No 1.

§1. Charge $-e'$, proper mass m' なる particle に対する Dirac の equation

$$\left(E + \frac{Ze \cdot e'}{r} + c\vec{\alpha}\vec{p} + \beta\mu'\right)\psi = 0 \qquad \mu = m'c^2$$

の solution を求める.

$$\psi^{(1)}_{E,j,u} = -i F_{E,j}(r)\, w^{(1)}_{j,u}(\theta,\phi)$$

$$\psi^{(2)}_{E,j,u} = -i F_{E,j}(r)\, w^{(2)}_{j,u}(\theta,\phi)$$

$$\psi^{(3)}_{E,j,u} = G_{E,j}(r)\, w^{(3)}_{j,u}(\theta,\phi)$$

$$\psi^{(4)}_{E,j,u} = G_{E,j}(r)\, w^{(4)}_{j,u}(\theta,\phi)$$

$$\begin{array}{lll}
 & j<0 & j>0 \\
w^{(1)}_{j,u} = & \xi(j,u)\,P^{u}_{-j} & \xi(j,u)(j+u)P^{u}_{j-1} \\
w^{(2)}_{j,u} = & \xi(j,u)\,P^{u+1}_{-j} & \xi(j,u)(-j+u+1)\,P^{u+1}_{j-1} \\
w^{(3)}_{j,u} = & \xi(j,u)(-j+u)\,P^{u}_{-j-1} & \xi(j,u)\,P^{u}_{j} \\
w^{(4)}_{j,u} = & \left.\xi(j,u)(j+u+1)\,P^{u+1}_{-j-1}\right\} & \left.\xi(j,u)\,P^{u+1}_{j}\right\}
\end{array}$$

$$\left.\begin{array}{l}
\left(A'^2 + \dfrac{\alpha' Z}{r}\right)F_{E,j} + \dfrac{dG_{E,j}}{dr} + \dfrac{1+j}{r}G_{E,j} = 0 \\
\left(B'^2 - \dfrac{\alpha' Z}{r}\right)G_{E,j} + \dfrac{dF_{E,j}}{dr} + \dfrac{1-j}{r}F_{E,j} = 0
\end{array}\right\}$$

$$A'^2 = \frac{2\pi}{h}\left(m'c + \frac{E}{c}\right) \qquad \alpha' = \frac{2\pi e e' Z}{hc}$$

$$B'^2 = \frac{2\pi}{h}\left(m'c - \frac{E}{c}\right)$$

or $A'^2 = \dfrac{1}{\Lambda}(\varkappa + \varepsilon)$ $\qquad B'^2 = \dfrac{1}{\Lambda}(\varkappa - \varepsilon)$

$$\Lambda = \frac{h}{2\pi m c} \qquad \varkappa = \frac{m'}{m} \qquad \varepsilon = \frac{E}{mc^2}$$

No 2.

$$P_j^u = \frac{(j-u)!}{2j\,j!}(\sin\theta)^u \frac{d^{j+u}}{(d\cos\theta)^{j+u}}(\cos^2\theta-1)^j e^{iu\phi}$$

$$\xi(j,u) = \frac{1}{2\sqrt{\pi}}\frac{1}{\sqrt{(|j|-u-1)!\,(|j|+u)!}}$$

$$\left.\begin{array}{l} F_{E,j}(r) = f_{E,j}\, r^{\gamma'-1} + \cdots\cdots \\ G_{E,j}(r) = g_{E,j}\, r^{\gamma'-1} + \cdots\cdots \end{array}\right\}$$

$E>0$

$$f_{E,j} = \mp N\sqrt{2(j+\gamma')(j\varepsilon-\gamma'\varkappa)} \qquad j \gtrless 0$$

$$g_{E,j} = N\sqrt{2(j-\gamma')(j\varepsilon-\gamma'\varkappa)}$$

$$N^2 = \frac{1}{8\pi mc^2}\left(\frac{2}{\Lambda}\right)^{2\gamma'+1}\frac{1}{\{\Gamma(2\gamma'+1)\}^2}\Phi(\varepsilon)$$

$$\Lambda = \frac{h}{2\pi mc^2}, \qquad \gamma' = \sqrt{j^2-\alpha'^2Z^2}$$

$$\Phi(\varepsilon) = e^{\pi b'}\eta^{2\gamma'-1}|\Gamma(\gamma'+ib')|^2$$

$$b' = \alpha' Z\frac{\varepsilon}{\eta} \qquad \eta = \sqrt{\varepsilon^2-\varkappa^2}$$

