$$\alpha^2 = \frac{2\mu}{\hbar^2} = \frac{2}{\hbar^2}\frac{M m}{M+m} \approx \frac{2m}{\hbar^2} = 2 \cdot 10^{54} \cdot 1.6 \cdot 10^{-24} \approx 3 \cdot 10^{30}$$

$$K = \alpha^2 (Z-1) e^2 = 3 \cdot 10^{30} \cdot 2.3 \cdot 10^{-19} (Z-1) = 0.7 \cdot 10^{12} (Z-1)$$

Energy ヲ MV 単位デ表ハセバ

$$E_0 = \alpha^2 E \cdot 10^6 \cdot 1.6 \cdot 10^{-12} = 5 \cdot 10^{24} E$$

$$R = \sqrt{E_0 + D_0}\, r$$

$$R_1 = \frac{K}{\sqrt{E_0 + D_0}} \approx 0.3 \frac{Z}{\sqrt{E+D}} \qquad E, D, \text{ in MV}$$

$$\frac{R_1}{R} = \frac{K}{(E_0 + D_0) r} = 0.14 \cdot 10^{-12} \frac{Z}{(E+D) r}$$

$$\approx 0.3 \frac{Z}{E+D} \quad (r = 5 \cdot 10^{-13}\,\text{cm})$$

$$\beta^2 = \frac{1}{R_1}, \quad \xi = R - R_1,$$

$$b = \sqrt{J_0}\, a, \quad \sqrt{E_0}\, a = \varepsilon,$$

$\underline{0 < E - D \ll 0.2\,Z}$ $\quad (E, D \text{ in } MV)$, $\quad \underline{E_0 + J_{10} + \lambda_2 J_0 < 0}$

$$\Phi_{dis} = 4\pi \frac{\left(k_1 \cos k_1 a \sinh k_2 a - k_2 \sin k_1 a \cosh k_2 a\right)^2}{\left| \lambda_1 k_2 P_1 B_2 - \lambda_2 k_1 P_2 B_1 \right|^2} \frac{e^{-2\psi(a)}}{k}$$

$$\left.\begin{matrix}\lambda_1 \\ \lambda_2\end{matrix}\right\} = -\frac{D_0 + J_{10} - J_{20}}{2J_0} \pm \sqrt{\left(\frac{D_0 + J_{10} - J_{20}}{2J_0}\right)^2 + 1}$$

$$k_1 = \sqrt{E_0 + J_{10} + \lambda_1 J_0} \qquad k = \sqrt{E_0}$$

$$k_2 = \sqrt{-(E_0 + J_{10} + \lambda_2 J_0)} \qquad k' = \sqrt{E_0 - D_0}$$

$$B_1 = \cos k_1 a - i\frac{k}{k_1} \sin k_1 a$$

$$B_2 = \cosh k_2 a - i\frac{k}{k_2} \sinh k_2 a$$

$$P_1 = \left(-G(a)\right)^{\frac{1}{4}} \sin k_1 a + k_1 \left(-G(a)\right)^{-\frac{1}{4}} \cos k_1 a$$

$$P_2 = \left(-G(a)\right)^{\frac{1}{4}} \sin k_2 a + k_2 \left(-G(a)\right)^{-\frac{1}{4}} \cos k_2 a$$

$$G(a) = k'^2 - \frac{K_0'}{a} = E_0 - D_0 - \frac{0.7 \cdot 10^{12}(Z-1)}{a}$$

$$\psi(a) = \int_a^{r_2} \sqrt{-G(r)}\, dr = \frac{K_0'}{k'} \cos^{-1} k' \sqrt{\frac{a}{K_0'}} - \sqrt{K_0' a - k'^2 a^2}$$

$$\approx \frac{\pi K_0'}{2k'} - 2\sqrt{K_0' a}$$

$$K_0' = 0.7 \cdot 10^{12}(Z-1)$$

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$0 < E - D \ll 0.2\,Z$ ノ假定ニ $l=0$ ノ part ノミ計算スレバ

$$\Phi_{dis} = \frac{4\pi}{k} \frac{(k_2 \tan k_1 a - k_1 \tanh k_2 a)^2}{(\lambda_1 k_2 \Gamma_1 - \lambda_2 k_1 \Gamma_2)^2 + k^2(\lambda_2 \tan k_1 a\, \Gamma_2 - \lambda_1 \tanh k_2 a\, \Gamma_1)^2} e^{-2\psi(a)}$$

$$\Gamma_1 = G_0^{\frac{1}{4}} \tan k_1 a + k_1 G_0^{-\frac{1}{4}}$$
$$\Gamma_2 = G_0^{\frac{1}{4}} \tanh k_2 a + k_2 G_0^{-\frac{1}{4}}$$

$k_2 = \sqrt{-(E_0 + J_{10} + \lambda_2 J_0)}$ ハ $E_0 + J_{10} + \lambda_2 J_0 > 0$ ノ場合ニハ imaginary ニナルガ 上ノ式ハソノマヽアテハマル.

Resonance ガアルタメニハ

$$\lambda_1 k_2 \Gamma_1 - \lambda_2 k_1 \Gamma_2 = 0.$$
$$\lambda_2 \tan k_1 a\, \Gamma_2 - \lambda_1 \tanh k_2 a\, \Gamma_1 = 0.$$

$\lambda_1 \Gamma_1$, 又ハ, $\lambda_2 \Gamma_2$ ノ中 zero デナイモノガアルハズデアルカラ 上ノ二ツノ condition ガ両立スルタメニハ

$$\begin{vmatrix} k_2 & k_1 \\ \tanh k_2 & \tan k_1 \end{vmatrix} = 0.$$

或ハ

$$k_2 \tan k_1 - k_1 \tanh k_2 = 0$$

之ハ cross-section ノ中ノ分子ガ zero ニナルコトデアルカラ Φ_{dis} ハ結局 resonance ハ持タナイコトニナル

k ガ k_1, $|k_2|$ ニ比シ小サイ場合ニ限リ resonance ガオコリ, ソノ condition ハ

$$\lambda_1 \frac{\tan k_1 a}{k_1} - \lambda_2 \frac{\tanh k_2 a}{k_2} = -(\lambda_1 - \lambda_2) G_0^{-\frac{1}{2}}$$

