared with m, as assumed by Fermi, it will be very rare that both two isobars with consecutive atomic numbers happen to be stable.[3]

In the following sections, we want to ~~treat~~ deal with this problem quantitatively and to show in what case the probability of occurrence of the above phenomenon becomes appreciable.

In case, when ΔW is larger than $mc^2+\mu c^2$, such a process will occur side by side with the emission of the positron. Hence, the ~~relative~~ ratio of the probabilities of occurrence of these two processes will also be calculated.

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3) This point was suggested by Prof. Beck, to whom the authors are much obliged.

§ 2. General Theory

According to Fermi's theory, β-disintegration can be considered as the emission of an electron and a neutrino at the same time with the transition of a heavy particle in the nucleus from a neutron state to a proton state.

We can alternatively assume that, in this case, the light particle jumps from a neutrino state of negative energy to an ~~electron~~ state of positive energy at the same time with the nuclear transition, thus an electron and an ~~//////~~ anti-neutrino being emitted.[4)] Similarly, in the case of the positron emission, we consider that the light particle jumps from an electron state of negative energy to a neutrino state of positive energy at the same time with the transition of a heavy particle in the nucleus from a proton state to a neutron state, thus a positron and a neutrino being emitted.

Further we want to assume that the light particle satisfies Dirac's wave equation, in which the charge and the proper mass are, however, not constant, but take the values $(-e, m)$ or $(0, \mu)$ according as the particle is in an electron or in a neutrino state. Thus the wave equation takes the form

$$\left\{ \frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \vec{\alpha}(c\vec{p} + e'\vec{A}) + \beta m'c^2 \right\} \psi = 0 , \qquad (1)$$

where e', m' are the matrices, which commute with each other and with all other quantities in (1). They can be written simultaneousely in the diagonal forms

$$\begin{pmatrix} e\cdot 1 & 0 \\ 0 & 0 \end{pmatrix} , \qquad \begin{pmatrix} m\cdot 1 & 0 \\ 0 & \mu\cdot 1 \end{pmatrix} , \qquad (2)$$

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4) H. Yukawa, Proc.Phys.Math.Soc.Japan, **17**, 48 , 1935. This paper will be referred to as I. This was also pointed out ~~by~~ recently by E. J. Konopinski & G.E.Uhlenbeck, Phys.Rev. **48**,7,1935. Compare also R.L. Doleceak, ~~Phys. Rev.~~ ibid. 48, 13, 1935.

where 1, 0 denotes unit and zero matrices respectively, with four rows and columns. Other notations are the same with those in the previous paper of the authors[5]. The wave function ψ has eight components, of which the first four refer to the electron state and the remaining four to the neutrino state, in the representation, in which e', m' are expressed by (2).

We want now to consider the case, when ΔW is larger than $-mc^2+\mu c^2$.

The change of the charge of the nucleus from Ze to (Z-1)e with the liberation of energy of amount ΔW can be considered as a periodic perturbation with the frequency $\nu = \frac{\Delta W}{h}$, which induces the transition of the light particle from an electron state to a neutrino state.

If we assume that this perturbation is described by a sort of field acting on the light particle[6] and further that this field can be derived from scalar and vector potentials, in analogy with the ordinary electromagnetic field, we have to add the term of the form

$$g'\tau'(U_0+\vec{\alpha}\vec{U})\exp(-2\pi i\nu t) \qquad (3)$$

to the operator in the left hand side of the wave equation (1), where τ' is an operator τ, which changes the electron state of the light particle into the neutrino state, having the form

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \qquad (4)$$

in the representation, in which e' and m' are expressed by (2). U_0, $\vec{U}$ are scalar and vector functions of the coordinates describing the potentials of the perturbing field due to the nuclear transition and g' is a constant analogous to the charge in the case of the ~~electric~~ electromagnetic interaction.