$E<0$

$$f_{E,j} = \overline{g}_{-E,-j}.$$

$$g_{E,j} = \overline{f}_{-E,-j}.$$

$$s_1' = \gamma_1'-1. \qquad \gamma_1' = \sqrt{1-\alpha'^2Z^2}$$

$$P_K = \frac{4\pi}{h}\,\frac{2\{\sqrt{2+s_1'}\sqrt{2+s_1}+\sqrt{s_1's_1}\}^2(\varepsilon'+\gamma_1'\varkappa)\Phi(\varepsilon')}{16\pi mc^2}$$

$$\times(\alpha Z)^{2\gamma_1+1}\left(\frac{2}{\Lambda}\right)^{2s_1}\left(\frac{4\pi mc}{h}\right)^6\frac{(\mathfrak{R}^2/4\pi)^2}{\{\Gamma(2\gamma_1+1)\}^2\,\Gamma(2\gamma_1'+1)}$$

$$\left.\begin{array}{l} e'=0 \\ m'=0 \end{array}\right\}\begin{array}{l} s_1'=0 \\ \gamma_1'=1 \end{array} = (1+\gamma_1)(\alpha Z)^{2\gamma_1+1}\frac{256\pi^5}{\{\Gamma(2\gamma_1+1)\}^2}\frac{m^5c^4}{h^7}\left(\frac{2}{\Lambda}\right)^{2\gamma_1-2}$$

$$\times\,\varepsilon'^2\,\mathfrak{R}^2$$

No 3.

$$\varepsilon' = \frac{\Delta E}{mc^2} + \gamma_1$$

$$\mathfrak{R} = \int r^{\gamma_1 - 1} \sqcup (r)\, d\vec{r}$$

$$= g_F \cdot \rho^{s_1} \int u_n v_m^* d\tau .$$

$\alpha Z << 1 \quad \gamma_1 \cong 1.$

$$P_K = 128 \pi^5 (\alpha Z)^3 \cdot \frac{m^5 c^4}{\hbar^7} \left(\frac{2}{\Lambda}\right)$$

$$\times \varepsilon'^2 \cdot g_F^2 \left(\frac{2\rho}{\alpha Z \Lambda}\right)^{2\gamma_1 - 2}$$

$\alpha Z << 1$, $\quad \left(\frac{2\rho}{\alpha Z \Lambda}\right)^{2\gamma_1 - 2} \simeq 1$

$g_F = 4 \times 10^{-50} \mathrm{cm}^3 \cdot \mathrm{erg}.$

$P_K \cong (\alpha Z)^3 \cdot \varepsilon'^2 \cdot 6 \times 10^{-5}.$

$\alpha Z = \frac{1}{10}.$ $\quad P_K \sim \varepsilon'^2 \cdot 6 \times 10^{-8}$

$\tau \sim \varepsilon'^{-2}\, 1{,}7 \cdot 10^{8}$ sec

1 year $\cong 3 \times 10^7$ $\qquad s_1 = \gamma - 1$

$$\sqrt{\frac{150}{200}} = \sqrt{\frac{3}{4}}$$

$\frac{200}{150}$

$0{,}75$

$\alpha Z = \frac{1}{5} = 0{,}2$

$\tau \sim \varepsilon'^2 \cdot 1{,}36 \cdot 10^7$ sec.

