On the Theory of β-Disintegration

§ General Form of the Perturbing Field

Heavy particle の transition の仕方はわからぬ. そこで.

i) Nucleus + Light Particle の Total Angular Momentum (或 Spin) は Conserve すること.

ii) Nucleus の initial 及び final state の Spin は夫々一定の値を有すること. 従つて ~~nucleus~~ light particle の initial 及び final state の spin も又夫々一定の値を有する

iii) perturbing field は

$$V = U_0 + \alpha \vec{U}$$

なる形に書き得ること.

等を假定する、

すると先. V の ~~行列~~ matrix element

$$(n, j'u' | V | n, j''u'')$$

は

$$j' - j'' \equiv \cancel{\text{const}}\ \Delta j$$

~~$u' - u'' \equiv \Delta u$ (± or 0)~~

が與へられた整数値に等しい時丈0でないといふ様なものでなければならぬ.

先 $\Delta j =$ ~~Δu~~ $= 0$ の時を考へると V は j, u と Commute せねばならぬ.

一方 ~~V は一般に~~ U_0. $\vec{U}$ は夫々 r, θ, φ の函数と考へられる.

$$j\hbar = \rho_3\{\vec{\sigma}\vec{m} + \hbar\}$$

$$j\hbar\, V - V j\hbar = \rho_3\{\vec{\sigma}\vec{m} + \hbar\}(U_0 + \rho_1 \vec{\sigma}\vec{U})$$

$$-(U_0 + \rho_1\vec{\sigma}\vec{U})\rho_3\{\vec{\sigma}\vec{m} + \hbar\}$$

$$= \rho_3\{(\vec{\sigma}\vec{m} + \cancel{\hbar})U_0 - U_0(\vec{\sigma}\vec{m} + \cancel{\hbar})\}$$

$$+ i\rho_2\{(\vec{\sigma}\vec{m} \cancel{+\hbar})(\vec{\sigma}\vec{U}) + (\vec{\sigma}\vec{U})(\vec{\sigma}\vec{m}) + 2\hbar\vec{\sigma}\vec{U}\}$$

~~$(1 \pm \rho_2)$~~ $(1 \cdot \rho_2)$

$$= \rho_3\{\vec{\sigma}(\vec{m}U_0 - U_0\vec{m})$$

$$+ i\rho_2 \cancel{\vec{\sigma}\vec{m}}\{i\vec{\sigma}(\vec{m}\times\vec{U}) + i\vec{\sigma}(\vec{U}\times\vec{m}) + 2\hbar\vec{\sigma}\vec{U}\}$$

$$= 0$$

$(1 - \rho_2)$ ~~$(1 + \rho_2)$~~ $\therefore \quad \vec{m}U_0 - U_0\vec{m} = 0,$

$$U_0 = U_0(r)$$

$$\vec{m}\times\vec{U} + \vec{U}\times\vec{m} \mp 2\hbar i\vec{U} = 0$$

$$m_y U_z - m_z U_y + U_y m_z - U_z m_y \mp 2\hbar i U_x$$

$$(\cancel{z} p_x - x p_z)U_z - U_z(\cancel{z} p_x - x p_z)$$

$$-(\cancel{x} p_y - y p_x)U_y + U_y(x p_y - y p_x) = 2\hbar i U_x$$

$x u_z$

$$-\cancel{z}\cancel{x}\frac{\partial U_z}{\partial x} + x\frac{\partial U_z}{\partial z} + x\frac{\partial U_y}{\partial y} \mp y\frac{\partial U_y}{\partial x}$$

$$= 2U_x$$

$$x\,\mathrm{div}\,U - (\vec{x}\tfrac{\partial}{\partial x})\vec{U}$$

$$\cancel{\mathrm{div}}(x\vec{U}) = \frac{\partial}{\partial x}(\vec{r}\vec{U})$$