5) H.Yukawa & S.Sakata, Proc.Phys.Math.Soc.Japan, 17, in press. This paper will be referred to as II.

6) Compare I, in which, however, only the scalar potential of the field was taken into account.

A possible theory of such a field, ~~formulated~~ proposed recently by one of the authors,[7] will not be discussed ~~later on~~ in this paper. Here we want to assume only that the potentials U_0 and $\vec{U}$ simply vanish outside of the nucleus.

Since the total angular momentum of the system consisting of the light particle and the nucleus is considered to be constant throughout the process of disintegration, the change of the total angular momentum $\vec{M}$ of the light particle is connected closely with that of the nuclear spin. Especially, when both the absolute value and the direction of the nuclear spin do not change during the disintegration, $\vec{M}$ ~~should~~ will be also constant, so that the perturbation operator (3) should commute with the operator

$$\vec{M} = \vec{m} + \frac{h}{4\pi}\vec{\sigma}, \tag{5}$$

where $\vec{m} = \vec{r} \times \vec{p}$ and $\vec{\sigma}$ is the spin vector. In such a case, if we assume U_0 and $\vec{U}$ to be the functions of coordinates $\vec{r}$ only, it follows that U_0 depends only on r, while $\vec{U}$ has the form

$$\frac{\vec{\sigma}\vec{r}}{r}U'$$

where U' is a certain function of r only. The proof of it is the same as in the previous paper.[8]

~~Thus~~ Hence the perturbed wave equation ~~becomes~~

$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \vec{\alpha}(c\vec{p} + e'\vec{A}) + \beta m'c^2 + g'\tau'(U_0 + \vec{\alpha}\vec{U})\exp(-2\pi i\nu t)\right\}\psi = 0 \tag{6}$$

can be transformed into the form[9]

$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \varepsilon p_r c + e'\vec{\alpha}\vec{A} + i\varepsilon\rho_3 j hc/2\pi r + \rho_3 m'c^2 + g'\tau'(U_0 + \varepsilon U')\exp(-2\pi i\nu t)\right\}\psi = 0, \tag{7}$$

7) See I.
8) See II, §4.

9) P.A.M.Dirac, Quantum Machanics, 2nd Ed. §73.

where
$$\left.\begin{aligned} \varepsilon &= \frac{\vec{\alpha}\vec{r}}{r} = \frac{\rho_1 \vec{\sigma}\vec{r}}{r} \\ \frac{jh}{2\pi} &= \rho_3 \left(\vec{\sigma}\vec{m} + \frac{h}{2\pi}\right) \\ p_r &= \frac{1}{r}\left(\vec{r}\vec{p} - \frac{ih}{2\pi}\right) = -\frac{ih}{2\pi}\left(\frac{\partial}{\partial r} + \frac{1}{r}\right). \end{aligned}\right\} \quad (8)$$

As the light particle is initially in the electron state moving in the central field of the nucleus with the charge Ze in our case, $\vec{A} = 0$ and A_0 is a function of r only, which is equal to $\frac{Ze}{r}$ except in the neighborhood of the nucleus.

Thus, there appear in (7) only the operators ε, ρ_3, j, p_r, and r, among which j commutes with all other ~~quantityies~~ quantities, while ε and ρ_3 commute with p_r and r, ~~and~~ satisfying the relations
$$\varepsilon^2 = \rho_3^2 = 1 \quad , \quad \varepsilon\rho_3 + \rho_3\varepsilon = 0$$
between themselves. They can be represented by the matrices[10)]
$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \quad (9)$$

If the commuting quantites ρ_3, j, r and
$$u = M_z = m_z + \frac{h}{4\pi}\sigma_z$$
are taken as the arguments of the wave functions, instead of r, θ, ϕ, ρ_3, ~~σ_3~~ and σ_3, in addition to the quantity such as e', which determines whether the light particle is in the electron state or in the neutrino state, the wave equation takes the form
$$\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t} + e'A_0 + \varepsilon p_r c + i\varepsilon\rho_3 j\, hc/2\pi r + \rho_3 m' c^2 + g'\tau'(U_0 + \varepsilon U')\exp(-2\pi i\nu t)\right\} \psi(\rho_3, j, r, u, e', t) = 0 \quad (10)$$