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Scattering ノ Cross-section

$E > 0, \quad E - D \ll a_1^2 Z$ ナラバ ($E-D<0$ デモヨイ)

$$\Phi_{scatt.} = \frac{4\pi}{k^2} \frac{\left[(\lambda_1\Gamma_1 k_2 - \lambda_2\Gamma_2 k_1)\tan ka - (\lambda_1\Gamma_1 \tanh k_2 a - \lambda_2\Gamma_2 \tan k_1 a)k\right]^2}{(\lambda_1\Gamma_1 k_2 - \lambda_2\Gamma_2 k_1)^2 + k^2(\lambda_1\Gamma_1 \tanh k_2 a - \lambda_2\Gamma_2 \tan k_1 a)^2} \cos^2 ka$$

Slow ~~neutron~~ neutron デハ $\tan ka \approx ka$ ト見ラレルカラ

$$\Phi_{scatt} = 4\pi a^2 \left(1 - \frac{\lambda_1\Gamma_1 \tanh k_2 a - \lambda_2\Gamma_2 \tan k_1 a}{\lambda_1\Gamma_1 k_2 a - \lambda_2\Gamma_2 k_1 a}\right)^2$$

故ニ. $\lambda_1\Gamma_1 k_2 - \lambda_2\Gamma_2 k_1 = 0$ デ且. $\lambda_1\Gamma_1 \tanh k_2 a - \lambda_2\Gamma_2 \tan k_1 a \neq 0$

ナラバ resonance ヲ示ス.

$\frac{D_0 + J_{10} - J_{20}}{2J_0} = \varepsilon$ トオケバ

$$\left.\begin{matrix}\lambda_1\\ \lambda_2\end{matrix}\right\} = -\varepsilon \pm \sqrt{1+\varepsilon^2}$$

デアルカラ

~~$k_2 \tan k_1 a - k_1 \tanh k_2 a$~~

$$k_1 \tanh k_2 a - k_2 \tan k_1 a = \Gamma_{(-)}$$

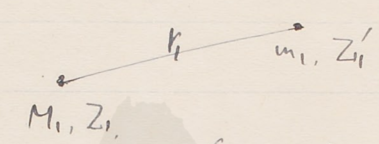
$$k_1 \tanh k_2 a + k_2 \tan k_1 a = \Gamma_{(+)}$$

トオケバ

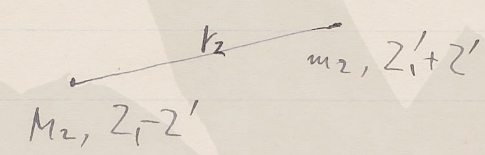
$$\lambda_1\Gamma_1 k_2 - \lambda_2\Gamma_2 k_1 = \left(\varepsilon\Gamma_{(-)} + \sqrt{1+\varepsilon^2}\,\Gamma_{(+)}\right) G_0^{\frac{1}{4}} + 2k_1 k_2 \sqrt{1+\varepsilon^2}\, G_0^{-\frac{1}{4}}$$

$$\lambda_1\Gamma_1 \tanh k_2 a - \lambda_2\Gamma_2 \tan k_1 a = \left(-\varepsilon\Gamma_{(-)} + \sqrt{1+\varepsilon^2}\,\Gamma_{(+)}\right) G_0^{-\frac{1}{4}} + 2\tan k_1 a \tanh k_2 a \sqrt{1+\varepsilon^2}\, G_0^{\frac{1}{4}}$$

$\begin{pmatrix}1\\0\end{pmatrix}$ mass M_1, charge Z_1e ノ Nucleus ト, mass m_1
charge $Z_1'e$ ノ particle トノ system (Energy ハ E')



$\begin{pmatrix}0\\1\end{pmatrix}$ mass M_2, charge $(Z_1-Z_1')e$ ノ Nucleus ト m_2, $(Z_1+Z_1')e$
ノ particle トノ system (Energy ハ $E'+D$)



$M_1+m_1 = M_2+m_2+D$ デアルトスレバ D ハ mass defect ヲ表ハス

Hamiltonian ハ

$$H = \begin{pmatrix} \frac{1}{2M_1}(p_{M_1})^2 + \frac{1}{2m_1}(p_{m_1})^2 & 0 \\ 0 & \frac{1}{2M_2}(p_{M_2})^2 + \frac{1}{2m_2}(p_{m_2})^2 \end{pmatrix}$$

$$+ \begin{pmatrix} J_{11}(r_1) & J_{12} \\ J_{21} & J_{22}(r_2) \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & -D \end{pmatrix}$$

- - - - (1)

トナル.
$J_{11}(r_1)$ ハ (M_1, m_1) ノ間ノ ordinary interaction ニヨル potential デ
$J_{22}(r_2)$ ハ $(M_2\ m_2)$ ノ間ノ ソレデアル.

J_{12}, J_{21} ハ Exchange type ノ force ニヨル potential デアルカ r_1, r_2 ダケノ function デアルカドウカハ 解ラナイ.

Eigenfunction ヲ $\begin{pmatrix} u_1' \\ u_2' \end{pmatrix}$ ト書キ. wave equation ヲ二ツノ聯立方程式ノ形ニ書ケバ,

$$\left.\begin{aligned} &\frac{1}{2M_1}\Delta_{M_1} u_1' + \frac{1}{2m_1}\Delta_{m_1} u_1' + \frac{1}{\hbar^2}\left\{(E'-J_{11})u_1' - J_{12}u_2'\right\} = 0 \\ &\frac{1}{2M_2}\Delta_{M_2} u_2' + \frac{1}{2m_2}\Delta_{m_2} u_2' + \frac{1}{\hbar^2}\left\{(E'+D) - J_{22}\right\}u_2' - J_{21}u_1'\} = 0 \end{aligned}\right\} \quad (2)$$

