



$l=0$

$$\begin{aligned}
 u_1 &= A_1 K \sqrt{k_1 r} \cdot \sqrt{\frac{2}{\pi k_1 r}} \sin k_1 r + A_2 J \sqrt{k_2 r} \sqrt{\frac{2}{\pi k_2 r}} \\
 &= \sqrt{\frac{2}{\pi}} A_1 K \sin k_1 r + \frac{-A_2 J \sqrt{2}}{i \sqrt{\pi}} \sinh k_2 r \\
 &= \sqrt{\frac{2}{\pi}} A_1 K \sin k_1 r + A_2 J \sinh k_2 r \\
 u_2 &= A_1 J \sinh k_1 r - A_2 K \sinh k_2 r
 \end{aligned}$$

$\sin(\tilde{a} k_2 r)$   
 $\frac{e^{-\tilde{a} k_2 r} - e^{\tilde{a} k_2 r}}{2i}$

$$\begin{aligned}
 u_1 &= B_1' e^{i k r} + B_2' e^{-i k r} \\
 u_2 &= C_1' W_{\eta, -\frac{1}{2}}(2i k' r) \\
 &\sim \mathcal{O}(D) e^{i k' r}
 \end{aligned}$$

$$\begin{aligned}
 u_1 &= \frac{1}{2ki} (e^{i k r} - e^{-i k r}) + C e^{i k r} \\
 u_2 &= \mathcal{O}(D) e^{i k' r} + \eta \log k' r
 \end{aligned}$$

$$\begin{aligned}
 k \left| \frac{1}{2ki} + C \right|^2 &= k \left\{ \left( \frac{1}{2k} \right)^2 + |D|^2 \right\} \\
 \left( \frac{1}{2k} \right)^2 + |C|^2 &+ \frac{1}{2ki} (\tilde{C} - C) = \left( \frac{1}{2k} \right)^2 + |D|^2 \\
 -\frac{1}{2ki} + \tilde{C} \cdot C &= |C|^2 e^{i\phi} \\
 |C|^2 + 2|C| \frac{\sin \phi}{k} &= |D|^2 \\
 \frac{1}{4k} &= k \left( \frac{1}{2ki} + C \right)^2 + k |D|^2 \\
 &= \frac{1}{4k} + \frac{1}{2i} (\tilde{C} - C) + k |C|^2 + k |D|^2
 \end{aligned}$$