~~10) These matrices have~~ ~~differ~~, ~~of course, from those of (2) and (4)~~

10) The symbols 1, ~~ρ_3~~, 0, ~~have not the~~ ~~differ~~ are not ~~the same~~, of course, ~~from~~ the same as those of (2) and (4).

Since j and u ~~commute with other operators~~ in (10), they are constants of motion, even when ~~the~~ perturbation of the above type exists, and can be considered as mere ~~numers~~, ~~so that~~ the components of the wave functions ~~becomes~~ can be written as

$$\psi^{(1)}_{j,u}(r,t)\ ,\quad \psi^{(2)}_{j,u}(r,t)\ ,\quad \varphi^{(1)}_{j,u}(r,t)\ ,\quad \varphi^{(2)}_{j,u}(r,t)\ ,$$

~~of which the first two refer to the electron state and the remaining two to the neutrino state, satisfying the equations~~

where ψ and φ correspond to $e'=e$ and 0 respectively, while the suffixes 1 and 2 ~~refer~~ to $\rho_3 = 1$ and -1 respectively. Thus (10) can be written as

$$\left.\begin{aligned}
&\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t}+eA_0+mc^2\right\}\psi^{(1)}_{j,u}(r,t)-\frac{hc}{2\pi}\left\{\frac{\partial}{\partial r}+\frac{1+j}{r}\right\}\psi^{(2)}_{j,u}(r,t)=0\\
&\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t}+eA_0-mc^2\right\}\psi^{(2)}_{j,u}(r,t)+\frac{hc}{2\pi}\left\{\frac{\partial}{\partial r}+\frac{1-j}{r}\right\}\psi^{(1)}_{j,u}(r,t)=0\\
&\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t}+\mu c^2\right\}\varphi^{(1)}_{j,u}(r,t)-\frac{hc}{2\pi}\left\{\frac{\partial}{\partial r}+\frac{1+j}{r}\right\}\varphi^{(2)}_{j,u}(r,t)\\
&\quad+g'U_0\exp(-2\pi i\nu t)\,\psi^{(1)}_{j,u}(r,t)-ig'U'\exp(-2\pi i\nu t)\,\psi^{(2)}_{j,u}(r,t)=0\\
&\left\{\frac{ih}{2\pi}\frac{\partial}{\partial t}-\mu c^2\right\}\varphi^{(2)}_{j,u}(r,t)+\frac{hc}{2\pi}\left(\frac{\partial}{\partial r}+\frac{1-j}{r}\right)\varphi^{(1)}_{j,u}(r,t)\\
&\quad+g'U_0\exp(-2\pi i\nu t)\,\psi^{(2)}_{j,u}(r,t)+ig'U'\exp(-2\pi i\nu t)\,\psi^{(1)}_{j,u}(r,t)=0
\end{aligned}\right\}\quad(11)$$

Since j and u commute with all other operators in (10), they are constants of motion ~~and~~ ... ~~so can be considered as~~ that

Now we consider a light particle initially in the electron state of energy E with the wave functions

$$\left.\begin{aligned}
&\psi^{(1)}_{j,u}(r,t)=\psi^{(1)}_{E,j,u}(r)\exp(-2\pi iEt/h)\\
&\psi^{(2)}_{j,u}(r,t)=\psi^{(2)}_{E,j,u}(r)\exp(-2\pi iEt/h)
\end{aligned}\right\}\quad(12)$$

$$\varphi^{(1)}_{j,u}(r,t)=\varphi^{(2)}_{j,u}(r,t)=0\ ,$$

where $\psi^{(1)}_{E,j,u}(r)$ and $\psi^{(2)}_{E,j,u}(r)$ satisfy the unperturbed equations