$\alpha Z = \frac{1}{2}$, $\quad 1 + \gamma_1 = 1{,}866 \qquad \left(Z = \frac{137}{2} = 68{,}5 \right.$ $\qquad \Lambda = \frac{\hbar}{mc} = \frac{10^{-27}}{0.9 \times 10^{-27} \times 3 \times 10^{10}}$

$\gamma_1 = \sqrt{1 - \frac{1}{4}}$ $\quad -s_1 = 2 - 2\gamma_1 = 0{,}268 \qquad \rho = 0.8 \times 10^{-13}$ $\qquad = \frac{1}{2{,}7} \cdot 10^{-10}$

$= \sqrt{0{,}75}$

$= 0{,}866$

$$P_K = 6 \cdot 10^{-5} \cdot \varepsilon'^2 \left(\frac{1}{2}\right)^3 \cdot \frac{1{,}866}{|\Gamma(2{,}732)|^2} \cdot \left(\frac{\alpha Z \Lambda}{2\rho}\right)^{0{,}268}$$

$$\frac{\alpha Z \Lambda}{2\rho} = \frac{1 \times 10^{-10}}{4 \times 0.8 \times 10^{-13} \times 2{,}7} = \frac{1}{8{,}64} \times 10^3$$

$$\left(\quad \right)^{0{,}268} = 1{,}9$$

$3{,}362$

$2 \times 0{,}866 = 1{,}732$

$3{,}2$
$\times 2{,}7$
$22{,}4$
$64.$
$8{,}624$

No. 4.

$\Gamma(2.732) = 1.59$

$[\Gamma(2.732)]^2 = 2.52$

$$\begin{array}{r} 0.20140 \\ 2 \\ \hline 0.40280 \end{array} \qquad \begin{array}{r} 1.866 \\ 1.9 \\ \hline \end{array}$$

6×1.8

$$P_K = \frac{\overset{3}{\cancel{6}} \times 1.9 \times 1.866}{\underset{10}{\cancel{2.52 \times 8}}} \varepsilon'^2 \times 10^{-5}$$

$\cong \varepsilon'^2 \times 10^{-5}$!!! for $a2 = \frac{1}{2}$.

$\tau \simeq 10^5 \mathrm{sec}$

$$\underline{\underline{1\ \mathrm{day} \neq 0.864 \times 10^5 \mathrm{sec}}}.$$

No 4.

$$128\pi^5 \cdot \frac{m^5c^4}{h^7} \cdot 16\times10^{-100} = 128\pi^5 \times 9^{5} \cdot \text{[illegible]} \cdot (6.65)^{-7} \cdot 16 \cdot 10^{-11}.$$

$m = 9\times10^{-28}$　　$m^5 = 9^5 \times 10^{-140}$

$c = 3\times10^{10}$　　$c^4 = 3^4 \times 10^{40}$

$h = 6.65\times10^{-27}$　　$h^{-7} = (6.65)^{-7} \times 10^{189}$

$$\frac{}{9^5 \times 3^4 \times (6.65)^{-7} \times 10^{89}}$$

$$\begin{array}{r} 27 \\ 7 \\ \hline 189 \end{array}$$

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log

128	2.10721
π^5	2.48575
9^7	6.67968
16	1.20412
	12.47676
	−11.
	1.47676
	−5.68974
	$\bar{5}$.78602

6×10^{-5}.

π : 0.497149 × 5 = 2.48575

9 : 0.95424 × 7 = 6.67968

6.65 0.81282 × 7 = 5.68974

3600

3.6×10^{3} sec

$$\begin{array}{r} 3.6 \\ 24 \\ \hline 144 \\ 72 \\ \hline 86.4\times10^{3} \end{array}$$

$$\begin{array}{r} 365 \\ 86.4 \\ \hline 1460 \\ 2190 \\ 2920 \\ \hline 31536.0 \end{array}$$

log 0.75 $\bar{1}$.87506,

$$\begin{array}{r} 2)\,-0.12494 \\ \hline -0.06247 \end{array}$$

$\bar{1}$.93753

0.866

$$\begin{array}{r} 3)\,-0.12494 \\ \hline -0.04165 \end{array}$$

$\bar{1}$.95835

0.9089

$$\begin{array}{r} 0.268 \\ 3 \\ \hline 0.804 \\ 0.526 \\ \hline 0.278 \end{array}$$