$U_x = x u_x$, $U_y = y u_y$, $U_z = z u_z$ とおく。

$$-z^2\frac{\partial u_z}{\partial x} + xz\frac{\partial u_z}{\partial z} + x\cancel{z}u_z + x u_y + xy\frac{\partial u_y}{\partial y} - y^2\frac{\partial u_y}{\partial x}$$

$$= 2x u_x$$

$$\vec{m}_x U_0 - U_0 \vec{m}_x = (y p_z - z p_y) U_0 - U_0 (y p_z - z p_y) = 0$$

$$y \frac{\partial U_0}{\partial z} - z \frac{\partial U_0}{\partial y} = 0$$

$z U_z^a$

$-z^2$

~~$m_y U_z -$~~ i $-i$

x $[m_y, U_z] - [m_z, U_y] = 2i\hbar U_x$　y　x

y $[m_z, U_x] - [m_x, U_z] = 2i\hbar U_y$　$-x$　y

z $[m_x, U_y] - [m_y, U_x] = 2i\hbar U_z$　z

$$-i[m_x + i m_y, U_z] + i[m_z, U_x + i U_y] = 2i\hbar (U_x + i U_y)$$

$$i[m_x - i m_y, U_z] - i[m_z, U_x - i U_y] = -2i\hbar (U_x - i U_y)$$

$$m_x U_y - U_y m_x - m_y U_x + U_x m_y = 2i\hbar U_z$$

$$i[m_x \pm i m_y, U_x + i U_y)$$

$$-i[m_x + i m_y, U_x - i U_y] = 4i\hbar U_z$$

$$[m_x - i m_y, U_x + i U_y] - [m_x + i m_y, U_x - i U_y]$$
$$= 4\hbar U_z$$

$[m_x - i m_y,$

$$[m_x + i m_y, U_z] - [m_z, U_x + i U_y] = -2\hbar (U_x + i U_y)$$

$$[m_x - i m_y, U_z] - [m_z, U_x - i U_y] = -2\hbar (U_x - i U_y)$$

$$[m_x - i m_y, [m_z, U_x + i U_y]] = (m_x - i m_y)\{m_z (U_x + i U_y)$$
$$-(U_x + i U_y) m_z\} - \{m_z (U_x + i U_y) - (U_x + i U_y) m_z\}(m_x - i m_y)$$
$$= \hbar [(m_x \pm i m_y), m_z](U_x + i U_y) + m_z [m_x - i m_y, U_x + i U_y]$$
$$- [m_x - i m_y, U_x + i U_y] m_z - [U_x + i U_y [m_x - i m_y, m_z]$$

$\cancel{zU_z}[xm_y - ym_x, U_z]$

$+\{ \qquad]$

$] = 2i\hbar\vec{r}\vec{U}$

$+\{$

$x(zp_x - \cancel{x}p_z) - y(yp_z - zp_y) = -r^2p_z + z\vec{r}\vec{p}$

y

$+z(xp_x+y$

$-r^2(\vec{p}\vec{U}) + \cancel{z}\vec{x}(\vec{r}\vec{p})\vec{U} = 2i\hbar\vec{r}\vec{U}$

$|||$

$-r^2(\vec{p}\vec{U}) + \vec{r}(\vec{r}\vec{p} - 2i\hbar)\vec{U} = 0$

$\cancel{r}\vec{p}\vec{U} + \frac{\vec{r}}{r}(\frac{\vec{r}}{r}\vec{p})\vec{U} = 2i\hbar\frac{\vec{r}}{r^2}\vec{U}$,