$$\left.\begin{aligned} &(E+eA_0+mc^2)\,\psi^{(1)}_{j,u}(r) - \frac{hc}{2\pi}\left(\frac{d}{dr}+\frac{1+j}{r}\right)\psi^{(2)}_{j,u}(r)=0 \\ &\frac{hc}{2\pi}\left(\frac{d}{dr}+\frac{1-j}{r}\right)\psi^{(1)}_{j,u}(r) + (E+eA_0-mc^2)\,\psi^{(2)}_{j,u}(r)=0 \end{aligned}\right\} \quad (13)$$

On account of the perturbation expressed by the terms involving g' in (11), the transition of the particle to the continuous neutrino state of positive energy E' with the wave functions

$$\left.\begin{aligned} &\psi^{(1)}_{j,u}(r,t) = \psi^{(2)}_{j,u}(r,t) = 0 \\ &\varphi^{(1)}_{j,u}(r,t) = \varphi^{(1)}_{E',j,u}(r)\exp(-2\pi i E't/h) \\ &\varphi^{(2)}_{j,u}(r,t) = \varphi^{(2)}_{E',j,u}(r)\exp(-2\pi i E't/h), \end{aligned}\right\} \quad (14)$$

satisfying the equations

$$\left.\begin{aligned} &(E'+\mu c^2)\,\varphi^{(1)}_{j,u}(r) - \frac{hc}{2\pi}\left(\frac{d}{dr}+\frac{1+j}{r}\right)\varphi^{(2)}_{j,u}(r)=0 \\ &\frac{hc}{2\pi}\left(\frac{d}{dr}+\frac{1-j}{r}\right)\varphi^{(1)}_{j,u}(r) + (E'-\mu c^2)\,\varphi^{(2)}_{j,u}(r)=0, \end{aligned}\right\} \quad (15)$$

can take place, if the condition

$$E' = E + \Delta W > \mu c^2$$

is fullfiled. since . . .

the

from ~~between~~ ~~two~~ the electron state with the fixed value of E, j, u ~~and~~ to the neutrino with $E' = E + \Delta W > \mu c^2$, j, u

If so, the probability of the transition per unit time is given by

$$P_{j,u} = \frac{4\pi^2 g'^2}{h}\sum_{j,u}\left|\int_0^\infty \{(\varphi^{(1)*}_{E',j,u}\psi^{(1)}_{E,j,u} + \varphi^{(2)*}_{E',j,u}\psi^{(2)}_{E,j,u})U_0 + i(\varphi^{(2)*}_{E',j,u}\psi^{(1)}_{E,j,u} - \varphi^{(1)*}_{E',j,u}\psi^{(2)}_{E,j,u})U'\}\, r^2 dr\right|^2, \quad (16)$$

where the wave functions are normalized with respect to the energy, i.e.

$$\int_0^\infty \{\psi^{*(1)}_{E_2,j,u}\psi^{(1)}_{E_1,j,u} + \psi^{*(2)}_{E_2,j,u}\psi^{(2)}_{E_1,j,u} + \varphi^{*(1)}_{E_2,j,u}\varphi^{(1)}_{E_1,j,u} + \varphi^{*(2)}_{E_2,j,u}\varphi^{(2)}_{E_1,j,u}\}\, r^2 dr = 1, \quad (17)$$

(summing ~~the (16)~~ $P_{j,u}$ for possible values of j, u. Thus

if E_1 and E_2 belong to the discrete energy spectrum, while the right hand side should be replaced by $\delta(E_1 - E_2)$, if they belong to the continuous spectrum. ~~In the latter case,~~ In this normalization, (16) expresses the probability per unit energy range, if the initial state is continuous, i.e. $|E| > mc^2$.