M_1 ノ座標ヲ $(\xi_{M_1}, \eta_{M_1}, \zeta_{M_1})$ ト書ケバ, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ノ state ト $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ノ state ト重心ヲ共通ニモタネバナラヌコトカラ

$$\left.\begin{aligned} &\frac{M_1\xi_{M_1} + m_1\xi_{m_1}}{M_1+m_1} = \frac{M_2\xi_{M_2} + m_2\xi_{m_2}}{M_2+m_2} = X \\ &\frac{M_1\eta_{M_1} + m_1\eta_{m_1}}{M_1+m_1} = \cdots = Y \\ &\text{—— —— —— ——} \end{aligned}\right\}$$

トオクコトガ出来ル. (XYZ) ハ重心ノ座標デアル.

$$\left.\begin{aligned} &\xi_{m_1} - \xi_{M_1} = x_1, \\ &\text{—— ——} \\ &\text{—— ——} \end{aligned}\right\} \qquad \left.\begin{aligned} &\xi_{m_2} - \xi_{M_2} = x_2 \\ &\text{—— ——} \\ &\text{—— ——} \end{aligned}\right\}$$

トオケバ

$$\frac{1}{M_1}\Delta_{M_1} + \frac{1}{m_1}\Delta_{m_1} = \frac{1}{M_1+m_1}\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}+\frac{\partial^2}{\partial Z^2}\right) + \frac{1}{\mu_1}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial y_1^2}+\frac{\partial^2}{\partial z_1^2}\right),$$
$$\left(\mu_1 = \frac{M_1 m_1}{M_1+m_1}\right)$$

デアルカラ

$$\begin{pmatrix} u_1' \\ u_2' \end{pmatrix} = u''(X, Y, Z)\begin{pmatrix} u_1(x_1, y_1, z_1) \\ u_2(x_2, y_2, z_2) \end{pmatrix},$$

$$E' = E + E''$$
(E'' ハ重心ノハコブ Energy)

トオイテ Separation ヲヤレバ
重心ノ wave equation トシテ

$$\frac{\hbar^2}{2M}\Delta_X u'' + E'' u'' = 0 . \qquad M = M_1+m_1 \approx M_2+m_2$$

u_1, u_2 ヲキメルモノトシテ

$$\left.\begin{aligned} &\Delta_{x_1} u_1 + \frac{2\mu_1}{\hbar^2}\Big[(E - J_{11})u_1 - J_{12}u_2\Big] = 0 . \\ &\Delta_{x_2} u_2 + \frac{2\mu_2}{\hbar^2}\Big[(E + D - J_{22})u_2 - J_{21}u_1\Big] = 0 . \end{aligned}\right\} (3)$$

$$\mu_1 = \frac{M_1 m_1}{M_1+m_1}, \quad \mu_2 = \frac{M_2 m_2}{M_2+m_2}$$

ヲ得ル.

$\begin{pmatrix}1\\0\end{pmatrix}$ ト $\begin{pmatrix}0\\1\end{pmatrix}$ ノ二ツノ system ノ移リカハリノトキニ重心ハ relative ニ system 全体ガズレルト考ヘ. ~~二ツノ part~~ Nucleus ト particle ノ相対的位置ハ変化シナイト考ヘレバ

$$x_1 = x_2, \quad y_1 = y_2, \quad z_1 = z_2$$

トオイテヨイ,

シカシ、コノ假定ハ、u_1 ガ neutron デ u_2 ガ proton デアルトキトカ、或ハ、wave function ガ spherical symetry デアルトキトカハ問題ニナラナイ。

結局、(3) ニ於テ $\Delta_{x_1} \equiv \Delta_{x_2}$.

$$\left\{\begin{aligned} u_1 &= \frac{f_1(r)}{r} Y_{lm} = \frac{f_1(r)}{r}\sqrt{\frac{(l-m)!}{(l+m)!}\frac{2l+1}{4\pi}}\, P_l^m(\cos\theta)\, e^{im\varphi} \\ u_2 &= \frac{g_1(r)}{r} Y_{lm} \end{aligned}\right.$$

ト assume スレバ (J_{12}, J_{21} ヲ r_1, r_2 ダケノ、從ッテ、r ダケノ function ト見テ)

$$\left.\begin{aligned} &\frac{d^2 f_1}{dr^2} + \left\{\alpha_1^2 (E - J_{11}) - \frac{l(l+1)}{r^2}\right\} f_1 - \alpha_1^2 J_{12}\, g_1 = 0 \\ &\frac{d^2 g_1}{dr^2} + \left\{\alpha_2^2 (E + D - J_{22}) - \frac{l(l+1)}{r^2}\right\} g_1 - \alpha_2^2 J_{21}\, f_1 = 0 \end{aligned}\right\} (4).$$

$$\left(\alpha_1^2 = \frac{2\mu_1}{\hbar^2},\ \alpha_2^2 = \frac{2\mu_2}{\hbar^2}\right)$$

コレデ問題ハ、適當ニ $\begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}$ ノ形ヲキメテ、(4) ヲ解クコトデアル。

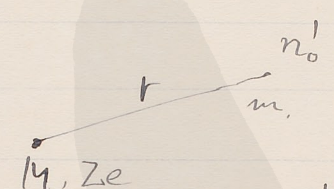
マヅ、neutron ~~produced~~ ニヨル proton-disintegration ヲ計算シテ見ル。

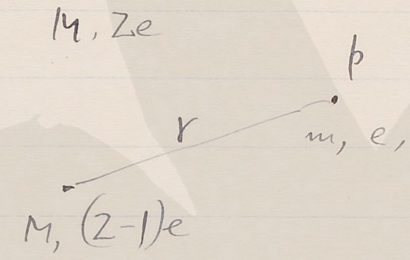
コノ場合ニハ、

$M_1 \approx M_2 \approx M$, $m_1 \approx m_2 \doteq m$

$Z_1 = Z$ (Nucleus ノ Atomic No.), $Z_1' = 0$, $Z' = 1$.

