

Interaction of the Heavy Particle
with the Nucleus

$$H = \frac{1}{2M} p^2 + \left(\frac{1-\tau_3}{2}\right)V + \left(\frac{\tau_1 + i\tau_2}{2}\right)J + \left(\frac{\tau_1 - i\tau_2}{2}\right)K + \left(\frac{1-\tau_3}{2}\right)D$$

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} \tau_1 + i\tau_2 \\ \tau_1 - i\tau_2 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \psi_2 \\ \psi_1 \end{pmatrix}$$

$$\frac{1}{2M} p^2 = -\frac{\hbar^2}{2M} \Delta = -\frac{\hbar^2}{2M} \left\{ \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} + \frac{l(l+1)}{r^2} \right\}$$

$$\frac{2M}{\hbar^2} E = E', \quad \frac{2M}{\hbar^2} V = V', \quad \frac{2M}{\hbar^2} J = J', \quad \frac{2M}{\hbar^2} D = D'$$

$$\left[\frac{\partial}{\partial r} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{l(l+1)}{r^2} + \left\{ E' - \frac{1-\tau_3}{2} (V' + D') + \frac{\tau_1 + i\tau_2}{2} J' - \tau_1 \frac{i\tau_2}{2} K' \right\} \right] \psi_{l,m}(r, \tau_3) = 0$$

$$\psi_{l,m}(r, \tau_3) = \frac{\chi}{r}$$

$$\left[\frac{d}{dr} - \frac{l(l+1)}{r^2} + \left\{ E' - \frac{1-\tau_3}{2} (V' + D') - \tau_1 \frac{i\tau_2}{2} J' - \tau_1 \frac{i\tau_2}{2} K' \right\} \right] \psi' = 0$$

$$\frac{d^2 \psi_1}{dr^2} - \frac{l(l+1)}{r^2} \psi_1' + \left\{ E' \psi_1' - J' \psi_1' \right\} = 0$$

$$\frac{d^2 \psi_2}{dr^2} - \frac{l(l+1)}{r^2} \psi_2' + E' \psi_2' - (V' + D') \psi_2' - K' \psi_2' = 0$$

$$\begin{aligned}
 & (A_0 + A_1 \tau_1 + A_2 \tau_2 + A_3 \tau_3) (a_0 + a_1 \tau_1 + a_2 \tau_2 + a_3 \tau_3) \\
 & = (a_0 + a_1 \tau_1 + a_2 \tau_2 + a_3 \tau_3) (\beta_0 + \beta_1 \tau_1) \\
 & (A_0 - \beta_0) a_0 + A_1 a_1 + A_2 a_2 + (A_3 - \beta_3) a_3 = 0 \\
 & A_1 a_0 + (A_0 - \beta_0) a_1 +
 \end{aligned}$$

$(\frac{1}{2}V + D)$

$$\left(\frac{d^2}{dr^2} (a_1 \psi_1 + a_2 \psi_2) + (E' a_1 - K' a_2) \psi_1' + (E' - V' + D') a_2 \right) - J a_1 \psi_2' = 0$$

$$a_1 \{ (E' - V' - D') a_2 - J a_1 \} = a_2 (E' a_1 - K' a_2)$$

$$J' a_1^2 + (V' + D') a_1 a_2 - K' a_2^2 = 0$$

$$a_1 = \frac{-(V' + D') \pm \sqrt{(V' + D')^2 + 4J'K'}}{2J'} a_2$$

$$\frac{E' a_1 - K' a_2}{a_1} = E' + \frac{2J'K'}{V' + D' \pm \sqrt{(V' + D')^2 + 4J'K'}}$$

$$L_1 = \frac{2J'K' E' + W}{\sqrt{(V' + D')^2 + 4J'K'} - (V' + D')}$$

$$L_2 = \frac{2J'K'}{\sqrt{(V' + D')^2 + 4J'K'} + (V' + D')}$$

$$\frac{d^2 X_1}{dr^2} + (E' + L_1) X_1 = 0 \quad \lambda_1 =$$

$$\frac{d^2 X_2}{dr^2} + (E' + L_2) X_2 = 0$$

$$\frac{d^2}{dr^2} - \ell(\ell+1) \frac{1}{r^2} + \frac{2m}{\hbar^2} (V(r) - E) = 0$$

$$\left(\frac{d^2}{dr^2} + E + \frac{2m}{\hbar^2} V(r) \right) f_1 + \frac{2m}{\hbar^2} V(r) f_2 = 0$$

$$\lambda_1 \left(\frac{d^2}{dr^2} + E - D + \frac{2m}{\hbar^2} V(r) \right) f_2 + \frac{2m}{\hbar^2} V(r) f_1 = 0$$

$$h_1 = f_1 + \lambda_1 f_2$$

$$h_2 = f_1 + \lambda_2 f_2$$

$$\lambda_1 \left((E + \frac{2m}{\hbar^2} V(r)) + \lambda_1 (E - D + \frac{2m}{\hbar^2} V(r)) \right) f_1$$

$$= 0 \left(\frac{2m}{\hbar^2} V(r) + \lambda_1 (E - D + \frac{2m}{\hbar^2} V(r)) \right) f_1$$

$$\frac{2m}{\hbar^2} V(r) + (D + \frac{2m}{\hbar^2} V(r) - \frac{2m}{\hbar^2} V(r)) \lambda_1 - \frac{2m}{\hbar^2} V(r) = 0$$

$$\lambda = \frac{-\left(D + \frac{2m}{\hbar^2} V(r) - \frac{2m}{\hbar^2} V(r) \right) \pm \sqrt{\left(D + \frac{2m}{\hbar^2} V(r) - \frac{2m}{\hbar^2} V(r) \right)^2 + 4 \left(\frac{2m}{\hbar^2} V(r) \right)^2}}{2 \left(\frac{2m}{\hbar^2} V(r) \right)}$$

~~Handwritten scribbles and corrections~~

$$\chi_1 = C_1 e^{i\sqrt{E-U_1}} + D_1 e^{-i\sqrt{E-U_1}}$$
$$\chi_2 = C_2 e^{i\sqrt{E+U_2}} + D_2 e^{-i\sqrt{E+U_2}}$$

$$\psi_1' = b_{11}\chi_1 + b_{12}\chi_2$$
$$\psi_2' = b_{21}\chi_1 + b_{22}\chi_2$$

$$V < V_0$$
$$V' = V_0$$