So the probability P per unit time of the transition from any electron ~~of the~~ state with energy E to the neutrino state

The above process corresponds to the emission of a positron of energy $-E = E_+$ and a neutrino of energy $E' = \Delta W - E_+$, when $E < -mc^2$, while it ~~corresponds to the disappearance of an electron of energy E and the emission of a neutrino of energy $E' = E + \Delta W$, when E belongs to is one of the~~ corresponds to the disappearance of ~~one of the~~ an orbital electron of energy E moving in the field of the nuclear charge Ze at the same time with the emission of a neutrino of energy $E' = E + \Delta W$, when E is one of the discrete energy values between mc^2 and 0. In both cases, the atomic number of the nucleus is reduced by one at the same time.

(of energy is given by) $P = \sum_{j,u} P_{j,u}$

If the nuclear static field is expressed everywhere by the Coulomb potential $A_0 = \frac{Ze}{r}$, the normalized wave functions of the continuous electron state of negative energy ~~an the~~ $-E_+$ become approximately, for small value of r,

$$\left.\begin{aligned} \psi^{(1)}_{-E_+,j,u}(r) &\cong N_+ \sqrt{2(j+\gamma')(j\varepsilon_+ + \gamma')} \cdot r^{\gamma'-1} \\ \psi^{(2)}_{-E_+,j,u}(r) &\cong + \text{ or } - N_+ \sqrt{2(j-\gamma')(j\varepsilon_+ + \gamma')} \cdot r^{\gamma'-1} \end{aligned}\right\} (18)$$

according as $j >$ or < 0, where

$$\left.\begin{aligned}
N_+^2 &= \frac{1}{8\pi mc^2}\frac{1}{\{\Gamma(2\gamma'+1)\}^2}\left(\frac{4\pi mc}{h}\right)^{2\gamma'+1}\eta_+^{2\gamma'-1} \\
&\times \exp\left(-\pi\alpha Z\frac{\varepsilon_+}{\eta_+}\right)\left|\Gamma\left(\gamma'+i\alpha Z\frac{\varepsilon_+}{\eta_+}\right)\right|^2, \\
\alpha &= \frac{2\pi e^2}{hc}, \quad \gamma' = \sqrt{j^2-\alpha^2 Z^2}, \\
\varepsilon_+ &= \frac{E_+}{mc^2}, \quad \eta_+ = \sqrt{\varepsilon_+^2-1}.
\end{aligned}\right\} \quad (19)$$

As the nuclear field is no more of Coulomb type in the neighborhood of the nucleus, the wave functions should be modified in this region. For simplicity, we take the values at $r = a_N$ of (18) as the modified functions in the nucleus, ~~where~~ a_N being the nuclear radius, whereas their values outside of the nucleus do not matter to our problem.

Similarly, for small value of r, the wave functions of the neutrino state of positive energy E can be expressed approximately by

$$\left.\begin{aligned}
\varphi^{(1)}_{E',j,u}(r) &\cong N_0\, 2j\,\sqrt{\varepsilon' \cancel{+\kappa} -\kappa}\; r^{j-1} \\
\varphi^{(2)}_{E',j,u}(r) &\cong 0
\end{aligned}\right\} \quad (20)$$

for $j > 0$ and

$$\left.\begin{aligned}
\varphi^{(1)}_{E',j,u}(r) &\cong 0 \\
\varphi^{(2)}_{E',j,u}(r) &\cong N_0\, 2j\,\sqrt{\varepsilon'+\kappa}\; r^{-j-1}
\end{aligned}\right\} \quad (21)$$

for $j < 0$, where

$$\left.\begin{aligned}
N_0^2 &= \frac{1}{8\pi mc^2}\frac{\{(|j|-1)!\}^2}{\{(2|j|)!\}^2}\left(\frac{4\pi mc}{h}\right)^{2|j|+1}\eta'^{\,2|j|-1}, \\
\varepsilon' &= \frac{E'}{mc^2}, \quad \eta' = \sqrt{\varepsilon'^2-\kappa^2}, \quad \kappa = \frac{\mu}{m}.
\end{aligned}\right\} \quad (22)$$