デアル

$\begin{pmatrix}1\\0\end{pmatrix} \longrightarrow$ 

$\begin{pmatrix}0\\1\end{pmatrix} \longrightarrow$ 

$$\alpha_1^2 = \cancel{\ } = \alpha_2^2 = \frac{2\mu}{\hbar^2} \quad \left(\mu = \frac{Mm}{M+m}\right)$$

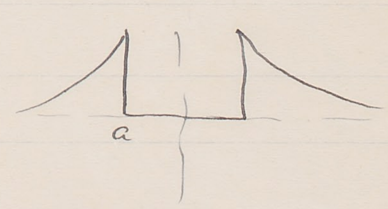
Potential トシテハ、~~neu~~ exchange type ノ force ニヨルモノト、Coulomb field トヲ、主トシテ考ヘル。

即チ、

一般ニハ
0デナイ

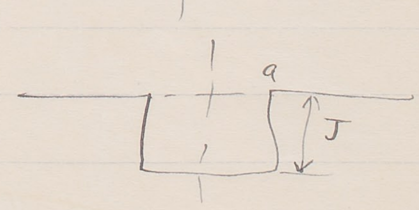
$$J_{11} = 0.$$

$$J_{22} = \begin{cases} 0 & r<a \\ \dfrac{(Z-1)e^2}{r} & r>a \end{cases}$$



一般ニ ～
$J_{12} = J_{21}$

$$J_{12} = J_{21} = \begin{cases} -J & r<a \\ 0 & r>a \end{cases}$$



$J_{12} = J_{21}$ トオイタノハ、proton disintegration ヲスル場合ハ Z ガ相当大キクテ $\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}0\\1\end{pmatrix}$ ノトキモ $\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}1\\0\end{pmatrix}$ ノトキモ、Nucleus 内ノ

核子ニハ大シテ変リガナイト見ラレルカラデアル

以上ノ potential ヲ (4) ニ入レルト、$(\alpha_1^2 = \alpha_2^2 = \alpha^2$ トオケバ$)$

<u>$r<a$</u> デハ

$$\left.\begin{aligned} f_1'' + \left[\alpha^2 E - \frac{l(l+1)}{r^2}\right] f_1 + \alpha^2 J g_1 &= 0. \\ g_1'' + \left[\alpha^2 (E+D) - \frac{l(l+1)}{r^2}\right] g_1 + \alpha^2 J f_1 &= 0 \end{aligned}\right\} \quad (5)$$

<u>$r>a$</u> デハ

$$\left.\begin{aligned} f_1'' + \left[\alpha^2 E - \frac{l(l+1)}{r^2}\right] f_1 &= 0. \\ g_1'' + \left[\alpha^2 \left(E+D - \frac{(Z-1)e^2}{r}\right) - \frac{l(l+1)}{r^2}\right] g_1 &= 0. \end{aligned}\right\} \quad (6)$$

Energy E ハアマリ大キクナク、高サ 10^6 volts 位ノ処デ $l=0$ ノ wave function ノミヲ計算シテ見ル。

J ニ対シテハ E ヲ省略スルコトニスレバ

(5) ハ $(l=0$ トオキ$)$

$$\left.\begin{aligned} f_1'' + J_0 g_1 &= 0. \\ g_1'' + J_0 f_1 &= 0. \end{aligned}\right\} \quad (5')$$

(6) ハ

$$\left.\begin{aligned} f_1'' + E_0 f_1 &= 0. \\ g_1'' + \left(E_0 + D_0 - \frac{K}{r}\right) g_1 &= 0 \end{aligned}\right\} \quad (6')$$

ココデ $\alpha^2 E = E_0$, $\alpha^2 J = J_0$ etc., $K = \alpha^2 (Z-1) e^2$

(5') ノ解ノ中 原点デ zero ニナルモノヲトレバ

$$\left.\begin{aligned} f_1 &= a_1 \sin\sqrt{E_0}\,r + a_2 \sinh\sqrt{E_0}\,r \\ g_1 &= a_1 \sin\sqrt{E_0}\,r - a_2 \sinh\sqrt{E_0}\,r \end{aligned}\right\} \quad (7)$$

$$\underline{r<a}$$

(6) カラ f_1 ヲ求メルノデアルガ, infinity デ plane wave e^{ikz} $(k=\sqrt{E_0})$ ノ $l=0$ ノ part ト, scattered wave トノ sum ノ形ニナル様ナモノヲトレバ

$$f_1 = \sqrt{\frac{4\pi}{E_0}}\sin\sqrt{E_0}\,r + C e^{i\sqrt{E_0}\,r} \quad (8)$$

$$\underline{r>a}$$

次ニ g_1 ヲ求メル

(6,2) ニオイテ $\sqrt{E_0+D_0}\,r = R$ トオケバ

$$g_1'' + \left(1-\frac{R_1}{R}\right) g_1 = 0 \quad (9)$$

ここニ $R_1 = \dfrac{K}{\sqrt{E_0+D_0}} \simeq 0.3\dfrac{Z}{\sqrt{E+D}}$　(E, D in MV)

(9) ノ asymptotic solution デ, 出テ行ク wave ヲ表ハスモノヲトレバ

$$g_1 = (F(R))^{-\frac{1}{4}} e^{i\int_{R_1}^{R}\sqrt{F(R)}\,dR} \quad (10)$$

$$\underline{R \gg R_1}$$

$R \approx \sqrt{E_0+D_0 a}$ ノ近傍デ (10) ニ対応スル (9) ノ solution ハドンナ形ニナルカシ調ベテ見ル.

$F(R) = 1 - \frac{R_1}{R} \approx 0$ ノ近傍ノ解ヲ求メテ見ル. コヽデハ $R \approx R_1$ デアルカラ, $F(R)$ ヲ $(R-R_1)$ ノ power series ニ expand スル

$$F(R) = \frac{1}{R_1}(R-R_1) - \frac{2}{2!R_1^2}(R-R_1)^2 + \cdots$$
$$= \left(\frac{R}{R_1}-1\right) - \left(\frac{R}{R_1}-1\right)^2 + \cdots$$

第一項ダケヲトリ, $\frac{1}{R_1} = \beta^2$, $R-R_1 = \xi$ トオケバ

$F(R) = \beta^2 \xi$ デアルカラ (9) ハ

$$g_1'' + \alpha^2 \xi g_1 = 0. \qquad (11)$$

寺寛 p.563 カラ

$$g_1 = \sqrt{\xi} \quad \begin{matrix} H^{(1)}_{\frac{1}{3}}\left(\frac{2\beta}{3}\xi^{\frac{3}{2}}\right) \\ H^{(2)}_{\frac{1}{3}}\left(\frac{2\beta}{3}\xi^{\frac{3}{2}}\right) \end{matrix}$$

コノ asymptotic form ガ (10) ト一致シナケレバナラナイ. 然ルニ

$$H^{(1)}_\nu(z) \to e^{i\left(z - \frac{2\nu+1}{4}\pi\right)}\sqrt{\frac{2}{\pi z}} \qquad (-\pi < \arg z < 2\pi)$$
$$H^{(2)}_\nu(z) \to e^{-i(\ \ ''\ \ )}\sqrt{\ ''} \qquad (-2\pi < \arg z < \pi)$$

デアルカラ $H^{(2)}_{\frac{1}{3}}$ ノ方ハトルコトガ出来ナイ.

