E21 060 P05

1

Theory of Disintegation of the Nucleus

by Neutron Impact

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Abstract

The cross section ~~of the elastic scattering and~~ Σ_{dis} of the nuclear disintegration by neutron impact with ~~the~~ emission ~~of the~~ proton was calculated on the assumption that the emitted proton can be identified with the impinging neutron. ~~Also~~ The expressions of ~~The~~ cross sections Σ_{cap} of the elastic scattering and the capture $\Sigma_{scat.}$ of the neutron was ~~calculated on the same assumption~~ altered by taking the above process into account. ~~The~~ Especially, the ratio $\Sigma_{cap}/\Sigma_{scat.}$ ~~is~~ can be either larger or smaller than 1, but the anomalous absorption ~~to~~ of the neutron without comparable scattering, ~~for~~ in the case of Cd ~~etc~~ for example, can ~~not~~ be hardly ~~explained~~ expected in our theory too.

§1. General Considerations

Among various processes caused by the collision of the heavy particle with the nucleus, there are two cases, in which the cross sections can be obtained by ~~the exact method of~~ straitforward application of the quantum mechanical method. They are, namely, the elastic scatering and the capture of the heavy particle with the emission of ~~a~~ the γ ray. Other processes such as the disintegration with the emission of another heavy particle or the ~~exit~~ inelastic scattering with the excitation of the nucleus have been considered to be so compicated that ~~only~~ a rough estimations we should be satisfied only with of their cross sections.

In this paper, the authors want to work out a simple method, though rather phenomenological, of treating some of them, for example, the disintegration by neutron impact with proton emission.

Now, according to the present theory of Heisenberg, Fermi and others, the atomic nucleus consists of ~~the~~ neutrons and ~~the~~ protons, β-disintegration being considered to be ~~accompanied~~ caused by the transformation of a neutron in the nucleus into a proton. On this point of view, it is possible that the neutron colliding with the neucleus of atomic number Z changes itself into the proton,

the atomic number of the nucleus being reduced to Z-1 simultaneously. Consequently, if the neutron and the proton are considered to be two states of the so called heavy particle, a part, at least of the neutron produced disintegration with proton emission, can be ~~attributed~~ reduced to ~~s~~ a sort of simple scattering of the ~~so called~~ heavy particle by the nucleus.

In order to formulate such an idea, we deal with a system consisting of a heavy particle and a nucleus, between which following types of forces act.

i) Ordinary short range force between the heavy particles and the nucleus.

ii) Coulomb force between the proton and the nucleus.

iii) The force which causes the transition of the heavy particle from the neutron to the proton state at the same time with the reduction of the atomic number of the nucleus by one.

iv) The force which causes the transition inverse to the above.

We ~~consider the case, in which a~~ neutron and a nucleus of the atomic number Z ~~existed initially transfer~~ changes into a proton and a nucleus of the atomic number Z-1

If we denote the state of the system, in which the ~~former~~ the heavy particle is neutron and the nucleus has the atomic number Z, by 1, and the state , in which the former is proton and the latter has the atomic number Z-1, by 2, there two states are linked together by the forces iii), iv), so that ~~The~~ the eigenfunction of the system should have ~~has~~, ~~then~~, two components, which will be written as $\begin{pmatrix} \psi_1' \\ \psi_2' \end{pmatrix}$ ~~$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$~~. Corresponding to this, the interaction energy becomes ~~the~~ a Hermitic matrix of the form

$$\begin{pmatrix} J & J \\ J & J \end{pmatrix} \quad \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \quad \cancel{\begin{pmatrix} V_1 & \tilde{J} \\ J & V_2 \end{pmatrix}},$$

~~where $V_{21} = \tilde{V}_{12}$ on account of the Hermitic~~

~~in which~~ where V_{11}, V_{22} mean the ordinary forces i), ii), while V_{21} and V_{12}, satisfying the condition $V_{21} = \tilde{V}_{12}$ the potentials mean the exchange forces ~~iv)~~ iii) and ~~iii)~~ iv) respectively.

In general those of V_1, V_2 ~~and~~ V and $\tilde{J}$ V's will depend on the coordinates, the moenta and the spins of all the particles constituting the system. We want, however, to deal with the problem as simply and as phenomenologically as possible. For this purpose ~~reason~~, we assume ~~J s~~ ~~them~~ V's to be functions only of the distance r between the heavy particle

and the centre of mass of the nucleus.

Next we denote the masses of the neutron, the proton, and the nuclei of atomic numbers Z and Z-1 by m_n, m_p, M_Z and M_{Z-1} respectively. The differences $m_n - m_p$ and $M_Z - M_{Z-1}$ are ~~being~~ so small that the kinetic energy of the system can be written approximately as

$$H = \frac{1}{2m'}\vec{p}'^2 + \frac{1}{2M'}\vec{P}'^2$$

where p' and P' are the momentum vectors of the heavy particle and the nucleus respectively and

$$m' = \frac{m_n + m_p}{2}, \qquad M' = \frac{M_Z + M_{Z-1}}{2}$$

The proper energy of the system has the form

$$\begin{pmatrix} (m_n + M_Z)c^2 & 0 \\ 0 & (m_p + M_{Z-1})c^2 \end{pmatrix}$$

or simply

$$\begin{pmatrix} 0 & 0 \\ 0 & +D \end{pmatrix},$$

where $D = (m_n + M_Z - m_p - M_{Z-1})c^2$ $(m_p + M_{Z-1} - m_n - M_Z)c^2$, as the difference of the energies of two states is only important.