Inserting these expressions into (16), we obtain the probability per unit time of disintegration of the nucleus with the emission of a positron of energy between E_+ and $E_+ + dE_+$ and a neutrino of energy between $\Delta W - E_+ - dE_+$ and $\Delta W - E_+$. We should notice hereupon that, in the summation with respect to j and u, the terms other than $j = \pm 1$ are ~~very~~ so small ~~as~~ that they can be neglected without serious error. Hence the result is

$$P_{E_+} = \frac{256\pi^4 m^5 c^4 g'^2}{\hbar^7 \{\Gamma(2\gamma+1)\}^2} \left(\frac{4\pi m c\, a_N}{\hbar}\right)^{2(\gamma-1)} \left\{ \frac{1+\gamma}{2} \left| \int U_0\, d\vec{r} \right|^2 + \frac{1-\gamma}{2} \left| \int U'\, d\vec{r} \right|^2 \right\}$$

$$\times F(\varepsilon_+, \kappa)\, d\varepsilon_+ \qquad (23)$$

where $\gamma = \sqrt{1-\alpha^2 Z^2}$, $\Delta w = \frac{\Delta W}{mc^2}$, $\kappa = \frac{\mu}{m}$, $d\varepsilon_+ = \frac{dE_+}{mc^2}$

and $$F(\varepsilon_+, \kappa) = \eta_+^{2\gamma-1} \exp\left(-\pi\alpha Z \frac{\varepsilon_+}{\eta_+}\right) \left| \Gamma\left(\gamma + i\alpha Z \frac{\varepsilon_+}{\eta_+}\right) \right|^2$$

$$\times \{\varepsilon_+ (\Delta w - \varepsilon_+)^{\cancel{2}} - \gamma \cdot \kappa\, \cancel{(\Delta w - \varepsilon_+)}\} \times \sqrt{(\Delta w - \varepsilon_+)^2 - \kappa^2}, \qquad (24)$$

which represents the energy disteibution of the positron.

The total probability per unit time of the positron emission is given by

$$P_+ = \int_1^{\Delta w - \kappa} P_{E_+}\, d\varepsilon_+ \qquad (25)$$

$$= \frac{256\pi^4 m^5 c^4 g'^2}{\hbar^7 \{\Gamma(2\gamma+1)\}^2} \left(\frac{4\pi m c\, a_N}{\hbar}\right)^{2(\gamma-1)} \left\{ \frac{1+\gamma}{2} \left| \int U_0\, d\vec{r} \right|^2 + \frac{1-\gamma}{2} \left| \int U'\, d\vec{r} \right|^2 \right\} \int_1^{\Delta w - \kappa} F(\varepsilon_+, \kappa)\, d\varepsilon_+$$

If we ~~t~~ make the mass of the neutrino zero, i.e. $\kappa = 0$, in the expressions ~~(24)~~ (23) and (25), they become identical with the corresponding expressions in the theory of Fermi and Wick,[11] except that the factor

$$g^2 \left| \int v_m^* u_n\, d\tau \right|^2 \qquad (26)$$

are replaced by

$$g'^2 \left\{ \frac{1+\gamma}{2} \left| \int U_0(r)\, d\vec{r} \right|^2 + \frac{1-\gamma}{2} \left| \int U'(r)\, d\vec{r} \right|^2 \right\} \qquad (27)$$

11) Fermi, loc.cit. G.C.Wick, Atti ~~dei~~ Accad. Lincei, 19,319,1934.

Similarly, we can calculate the probability per unit time of any one of the orbital electrons by the nucleus, provided that ~~[illegible]~~ ΔW is larger than $-mc^2 + \mu c^2$. Since it can be seen easily that the probability, when the electron is initially in one of the K states, is large compared with other cases, ~~only this case~~, it will be enough for us to deal with this case only.