Ep 4

$$g_1 = A\sqrt{\xi}\,H^{(1)}_{\frac{1}{3}}\left(\frac{2\beta}{3}\xi^{\frac{3}{2}}\right) \approx A\sqrt{\frac{3}{\pi\beta}}\,\xi^{-\frac{1}{4}}e^{i\left(\frac{2\beta}{3}\xi^{\frac{3}{2}}-\frac{5}{12}\pi\right)}$$

(10) ニオイテ $F(R)=\beta^2\xi$ トオキ 積分ヲ遂行スレバ

$$g_1 = \frac{1}{\sqrt{\beta}}\,\xi^{-\frac{1}{4}}e^{i\frac{2\beta}{3}\xi^{\frac{3}{2}}} \qquad (12)$$

コノ二ツヲ比較シテ $A=\sqrt{\frac{3}{\pi}}\,e^{i\frac{5\pi}{12}}$

故ニ $R\approx R_1$ デ [illegible], $R\gg R_1$ ノ (10) ニ対応スル解ハ

$$g_1 = \sqrt{\frac{\pi}{3}}\,e^{i\frac{5\pi}{12}}\sqrt{\xi}\,H^{(1)}_{\frac{1}{3}}\left(\frac{2\beta}{3}\xi^{\frac{3}{2}}\right) \qquad (13)$$

(13) ハ ξ ガ positive デモ negative デモアテハマル。ξ ガ neg. ノ場合ニハ

$$\arg\xi = \cancel{e^{i}}\,\pi$$

トシテモ

$$\arg\xi = \cancel{e^{i}}\,-\pi$$

トシテモ (13) ハ全ク同ジ式ヲ与ヘル。

ξ ガ neg. ノトキニ

$$\arg\xi = \cancel{e^{i}}\,\pi$$

トレバ

$$\arg\xi^{\frac{3}{2}} = \frac{3}{2}\pi$$

デアルカラ、前ニ示シタ $H^{(1)}_{\nu}$ ノ asymptotic solution ガソノママアテハマルカラ。従ッテ (12) モ ξ ノ正負ニカヽハラズ成立スルコトニナル。

従ッテ $\xi<0$ ノ部分（$R<R_1$ ノ部分）デハ (13) トツナガル解ハ (10) ト全ク同ジ形デナケレバナラナイ

$$g_1 = (F_1(R))^{-\frac{1}{4}} e^{i\int_{R_1}^{R}\sqrt{F_1(R)}\,dR}$$

$$= e^{-i\frac{\pi}{4}}(-F_1(R))^{-\frac{1}{4}} e^{-\int_{R_1}^{R}\sqrt{-F_1}\,dR} \qquad (14)$$

$\underline{R<R_1}$

コヽデ $F_1(R)$ ノ偏角ハ $F_1(R)=\alpha^2\xi$ デアルカラ ヤハリ π トオイタノデアル.

(14) ハ R ガアマリ小サクナルト $F_1(R)$ ノ変化ガハゲシクナルノデ又アテハマラナクナル

R ノ充分小サナ所デハ (9) ハ

Barrier 大が高い

$$g_1'' - \frac{R_1}{R} g_1 = 0$$

とした

寺寛 p. 563 カラ

$H_{\frac{1}{3}}$ と H_1 とchigau

$$g_1 = \sqrt{R}\ \begin{matrix} H_1^{(1)}(2i\sqrt{R_1R}) \\ H_1^{(2)}(\ \text{''}\ ) \end{matrix}$$

コレガ asymptotic ニ (14) ト一致シナケレバナラナイコトカラ $H_1^{(2)}$ ノ方ハステヽ

$$g_1 = A\sqrt{R}\,H_1^{(1)}(2i\sqrt{R_1R}) \approx e^{-i\frac{\pi}{4}}\sqrt{\frac{1}{\pi}}\left(\frac{1}{R_1}\right)^{\frac{1}{4}} R^{\frac{1}{4}} e^{i\left(2i\sqrt{R_1R}-\frac{3\pi}{4}\right)} A.$$

(14) ニオイテ $F_1(R) = 1-\frac{R_1}{R} \approx -\frac{R_1}{R}$ ト置テ ~~精密~~ 計算スレバ*

$$g_1 = e^{-i\frac{\pi}{4}}\left(\frac{R_1}{R}\right)^{-\frac{1}{4}} e^{2R_1\left(1-\sqrt{\frac{R}{R_1}}\right)}$$

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* コレハ適當デナイ. 詳シク計算スレバ first approximation トシテ $e^{2R_1\left(1-\sqrt{\frac{R}{R_1}}\right)}$ ノ代リニ $e^{2R_1\left(\frac{\pi}{2}-\sqrt{\frac{R}{R_1}}\right)}$ ガ入ッテクル. シカシ. コヽデハ. コノマヽニシテオク

~~[illegible]~~ コノ二ツノ式ヲ比較シテ

$$A = \sqrt{\pi}\, e^{2R_1} e^{i\frac{3\pi}{4}}$$

故ニ

$$g_1 = e^{i\frac{3\pi}{4}} \sqrt{\pi}\, e^{2R_1} \sqrt{R}\, H_1^{(1)}\left(2i\sqrt{R_1 R}\right) \qquad \text{(15)}$$

$$\underline{R \ll R_1}$$

以上デ solution ハ皆求マッタ

<u>$r<a$</u>

$$f_1 = a_1 \sin\sqrt{J_0}\, r + a_2 \sinh\sqrt{J_0}\, r$$
$$g_1 = a_1 \sin\sqrt{J_0}\, r - a_2 \sinh\sqrt{J_0}\, r \qquad \text{(16)}$$