8.64

$$\begin{array}{r} 1.93651 \\ \times\,0.268 \\ \hline 0.5266 \end{array}$$

3.362

log 1.93651　0.29336

log 0.268　$\bar{1}$.42813

$\bar{1}$.72149

No. 6.

$$P_{K,abs} = 256\pi^5 \cdot \frac{m^5c^4}{h^7} \cdot g^2 \cdot$$

$$\times \frac{1+\gamma}{2} \cdot (\alpha Z)^3 \cdot \left(\frac{4\pi m c a_N}{\alpha Z h \cdot a_N}\right)^{2\gamma-2} \frac{1}{\Gamma(2\gamma+1)}$$

$$\times \left(\Delta w + \gamma\right)^2.$$

$$= 1.2 \times 10^{-4}$$

$$\times \frac{1+\gamma}{2} (\alpha Z)^3 \left(\frac{4\pi\, m c\, a_N}{\alpha Z h \cdot a_N}\right)^{2\gamma-2} \frac{1}{\Gamma(2\gamma+1)}$$

$$\times \left(\Delta w + \gamma\right)^2$$

256	2,40824
π^5	2,48575
$10^{200} \times m^5c^4g^2$	6,67968
9^7	
16	1,20412
	12,77779
	5,71268
	7,06511
	−11.
	$\overline{4}$,06511

1.16×10^{-4}
$\parallel$
$256\pi^5 \cdot \frac{m^5c^4}{h^7} \cdot g^2$

$\log_{10}\pi = 0.49715$
5
2,48575

$m = 9 \times 10^{-28}$
$c = 3 \times 10^{10}$
$g = 4 \times 10^{-50}$

$\log_{10} 9 = 0,95424$
7
6,67968

$h = 6.55 \times 10^{-27}$

27
7
189

$m^5 = 9^5 \times 10^{-140}$
$c^4 = 9^2 \times 10^{40}$
$g^2 = 16 \times 10^{-100}$
$m^5c^4g^2 = 9^7 \times 16 \times 10^{-200}$

$\log_{10} 16 = 1,20412$

$\log_{10} 6.55 = 0,81624$
7
$\overline{5}$,71268

$\alpha Z = \frac{1}{10}$, $(Z \sim 13.7)$ $\gamma = 1$. $\Gamma(2\gamma+1) = 2$.

$$P_{K,abs} = 1.2 \times 10^{-4} \times 10^{-3} \times \frac{1}{2} \times (\Delta w + \gamma)^2$$

$$= 0.6 \times 10^{-7} \times (\Delta w + \gamma)^2$$

$864 \overline{)1700}$ 0.2

0.166

$$\tau = \frac{0.17 \times 10^8}{(\Delta w + 1)^2} \text{sec} \cong \frac{200}{(\Delta w + 1)^2} \text{day}$$

$1 \text{ day} = 0.864 \times 10^5$

$Z = 1$;　　$137^3 = 2.57 \times 10^6$

$$P_{K,abs} = \frac{1.2}{2.57 \times 2} \times 10^{-4} \times 10^{-6} \times (\Delta w + 1)^2$$

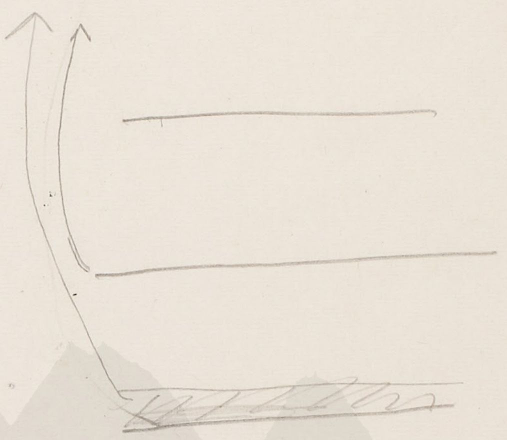
$$\tau = \frac{\overset{4.3}{\cancel{2.57}}}{0.6} \times 10^{10} \times \frac{1}{(\Delta w+1)^2} \text{ sec}$$

$$5 \quad \frac{4.3 \times 10^{10}}{0.864 \times 10^5} = \frac{5 \times 10^5}{(\Delta w + 1)^2} \text{ day} = \frac{1370}{(\Delta w+1)^2} \text{ year}$$

$$\begin{array}{r} 1.37 \\ 365 \overline{)500} \\ 365 \\ \hline 1350 \\ 1095 \\ \hline 2550 \end{array}$$