The wave functions of ~~a~~ either of K states are given approximately by

$$\left.\begin{aligned} \psi_K^{(1)} &= -N_K\sqrt{1-\gamma}\; a_N^{\gamma-1} \\ \psi_K^{(2)} &= N_K\sqrt{1+\gamma}\; a_N^{\gamma-1} \end{aligned}\right\} \qquad \begin{matrix}(30)\\(28)\end{matrix}$$

$$N_K^2 = \frac{1}{2\Gamma(2\gamma+1)}\left(\frac{2\pi^2 m e^2 Z}{h^2}\right)^{2\gamma+1} \qquad (31).$$

in the nucleus, where ~~so that the requred probability per unit time of absorption of any one of the K electrons becomes fin~~

~~in the nucleus:~~ Inserting these expressions and the equations (20), (21) into (16), the required probability per unit time of absorption of any one of the K electrons by the nucleus becomes finally

$$P_K = (\alpha Z)^{2\gamma+1}\frac{256\pi^5 m^5 c^4 g^2}{h^7\,\Gamma(2\gamma+1)}\left(\frac{4\pi m c\, a_N}{h}\right)^{2(\gamma-1)}$$

$$\times\left\{\frac{1+\gamma}{2}\left|\int U_0 d\vec{r}\right|^2 + \frac{1-\gamma}{2}\left|\int U' d\vec{r}\right|^2\right\}(\Delta W+\gamma+\kappa)\sqrt{(\Delta W+\gamma)^2-\kappa^2} \qquad \begin{matrix}(29)\\(32)\end{matrix}$$

In case, when ~~the~~ both types of process can occur, the ratio ~~r~~ of their probabilities is given by

$$\sigma = \frac{P_+}{P_K} = \frac{\int_1^{\Delta W-\kappa} F(\varepsilon_+,\kappa)\,d\varepsilon_+}{\pi(\alpha Z)^{2\gamma+1}\,\Gamma(2\gamma+1)\,(\Delta W+\gamma+\kappa)\sqrt{(\Delta W+\gamma)^2-\kappa^2}} \qquad \begin{matrix}(30)\\(33)\end{matrix}$$

which does not depend~~s~~ on U_0 and $\vec{U}$, so that it can be determined in any special case without detailed ~~special~~ assumptions on the nuclear transition.

The mean life time of the isobar becomes

$$\tau_K = \frac{1}{P_K} \tag{31}$$

if Δw lies between $-\gamma+\kappa$ and $1+\kappa$, whereas it becomes

$$\tau = \frac{1}{P_+ + P_K} \tag{32}$$

if Δw is larger than $1 + \kappa$.

In the ~~ab~~ove calculations, ~~we~~ it should be noticed that the normal states of two isobars was always assumed to have the same spin value. If it is not the case, the probabilit~~y~~ies P_K, P_+ will be both much smaller.

§ 3. Numerical Results

We want now to obtain the numerical values of the mean life time of the isobar Z for special values of Z and ΔW.

By comparing with the experiment in the case of natural β-disintegration, Fermi took μ equal to zero and the constant g equal to

$$4 \times 10^{-50}\ \mathrm{cm}^3\,\mathrm{erg}\,, \tag{33}$$

assuming

$$\left|\int v_m^* u_n\, dx\right|^2$$

in the formulae (23), (25), (29)), to be of the order of 1. Accordingly, we put κ equal to ~~zero~~ and the factor given by (27) equal to the above value 4×10^{-50} cm³ erg, when the nuclear spin does not change during the disintegration. In this case τ_K has the ~~following~~ general form

~~Thus, we have the following results~~ for the value of Δw between $-\gamma$ and ~~1.~~ $\tau_K = \dfrac{\text{const}}{(\Delta w+\gamma)^2}$, the numerical values for several cases being given in the following table.