<u>$r>a$</u>

$$f_1 = \sqrt{\frac{4\pi}{E_0}} \sin\sqrt{E_0}\, r + C e^{i\sqrt{E_0}\, r}$$
$$g_1 = -D\left(\frac{R}{R_1}\right)^{\frac{1}{4}} e^{2R_1 - 2\sqrt{R_1 R}} \qquad R_0 < R_1 \qquad \text{(17)}$$
$$= D e^{-i\frac{3\pi}{4}} \left(F(R)\right)^{-\frac{1}{4}} e^{i\int_{R_1}^{R}\sqrt{F}\, dR} \qquad R > R_1$$
$$\left(F(R) = 1 - \frac{R_1}{R}\right)$$

<u>$r>a$</u> ノ部分ノ g_1 ニハ色々ノ解ガアッテ $r \approx a$ ノ所デ
一番ヨクアッテイル解ヲエラバネバナラナイガ、コレデ簡単ノ
タメニ上ノ様ニエランダ。コノ解ガ一番ヨクアテハマルノハ

$\frac{R_1}{R} \gg 1, \quad 2\sqrt{R_1 R_0} \gg 1$

(~~…~~ トキ デアレバ。 書キカヘレバ

$$\frac{R_1}{R} = \frac{Ic}{(E_0+D_0)r} \approx 0{,}3\,\frac{Z}{E+D} \qquad \left(\begin{array}{l} r = 5\cdot 10^{-13} \\ E, D, \text{ in MV} \end{array}\right)$$

$$2\sqrt{R_1 R} = 2\sqrt{Ic\,r} \approx \sqrt{Z} \qquad (r = 5\cdot 10^{-13})$$

デアルカラ

$$\boxed{\frac{Z}{E+D} \gg 3.}$$

ナラバヨイコトニナル.

$r = a$ デ 両側ノ wave function ヲツナゲル.

マヅ f_1 ヲツナゲル ⑯, ⑰ カラ

$$a_1 \sin\sqrt{J_0}\,a + a_2 \sinh\sqrt{J_0}\,a = \sqrt{\frac{4G}{E_0}}\sin\sqrt{E_0}\,a + Ce^{i\sqrt{E_0}a}$$

$$a_1 \cos\sqrt{J_0}\,a + a_2 \cosh\sqrt{J_0}\,a = \sqrt{\frac{E}{J_0}}\left(\sqrt{\frac{4G}{E_0}}\cos\sqrt{E_0}\,a + iCe^{i\sqrt{E_0}a}\right)$$

第2式右辺ノ $i\sqrt{\frac{E}{J_0}}Ce^{i\sqrt{E_0}a}$ ノ term ハ、第一式右辺ノ $Ce^{i\sqrt{E_0}a}$ ニ比ベテ小サイト見テ省略スル.

$\sqrt{J_0}\,a = \varphi$. $\sqrt{E_0}\,a = \varepsilon$ トオケバ

$$a_1 \sin\varphi + a_2 \sinh\varphi = \sqrt{\frac{4G}{E_0}}\sin\varepsilon + Ce^{i\varepsilon}$$

$$a_1 \cos\varphi + a_2 \cosh\varphi = \sqrt{\frac{4G}{J_0}}\cos\varepsilon$$

コレカラ a_1, a_2 ヲトケバ

$$a_1 = \frac{\cosh\varphi\left(\sqrt{\frac{4\xi}{E_0}}\sin\varepsilon + Ce^{i\varepsilon}\right) - \sinh\varphi\sqrt{\frac{4\xi}{T_0}}\cos\varepsilon}{\cosh\varphi\sin\varphi - \sinh\varphi\cos\varphi}$$

$$a_2 = \frac{\sinh\varphi\sqrt{\frac{4\xi}{T_0}}\cos\varepsilon - \cos\varphi\left(\sqrt{\frac{4\xi}{E_0}}\sin\varepsilon + Ce^{i\varepsilon}\right)}{\cosh\varphi\sin\varphi - \sinh\varphi\cos\varphi}$$

$$\left.\begin{aligned} B &= \sqrt{\frac{4\xi}{E_0}}\sin\varepsilon + Ce^{i\varepsilon} \\ A_1 &= \cosh\varphi\sin\varphi - \sinh\varphi\cos\varphi \end{aligned}\right\}$$ トオケバ

$$a_1 = \frac{1}{A_1}\left(B\cosh\varphi - \sqrt{\frac{4\xi}{T_0}}\sinh\varphi\cos\varepsilon\right)$$

$$a_2 = -\frac{1}{A_1}\left(B\cos\varphi - \sqrt{\frac{4\xi}{T_0}}\sinh\varphi\cos\varepsilon\right)$$

(18)

次ニ g_1 ヲツナゲバ (17) ノ g_1 ハ 書キカヘルト

$$g_1 = -D\left(\frac{R}{R_1}\right)^{\frac{1}{4}} e^{2R_1 - 2\sqrt{R_1 R}}$$

$$= -D\left(\frac{\sqrt{E_0+D_0}}{K}\right)^{\frac{1}{2}}(Kr)^{\frac{1}{4}} e^{2\frac{K}{\sqrt{E_0+D_0}} - 2\sqrt{Kr}}$$

トナル. コレノ derivative ハ

$$g_1' = -D\left(\frac{\sqrt{E_0+D_0}}{K}\right)^{\frac{1}{2}}(Kr)^{\frac{1}{4}} e^{\,''}\left(\frac{1-4\sqrt{rK}}{4r}\right)$$

之ヲ用ヒテ $r<a$ ノ g_1 ノ solution トツナゲバヨイ.