137×10^3

$$\boxed{-1 < \Delta w < 1}$$



$\alpha Z = 0.2$　　$(Z = 27.5)$

$$\tau \cong \frac{25}{(\Delta w+1)^2} \text{ day}$$

$\Delta w = 2mc^2$　　$\tau \cong 2.5$ day

$\Delta w = mc^2$　　$\tau \cong 6$ day

$\Delta w = 0$　　$\tau \cong 25$ day

αZ　0.5

No. 8.

$\alpha Z = 0.5$ $\quad \gamma = 0.866$

$$P_{K,abs} = 1.2 \times 10^{-4} \times \frac{1.866}{2} \left(\frac{1}{2}\right)^3 \cdot \frac{1}{\Gamma(2.732)} (\Delta w + \gamma)^2$$

$$\times \left(\frac{8\pi \times 0.9 \times 10^{-29} \times 3 \times 10^{10}}{6.55 \times 10^{-27} \; (\times 0.8 \times 10^{-12})} \right)^{-0.268}$$

$$= \frac{1.2 \times 1.866 \times 10^{-5}}{1.6 \times 1.5824} \left(\frac{6.55 \times 10^2}{17.28} \right)^{0.268}$$

$\parallel$

2.65

6.55×10^2 $\quad$ 2.81624
17.28 $\quad$ 1.23704
log $\quad$ 1.57920
log $\quad$ 0.19838
log 0.268 $\quad$ $\bar{1}.42813$
$\bar{1}.62651$
0.268 log( ) $\quad$ 0.4232
2.650

5.76
3
17.28

2.65
1.2
530
265
3.180

1.866
3.18
14928
1866
5598
5.93388

1.58
1.6
948
158
2.528

$$= \frac{1.2 \times 1.866 \times 10^{-5} \times 2.65}{1.6 \times 1.58}$$

$$\cong 2 \times 10^{-5}$$

$$\tau = \frac{0.5 \times 10^5 \; \text{sec}}{(\Delta w + 0.866)^2} = \frac{14 \; \text{hour}}{(\Delta w + 0.866)^2}$$

1 hour $= 0.36 \times 10^4$

$$36 \overline{)50}$$
14
36
140

0.43
5.934) 2.5280
23736
15440

12
36) 43
36
70

$$P_{\beta+} = \frac{1+\gamma}{2} \frac{256\pi^4}{\{\Gamma(\underset{2\gamma+1}{\cancel{2s+3}})\}^2} \cdot \frac{m^5c^4}{\cdot\, \hbar^7} \cdot \left(\frac{2}{\Lambda}\right)^{\overset{2\gamma-2}{\cancel{2s}}} \cdot \mathcal{R}_+^2 \cdot \int F_+(\varepsilon_+)\,d\varepsilon_+$$

$$P_{K.ab.} = \frac{1+\gamma}{2} \cdot (\alpha Z)^{\overset{2\gamma+1}{\cancel{2s+3}}} \frac{256\pi^5}{\Gamma(\underset{2\gamma+1}{\cancel{2s+3}})} \frac{m^5c^4}{\hbar^7} \left(\frac{2}{\Lambda}\right)^{\overset{2\gamma-2}{\cancel{2s}}} \cdot \mathcal{R}_+^2 \cdot (\Delta w + \gamma)^2$$

$$F_+(\varepsilon_+) = \eta_+^{2\gamma-1} \cdot e^{-\pi b_+} \cdot |\Gamma(\gamma + i b_+)|^2 \cdot \varepsilon_+ \cdot (\Delta w - \varepsilon_+)^2$$

$$\sigma = \frac{P_{\beta+}}{P_{K.ab.}} = \frac{\int F_+(\varepsilon_+)\,d\varepsilon_+}{\pi(\alpha Z)^{\frac{\cancel{2s+3}}{2\gamma+1}}\ \Gamma(\underset{2\gamma+1}{\cancel{2s+3}}) \cdot (\Delta w + \gamma)^2}$$