$ym_zU_y - U_yym_z = {}_{m_z}yU_y \cancel{+x}U_y$

$= U_ym_z \ldots xU_y$

$(ym_y + xm_x, U_z] - [ym_z \cancel{+xm_z}, U_y] - [xm_z, U_x] =$

$= \boxed{2}i\hbar(yU_x - xU_y)$

$y(zp_x - xp_z) + x(yp_z - zp_y)$

$\cancel{z}-[zm_z, U_z] - [m_z, yU_y] - [m_z, xU_x]$

$= 3i\hbar(yU_x - xU_y)$

$[xm_y - ym_x, U_z] = xm_y - ym_x,$

$[\vec{m}_x, \vec{r}\vec{U}] = -3i\hbar\,\vec{r}\times\vec{U}$

$-[\vec{m}_x, \vec{r}\times\vec{U}] = 2i\hbar\,\vec{r}\vec{U}$

$\{\vec{m}[\vec{m}, \vec{r}\vec{U}]\} = 2i\hbar\,\vec{r}\vec{U}$

$m_x (m_x \vec{\nabla} \vec{U} - \vec{\nabla} \vec{U} m_x) - (\quad) m_x$

$+ m_y (m_y \vec{\nabla} \vec{U} - \vec{\nabla} \vec{U} m_y)$

$+ m_z (m_z \vec{\nabla} \vec{U} - \vec{\nabla} \vec{U} m_z) = 6(i\kappa)^2 \vec{\nabla} \vec{U}$

~~$(m_x^2 + m_y^2 + m_z^2, \vec{\nabla} \vec{U})$~~

$m_x m_y U_z - m_x U_z m_y - m_x U_z m_y + m_y U_z m_x$

$i\hbar m_z U_z + m$

~~$m_x + \frac{1}{2}$~~ $(\vec{m} + \frac{\hbar}{2} \vec{\sigma})(U_0 + \vec{\alpha} \vec{U}) - (U_0 + \vec{\alpha} \vec{U})(\vec{m} + \frac{\hbar}{2} \vec{\sigma})$

$\vec{U} =$

$U_0 =$

$\vec{\sigma}_y (\sigma_z U_z) \quad \sigma_y (\sigma$

$\vec{\sigma}_x (\sigma_y U_y) \quad \vec{\sigma}_z (\sigma_y U_y)(\sigma_z U_y -$

$\vec{m} U_0 - U_0 \vec{m} = 0,$

$\vec{m}(\vec{\sigma} \vec{U}) - (\vec{\sigma} \vec{U}) \vec{m} + \frac{\hbar}{2} \{ \vec{\sigma}(\vec{\sigma} \vec{U}) - (\vec{\sigma} \vec{U}) \vec{\sigma} \}$

$\vec{m}_x U_x - U_x \vec{m}_x + \frac{\hbar}{2} i (\vec{\sigma} \times \vec{U}) = 0.$

~~$(U_z - U_y)$~~

$(z\,0$

$m_y U_x - U_x m_y + -$

$j\hbar = \rho_3\{\vec{\sigma}\vec{m} + \hbar\}$

$j\hbar(U_0 + \vec{\alpha}\vec{U}) - (U_0 + \vec{\alpha}\vec{U})j\hbar = 0,$

$\rho_3(\vec{\sigma}\vec{m}+\hbar)(U_0+\rho_1\vec{\sigma}\vec{U}) - (U_0+\rho_1\vec{\sigma}\vec{U})\rho_3(\vec{\sigma}\vec{m}+\hbar) = 0$

○ $\vec{m}U_0 - U_0\vec{m} = 0,$

$(\vec{\sigma}\vec{m})(\vec{\sigma}\vec{U}) + (\vec{\sigma}\vec{U})(\vec{\sigma}\vec{m}) + 2\hbar\vec{\sigma}\vec{U} = 0,$

$i\vec{\sigma}(\vec{m}\times\vec{U}) + i\vec{\sigma}(\vec{U}\times\vec{m}) + 2\hbar\vec{\sigma}\vec{U} = 0,$

○ $(\vec{m}\times\vec{U}) + (\vec{U}\times\vec{m}) = 2i\hbar\vec{U},$

$\{\vec{r}\times(\vec{m}\times\vec{U})\} + \{\vec{r}\times(\vec{U}\times\vec{m})\} = 2i\hbar(\vec{r}\times\vec{U})$

$\vec{r}(\vec{m}\times\vec{U}) + \vec{r}(\vec{U}\times\vec{m}) = 2i\hbar\vec{r}\vec{U},$