$$a_1 \sin\varphi - a_2 \sinh\varphi = -D\left(\frac{\sqrt{F_0+D_0}}{K}\right)^{\frac{1}{2}}(Ka)^{\frac{1}{4}} e^{2\frac{K}{\sqrt{F_0+D_0}} - 2\sqrt{K}a}$$

$$a_1 \cos\varphi - a_2 \cosh\varphi = -D(\ )^{\frac{1}{2}}(\ )^{\frac{1}{4}} e^{\ ''} \cdot \frac{1-4\sqrt{K}a}{4\sqrt{J_0}a}$$

$\dfrac{1-4\sqrt{K}a}{4\sqrt{J_0}a} \approx \dfrac{1-2.5\sqrt{2}}{40}$ デアルカラ 2ガ大キクトキハ 1ノorderニナル。

省略出来ルノハ $4\sqrt{K}a \approx 2.8$ ニ 1ノ term デアル。

$$\left(\frac{\sqrt{F_0+D_0}}{K}\right)^{\frac{1}{2}} e^{2\frac{K}{\sqrt{F_0+D_0}}} = H$$
$$(Ka)^{\frac{1}{4}} e^{-2\sqrt{K}a} = \gamma$$
トオケバ

$$a_1 \sin\varphi - a_2 \sinh\varphi = -DH\gamma$$
$$a_1 \cos\varphi - a_2 \cosh\varphi = DH\gamma\sqrt{\frac{K}{J_0 a}}$$

~~故ニ~~ コレカラ a_1 a_2 ヲトケバ

$$a_1 = -\frac{DH\gamma}{A_1}\left(\cosh\varphi + \sqrt{\frac{K}{J_0 a}}\sinh\varphi\right)$$
$$a_2 = -\frac{DH\gamma}{A_1}\left(\cos\varphi + \sqrt{\frac{K}{J_0 a}}\sin\varphi\right) \qquad (19)$$

(18) ト (19) ハ 一致セネバナラヌカラ。

$$B\cosh\varphi - \sqrt{\frac{4q}{J_0}}\sinh\varphi\cos\varepsilon = -DH\gamma\left(\cosh\varphi + \sqrt{\frac{K}{J_0 a}}\sinh\varphi\right)$$
$$B\cos\varphi - \sqrt{\frac{4q}{J_0}}\sin\varphi\cos\varepsilon = DH\gamma\left(\cos\varphi + \sqrt{\frac{K}{J_0 a}}\sin\varphi\right) \qquad (20)$$

コレカラ B ヲ消去スレバ

$$\sqrt{\frac{K\xi}{J_0}}\cos\varepsilon\cdot\left(\cosh\varphi\sin\varphi-\sinh\varphi\cos\varphi\right)$$
$$=DH\gamma\left(-2\cosh\varphi\cos\varphi\mp\sqrt{\frac{K}{J_0a}}\left(\cosh\varphi\sin\varphi+\sinh\varphi\cos\varphi\right)\right)$$

$$\frac{1}{2}\sqrt{\frac{K}{J_0a}}\approx\frac{2.5\sqrt{2}}{80}$$ ヲ 1 ニ対シ省略スレバ

$$D=-\sqrt{\frac{\xi}{J_0}}\frac{A_1\cos\varepsilon}{H\gamma\cosh\varphi\cos\varphi}$$

H, γ ノ値ヲ入レレバ

$$D=-\sqrt{\frac{\pi}{J_0}}\frac{A_1\cos\varepsilon}{\cosh\varphi\cos\varphi}\cdot(Ka)^{-\frac{1}{4}}e^{2\sqrt{Ka}}\left(\frac{K}{\sqrt{E_0+D_0}}\right)^{\frac{1}{2}}e^{-2\frac{K}{\sqrt{E_0+D_0}}}$$

即チ

$$D=-\sqrt{\pi}\frac{A_1}{\sqrt{J_0}\cosh\sqrt{J_0}a\cos\sqrt{J_0}a}\cdot\frac{e^{2\sqrt{Ka}}}{(\sqrt{Ka})^{\frac{1}{2}}}\left(\frac{K}{\sqrt{E_0+D_0}}\right)^{\frac{1}{2}}e^{-\frac{2K}{\sqrt{E_0+D_0}}}\cos\varepsilon \quad \text{------} \quad (21)$$

(17) カラ直チニ知レル様ニ proton emission ノ total cross section ハ

$$|D|^2\frac{\sqrt{E_0+D_0}}{\sqrt{E_0}}$$

デアル. 勿論コレハ $E_0+D_0>0$ ノトキニ限リ意味ヲモツ, ~~(20) ヨリ~~
(20) カラ

$$|D|^2\frac{\sqrt{E_0+D_0}}{\sqrt{E_0}}=\frac{\pi A_1^2}{J_0\cosh^2\sqrt{J_0}a\cos^2\sqrt{J_0}a}\frac{e^{4\sqrt{Ka}}}{\sqrt{Ka}}\frac{\sqrt{K}}{\sqrt{E_0}}e^{-\frac{4K}{\sqrt{E_0+D_0}}}\cos^2\sqrt{E_0}a \quad \text{------} \quad (22)$$

故ニ $\cos\sqrt{J_0}a = 0$ ノ近傍デハ cross section ハ非常ニ大キクナルワケデアル.

E, D ヲ MV unit デ表ハシ, a ヲ 10^{-12} cm unit デ表ハセバ

$$|D|^2\frac{\sqrt{E_0+D_0}}{\sqrt{E_0}} = 0{,}2\cdot 10^{-24}\frac{e^{3{,}3\sqrt{2a}}}{J\sqrt{a}}\sqrt{\frac{2}{E}}\,e^{-1{,}35\frac{2}{\sqrt{E+D}}}\cos^2(2{,}2\sqrt{E}a)\times\frac{A_1^2}{\cosh^2\sqrt{J_0}a\cos^2\sqrt{J_0}a}$$

コヽニ
$$e^{3.3\sqrt{2a}} \approx 10^{1.45\sqrt{2a}}$$
$$e^{-1{,}35\frac{2}{\sqrt{E+D}}} \approx 10^{-0{,}6\frac{2}{\sqrt{E+D}}}$$
デアル