αZ	Z	τ_K
1/ 137	1	$200(\Delta w+1)^{-2}$ days
0.1	14	$1370(\Delta w+1)^{-2}$ years
0.2	27	$25(\Delta w+1)^{-2}$ days
0.5	69	$14(\Delta w+1)^{-2}$ hours

~~These results shows that~~ Thus, the life time of isobar Z becomes shorter and

and will ~~changing~~ disintegating into the neutron ~~with~~ absorbing the orbital electron with the mean life time of –

~~The special case Z=1 is a little interesting, as either both of the neutron and the proton can~~

Thus ~~the~~ both of the neutron it is ~~not~~ impossible that ~~the~~ both of the neutron ~~of~~ and the proton are stable "~~if~~ unless the neutrino mass is ~~not small~~ zero.

~~Com~~

* It should be noticed, however, that the general theory above can not be safely extended ~~to~~ upto $Z=1$, so that the result need not to be taken too seriously.

shorter as Z increases for ~~the~~ a given value of $\Delta w + \gamma$. It is only of several hours for Z larger than 70, unless Δw ~~$\sqrt{1}$~~ Z is very near to $-\gamma = -\sqrt{1-\alpha^2 Z^2}$. These results seem to be in conflict with the fact that several pairs of stable isobars with consecutive atomic numbers are found in nature , especially for heavier elements.

Hence, it seems necessary to assume either that such a pair has always different nuclear spins, or that the mass of the neutrino is not small compared with that of the electron. If the first alternative happen to be excluded by future experiments determining the nuclear spins, the second alternative will have to be adopted, necessitating an essential modification of Fermi's theory. The recent proposal of Uhlenbeck and Konopinski[12] is not sufficient for this purpose, as the distribution curve of β-ray does not agree with the experiment near the upper limit of the energy of the electron in this case also, if the mass of the neutrino is not small compared with that of the electron.†

Next, for the case, when Δw is larger than 1, the ratio σ of the probabilites of the disintegration by positron emission and by K electron absorption are calculated for several values of Z and Δw, the results being summarized in the following table.

αZ	$\Delta w = \frac{\Delta W}{mc^2}$	$\sigma = \frac{P_+}{P_K}$	αZ	Δw	σ
0.1	2	2.9	0.2	2	0.2
0.1	2.4	9.6(~~21~~)	0.2	3	2.6
0.1	3	31.3	0.2	5	20.7
0.1	5	250	0.6	2	0.8×10^{-3}

αZ	Δw	σ
0.6	5	0.1

† The special case $Z=1$ is a little interesting, as it shows that the proton is not stable, having mean life time of order of thousand years, if ~~the di its mass~~ the difference of the masses of the proton and the neutron is not smaller $-mc^2$,* very near or not than the hydrogen atom will disintegrate into

12) Uhlenbeck and Konopinski, loc. cit.

From these results we see that, for ~~almost~~ ~~all the~~ artificially produced radio-elements ~~ever known~~ of positron type, for which αZ is small ~~are smll~~ and Δw is larger than 2 always, the ratio is no large ~~far larger than 1,~~ so that the disintegration by absorption ~~the~~ of orbital electrons does not seriously affect the mean life time. ~~has no serious effects in these cases.~~ ~~can be ignored without~~ On the contrary, for large values of Z ~~and~~ such a process ~~is~~ becomes ~~overwhelmingly~~ much more frequent than the ordinary process of positron emission, ~~large compared~~ with as long as Δw is not too large compared with 1.

At any rate

From these results we see that, on the one hand, ~~for~~ for small value where

In our case, as ~~i~~ the electron is moving initially in the central field of the nucleus with the charge Ze, $A = 0$, Inserting these expressions into (16), we obtain the probability of disintegration of the nucleus with the emission of apositron of energy between E , E +dE and~~d~~ a neutrino