$y(m_xU_y - m_yU_x) - z(m_zU_x - m_xU_z) = 2i\hbar$

$+ y(U_xm_y - U_ym_x) - z(U_zm_x - U_xm_z) = 2i\hbar(\vec{r}\times\vec{U})_x$

$$\begin{array}{l|l}
m_zx - xm_z = i\hbar y\,; & m_x yU_y - yU_y m_x - i\hbar zU_y \\
m_xy - ym_x = i\hbar z. & \pm\, m_y yU_x + yU_x m_y \\
m_yx - xm_y = -i\hbar z & -\ m_z zU_x + zU_x m_z \\
 & + m_x zU_z - zU_z m_x + i\hbar yU_z
\end{array}$$

$= 2i\hbar(yU_z - zU_y)$

$m_x(\vec{r}\vec{U}) - (\vec{r}\vec{U})m_x$

$* - m_x(xU_x) + (xU_x)m_x$

$- m_y(yU_x) + \cdots\cdots = i\hbar(\cancel{yU}(\vec{r}\times\vec{U}))_x$

$- m_x(zU_x) + \cdots$

$$\boxed{\vec{m}(\vec{r}\vec{U}) - (\vec{r}\vec{U})\vec{m} = i\hbar(\vec{r}\times\vec{U})}$$

$x(m_yU_z - m_zU_y) + y(m_zU_x - m_xU_z) + z(m_xU_y - m_yU_x)$

$+ x(U_ym_z - U_zm_y) + y(U_zm_x - U_xm_z) + z(U_xm_y - U_ym_x)$

$= 2i\hbar(xU_x + yU_y + zU_z)$

$m_yxU_z - xU_zm_y + i\hbar zU_z + \cdots +$

$$\boxed{\vec{m}\cdot(\vec{r}\times\vec{U}) = \cancel{i\hbar\ \vec{r}}\vec{U}0}$$

$$\underbrace{\qquad}_{-(\vec{r}\times\vec{U})\vec{m}}$$

$$\boxed{\vec{m}\{\vec{m}(\vec{r}\times\vec{U}) - (\vec{r}\times\vec{U})\vec{m}\} - \{\vec{m}(\vec{r}\times\vec{U}) - (\vec{m}\times\vec{U})\vec{m}\}\vec{m}}$$

$$\boxed{= -i\hbar(i\hbar)(\vec{r}\times\vec{U}) = \hbar^2(\vec{r}\times\vec{U})}$$

$$\boxed{\vec{m}\cdot(\vec{m}_x(\vec{r}\vec{U}) - (\vec{r}\vec{U})\vec{m}_x) - (\qquad)\vec{m} = \cancel{\hbar^2}\,0\,\cancel{\vec{m}}\vec{r}\vec{U}}$$

initial state と final state の total angular momentum
が等しいためには

$$(\vec{m} + \tfrac{\hbar}{2}\vec{\sigma})(U_0 + \vec{\alpha}\vec{U}) - (U_0 + \vec{\alpha}\vec{U})(\vec{m} + \tfrac{\hbar}{2}\vec{\sigma}) = 0$$

or $\vec{m}U_0 - U_0\vec{m} = 0$, $\quad U_0 = U_0(r)$

$$(\vec{m} + \tfrac{\hbar}{2}\vec{\sigma})(\vec{\sigma}\vec{U}) - (\vec{\sigma}\vec{U})(\vec{m} + \tfrac{\hbar}{2}\vec{\sigma}) = 0$$