次ニ B ヲ求メルト (20) カラ

$$\frac{B\cosh\varphi-\sqrt{\frac{4\xi}{J_0}}\sinh\varphi\cos\xi}{B\cos\varphi-\sqrt{\frac{4\xi}{J_0}}\sin\varphi\cos\xi} = -\frac{\cosh\varphi+\sqrt{\frac{K}{J_0a}}\sinh\varphi}{\cos\varphi+\sqrt{\frac{K}{J_0a}}\sin\varphi}.$$

$$B = \sqrt{J_2}\,\frac{A_2+2\sqrt{\frac{K}{J_0a}}\sinh\varphi\sin\varphi}{\sqrt{J_0}\cosh\varphi\cos\varphi}\cdot\cos\xi \qquad (23)$$

コヽニ, $A_2 = \cosh\varphi\sin\varphi+\sinh\varphi\cos\varphi.$

故ニ

$$Ce^{i\sqrt{E_0}a} = B - \sqrt{\frac{4\pi}{E_0}}\sin\sqrt{E_0}a$$

$$= \frac{\sqrt{\pi}}{\sqrt{J_0}}\left\{\frac{A_2 + 2\sqrt{\frac{I_C}{J_0 a}}\sinh\sqrt{J_0}a\sin\sqrt{J_0}a}{\cosh\sqrt{J_0}a\cos\sqrt{J_0}a}\cos\sqrt{E_0}a - 2\sqrt{\frac{J_0}{E_0}}\sin\sqrt{E_0}a\right\}$$

$|C|^2$ ガ scattered neutron ノ cross section ニナルカラ．上ノ C ノ expression カラ．コレモ

$$\cos\sqrt{J_0}a = 0 \quad , \sqrt{J_0}a = \left(n+\frac{1}{2}\right)\pi$$

デ大キクナルコトガ知レル．

$$|C|^2 = \frac{\pi}{J_0}\left[\left(\tan\varphi + \tanh\varphi + 2\sqrt{\frac{I_C}{J_0 a}}\tan\varphi\tanh\varphi\right)\cos\sqrt{E_0}a - 2\sqrt{\frac{J_0}{E_0}}\sin\sqrt{E_0}a\right]^2$$

Slow neutron デハ

$$|C|^2 = \frac{\pi}{J_0}\left[\left(\tan\varphi + \tanh\varphi + 2\sqrt{\frac{I_C}{J_0 a}}\tan\varphi\tanh\varphi\right) - 2a\sqrt{J_0}\right]^2$$

$$= 4\pi\left[a - \frac{1}{\sqrt{J_0}}\frac{\tan\varphi + \tanh\varphi + 2\sqrt{\frac{I_C}{J_0 a}}\tan\varphi\tanh\varphi}{2}\right]^2$$

之ヲ Bethe ノ結果ト比較スルト．

Bethe デハ．$\Phi_{el} = 4\pi\left(\lambda_0\cot\varphi_0 + \frac{r_0}{a}\right)^2$

$$= 4\pi\left(a - \frac{1}{\sqrt{J_0}}\tan\sqrt{J_0}a\right)^2$$

デアルカラ．

相異ハ $\tan\varphi$ ノ代リニ $\dfrac{\tan\varphi+\tanh\varphi+2\sqrt{\frac{K}{J_0 a}}\tan\varphi\tanh\varphi}{2}$

ノ入レコトデアル.

Disintegration ノ cross section ヲ Bethe ノトヲ比較シテ見ル

Bethe デハ

$$\Phi_{dis}'=\frac{2\pi}{k}\frac{2\pi^2\hbar^2}{\mu v}\frac{\frac{1}{\sqrt{J_0}}}{\cos^2\varphi+\frac{E}{E+D}}\frac{1}{4e^{2G}\sin^2\varphi_\alpha+\frac{1}{4}e^{-2G}\cos^2\varphi_\alpha}\quad I$$

デアル. ココニ

$$G=\int\sqrt{\frac{K}{r}-(E_0+D_0)}\,dr=\int\sqrt{\frac{R_1}{R}-1}\,dR$$

$$\approx\sqrt{R_1}\int R^{-\frac{1}{2}}dR=\sqrt{R_1}\left|2R\right|_{r=a}^{R=R_1}$$

$$=2\left(\frac{K}{\sqrt{E_0+D_0}}-\sqrt{Ka}\right)$$

デアルカラ

$$\Phi_{dis}'\approx\hbar^3\frac{1}{\sqrt{E_0}}\frac{1}{\sqrt{J_0}}\frac{1}{\cos^2\varphi+\frac{E_0}{E_0+D_0}}\frac{\frac{1}{4}e^{-4\left(\frac{K}{\sqrt{\ }}-\sqrt{\ }\right)}}{\sin^2\varphi_\alpha}\quad I$$

我々ノ場合ニハ (22) カラ

$$\Phi_{dis}'=\hbar\frac{\sqrt{K}}{J_0\sqrt{a}\sqrt{E_0}}(\tan\varphi-\tanh\varphi)^2e^{-4\left(\frac{K}{\sqrt{\ }}-\sqrt{\ }\right)}\cos^2\sqrt{E_0 a}.$$

コノ様ニハ, Bethe ノ式ノ φ_α ~~ニ~~ ニ対応スルモノガナイガ

アツ".

$$\left(\cos\varphi - \cosh\varphi\right)^2 \longrightarrow \frac{1}{\cos^2\varphi + \frac{E_0}{E_0+D_0}}$$

$$e^{-4(\ \ )} \longrightarrow \frac{e^{-4(\ \ )}}{\sin^2\varphi_\alpha}$$

ト対応シテヰル

$$I = \frac{1}{\hbar^2}\sqrt{\frac{K}{J_0 a}}\,\cos^2\sqrt{E_0}\,a$$

$$\frac{1}{\hbar^2}\sqrt{\frac{K}{J_0 a}} \cancel{\approx \frac{1}{10}\sqrt{\frac{2}{100}}} \qquad \left(\begin{matrix} J_0 = 10^8\ \text{volt} \\ a = 5\cdot 10^{-13}\ \text{cm} \end{matrix}\right)$$

$$\approx \frac{\sqrt{2}}{100}$$

コレハ Betheガ 実験トヨク比較シテ出シテヰル

$$I = \frac{1}{60}$$

ト comparable デアルト考ヘラレル.