ρ^s

$$\mathcal{R}_+ = \int r^{\text{[illegible]}\gamma-1}\, U(r)\, d\vec{r} \quad \fallingdotseq \quad g \cdot a_N^s$$

$$P_{pair} = 4\gamma^2 \frac{256\pi^4}{\{\Gamma(\underset{2\gamma+1}{\cancel{2s+3}})\}^4} \frac{m^5c^4}{\hbar^7} \cdot \left(\frac{2}{\Lambda}\right)^{\overset{4\gamma-4}{\cancel{4s}}} \cdot \mathcal{R}_p^2 \cdot \int F_p(\varepsilon_+)\,d\varepsilon_+$$

$$P_{K.em.} = 4(\alpha Z)^{\frac{\cancel{2s+3}}{2\gamma+1}} \frac{256\pi^5}{\{\Gamma(\underset{2\gamma+1}{\cancel{2s+3}})\}^3} \cdot \frac{m^5c^4}{\hbar^7} \cdot \left(\frac{2}{\Lambda}\right)^{\overset{4\gamma-4}{\cancel{4s}}} \cdot \mathcal{R}_p^2 \cdot \Phi(\varepsilon'_-) \cdot (\varepsilon'_- + \gamma)$$

$$\varepsilon'_- = \Delta w + \gamma$$

$$F_p(\varepsilon_+) = \eta_+^{2\gamma-1} e^{-\pi b_+} |\Gamma(\gamma + i b_+)|^2 \cdot \eta_-^{2\gamma-1} e^{\pi b_-} |\Gamma(\gamma + i b_-)|^2 \cdot (\varepsilon_+\varepsilon_- - \gamma^2)$$

$$(\varepsilon_- = \Delta w - \varepsilon_+)$$

$$\mathcal{R}_p = \int r^{2\gamma-2}\, V(r)\, d\vec{r}$$

$$\rho = \frac{P_{pair}}{P_{K.em}} = \frac{\gamma^2 \cdot \int F_p(\varepsilon_+)\,d\varepsilon_+}{\pi(\alpha Z)^{\frac{\cancel{2s+3}}{2\gamma+1}}\ \Gamma(\underset{2\gamma+1}{\cancel{2s+3}}) \cdot \Phi(\varepsilon'_-)(\varepsilon'_- + \gamma)}$$

electron

$$\alpha Z \, C^{(1)}_{\gamma'-1} - (\gamma'-1+1+j) C^{(2)}_{\gamma'-1} = 0$$

$$(\gamma'-1+1-j) C^{(1)}_{\gamma'-1} + \alpha Z \, C^{(2)}_{\gamma'-1} = 0$$

$$C^{(1)}_{\gamma'-1} : C^{(2)}_{\gamma'-1} = \gamma'+j : \alpha Z$$

$$\gamma' = \sqrt{j^2 - \alpha^2 Z^2}$$

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neutrino

$C^{(1)} : C^{(2)} = 1 : 0$ for $j > 0$

$C^{(1)} : C^{(2)} = 0 : 1$ for $j < 0$

$j>0$ $\quad |(\gamma'+j) U_0 + i\alpha Z \, U'|^2 = (\gamma'+j)^2 U_0^2 + \alpha^2 Z^2 U'^2$

$j<0$ $\quad |\alpha Z \, U_0 - i(\gamma'+j) U'|^2 = \alpha^2 Z^2 U_0^2 + (\gamma'+j)^2 U'^2$

$\left. \begin{array}{l} j=1 \\ j=-1 \end{array} \right\}$ $\quad \{(\gamma'+1)^2 + \alpha^2 Z^2\} U_0^2 + \{(\gamma'-1)^2 + \alpha^2 Z^2\} U'^2$

$$1-\alpha^2 Z^2 + 2\sqrt{1-\alpha^2 Z^2} + 1 + \alpha^2 Z^2 = 2(1+\gamma')$$

$$2(1-\gamma')$$