$(m_y + \tfrac{\hbar}{2}\sigma_y)(\sigma_z U_z)$

σ_x: $\quad m_x U_x - U_x m_x = 0$

$m_y U_x - U_{~~x~~} m_{~~x~~y} = -i\hbar U_z$ etc

$$(y p_z - z p_y) U_x - U_x (y p_z - z p_y) = 0$$

$$(z p_x - x p_z) U_x - U_x (z p_x - x p_z) = -i\hbar U_z$$

$~~m_x~~ U_x = x u_x$

$m_x x u_x - x u_x m_x = (m_x x - x m_x) u_x$

$~~+ u_x~~ \; x(m_x u_x - u_x m_x) = 0.$

○ $m_x U_x - U_x m_x = 0$

$m_y U_x - U_x m_y = -i\hbar U_z$

$m_z U_x - U_x m_z = i\hbar U_y$

$m_x U_y - U_y m_x = i\hbar U_z$

○ $m_y U_y - U_y m_y = ~~i\hbar~~ 0$

$m_z U_y - U_y m_z = -i\hbar U_x$

$m_x U_z - U_z m_x = -i\hbar U_y$

$m_y U_z - U_z m_y = i\hbar U_x$

$m_z U_z - U_z m_z = 0$

$m_z U_z - U_z m_z = 0.$

$(m_x - i m_y)$

$(m_x + i m_y) U_z - U_z (m_x + i m_y) = -\hbar (U_x + i U_y)$

$(m_x - i m_y) U_z - U_z (m_x - i m_y) = \hbar (U_x - i U_y)$

$m_x (U_x + i U_y) - (U_x + i U_y) m_x = -\hbar U_z$

$i m_y (U_x + i U_y) - (U_x + i U_y) i m_y = \hbar U_z$

$(m_x - i m_y)(U_x + i U_y) - (\quad)(m_x - i m_y)$

$= -2\hbar U_z$

$(m_x - i m_y)(m_x + i m_y) U_z$

$- ~~2~~ (m_x - i m_y) U_z (m_x + i m_y) + U_z (m_x + i m_y)(m_x - i m_y)$

$- (m_x + i m_y) U_z (m_x - i m_y)$

$= 2\hbar^2 U_z$

$z p_x$

$m_y(xU_y - yU_x) - (xU_y - yU_x)m_y$

$= -i\hbar\, z\, U_y - y\, \cancel{m}(-i\hbar U_z) = (+i\hbar)(yU_z - zU_y)$

一方、 $p_x(xU_y - yU_x) - (xU_y - yU_x)p_x$

$= (-i\hbar)U_y + x(p_x U_y - U_y p_x)$

$- y(p_x U_x - U_x p_x)$

$= \cancel{(-i\hbar)U_y} + x(p_x U_y - U_y p_x)$

$-\cancel{\hbar}(x p_y - y p_x)U_x - U_x(\quad) = i\hbar U_y$

$= -(x p_y U_x - U_x x p_y) + \cancel{i\hbar U_y}$

$= \cancel{-x(p}$

$= \cancel{p_z}\, x(p_x U_y - U_y p_x - p_y U_x + U_x p_y)$

$p_z(xU_y - yU_x) - (xU_y - yU_x)p_z$

$= x\{p_z U_y - U_y x p_z - y p_z U_x + U_x y p_z$

$= \cancel{z} p_x U_y - U_y z p_x - \cancel{y} z p_y U_x + U_x z p_y$

$= z(p_x U_y - U_y p_x - p_y U_x + U_x p_y)$

$\therefore (z p_x - x p_z)(xU_y - yU_x) - (xU_y - yU_x)(z p_x - x p_z) = 0$

$\therefore \boxed{yU_z - z\cancel{e}U_y = 0,}$

$x \cdot m_x U_x - U_x x m_x = 0$　　$-i\hbar z$

$y\, m_x U_y - U_y y m_x = i\hbar y U_z$　　$y m_x -$

$z\, m_x U_z - U_z z m_x = -i\hbar\, z U_y$

$m_x \vec{r}\vec{U} - \vec{r}\vec{U} m_x - i\hbar z U_y + i\hbar y U_z = i\hbar y U_z - i\hbar z U_y$

$$\boxed{\vec{m}_x(\vec{r}\vec{U}) - (\vec{r}\vec{U})\vec{m}_x = 0}$$

$U_x = x\, u$　　$(x^2+y^2+z^2)u = f(r),$

$U_y = y \cdot u$

$U_z = z \cdot u$　　$u = f(r)$

---

nucleus の initial state、final state の spin を それ I ≠ 0 ならば、

$$\vec{m}_x + \vec{I} = \vec{m}' + \vec{I}'$$

に対して、$\boxed{\vec{I} = \vec{I}' = 0}$ のときは、

$\vec{m} = \vec{m}'$、