Thus the Hamiltonian of the system becomes

$$H = \frac{1}{2m'}\vec{p}'^2 + \frac{1}{2M'}\vec{P}'^2 + \begin{pmatrix} V_{11}^{r}(r) & J_{12}^{r}(r) \\ V_{21}^{r}(r) & V_{22}^{r}(r) \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & +D \end{pmatrix} \qquad (1)$$

Which can be transformed into the form

$$H = \frac{1}{2m}\vec{p}^2 + \frac{1}{2M}\vec{P}^2 + \begin{pmatrix} V_1(r) & \tilde{V}_{12} \\ V_{21}(r) & V_{22}(r) \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & +D \end{pmatrix} \qquad (2)$$

where $\vec{p}$ and $\vec{P}$ are the relative momentum and the momentum of the centre of mass respectively and m and M are the reduced mass $\frac{mM'}{m'+M'}$ and the total mass $m'+M'$ respectively.

In the Schrödinger equation formed from this Hamiltonian, the relative coordinates $\vec{r}$ and the coordinates $\vec{R}$ of the centre of mass are already separated ~~already~~, so that the eigenfunction $\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$ can be written in the form $\chi(\vec{R})\begin{pmatrix}\psi_1(\vec{r})\\ \psi_2(\vec{r})\end{pmatrix}$, ~~whereas~~ the ~~total~~ energy E of the system being ~~can be~~ decomposed into the kinetic energy E_M and the energy E of the relative motion.

The functions X, ψ_1 and ψ_2 ~~where in which u, u and U~~ satisfy the differential equations

$$\frac{h^2}{8\pi^2 M}\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}+\frac{\partial^2}{\partial Z^2}\right)X + E_M X = 0, \qquad (3)$$

and

$$\left.\begin{aligned} &\frac{h^2}{8\pi^2 m}\Delta\psi_1 + (E - V_{11})\psi_1 - V_{12}\psi_2 = 0 \\ &\frac{h^2}{8\pi^2 m}\Delta\psi_2 + (E - V_{22})\psi_2 - V_{21}\psi_1 = 0 \end{aligned}\right\} \qquad (4)$$

As V's are assumed to be functions of r only, ψ_1 and ψ_2 can take the forms

$$\left.\begin{aligned} \psi_1 &= \frac{u_1(r)}{r} Y_{l,m}(\theta,\phi) \\ \psi_2 &= \frac{u_2(r)}{r} Y_{l,m}(\theta,\phi) \end{aligned}\right\},$$

where Y_{lm} is the spherical harmonic, while u_1 and u_2 should satisfy the simultaneous equations

$$\left.\begin{aligned} &\frac{d^2 u_1}{dr^2} + \left\{\kappa^2(E - V_{11}) - \frac{l(l+1)}{r^2}\right\}u_1 - \kappa^2 V_{12} u_2 = 0 \\ &\frac{d^2 u_2}{dr^2} + \left\{\kappa^2(E - D - V_{22}) - \frac{l(l+1)}{r^2}\right\}u_2 - \kappa^2 V_{21} u_1 = 0 \end{aligned}\right\} \quad (5)$$

where $\kappa^2 = \frac{8\pi^2 m}{h^2}$.

Thus the problem of ~~the~~ collision ~~interaction~~ of the heavy particle with the nucleus is reduced to that of solving ~~these simultaneous equations~~ (5) under ~~special~~ suitable conditions.

§ 2. ~~General Expressions of the cross sections of~~ Scattering and Disintegration by Neutron Impact.

Now we want to apply the above general method to the calculation of the cross section of the ~~nuclear~~ disintegration of the nucleus with the atomic number Z by neutron impact with proton emission.

As the simplest form of the interaction potential, we take

$$\cancel{V = J = -J} \quad V_{12} = V_{21} = J \;^{1)}$$

$$\cancel{V_{12} = J e^{i\theta} \qquad V_{21} = J e^{-i\theta},}$$

1) More generally for a case $V_{12} = Je^{i\theta}$, $V_{21} = Je^{-i\theta}$, ~~this~~ can be reduced the above, by taking $e^{i\theta}\psi_2$ instead of ψ_2.

for r < a and

$$V_{12} = V_{21} = 0$$

for r > a, where J is a positive ~~and a real~~ ~~quantity~~ constant ~~and a real number~~, respectively and a is the nucleus radius. ~~As the phase θ can be reduced to zero without further restriction, we write simply $V_{12} = V_{21} = J$ hereafter,~~

Similarly, we take

$$V_{11} = -V_1 \text{ and } V_{22} = -V_2$$

for r < a and

$$V_{11} = 0 \quad \text{and} \quad V_{22} = \frac{(Z-1)e^2}{r}$$

for $r > a$, where V_1 and V_2 are positive constants. ~~and~~

In this case, the simultaneous equations (5) become

$$\left.\begin{aligned} &\frac{d^2u_1}{dr^2} + \{\kappa^2(E+V_1) - \frac{l(l+1)}{r^2}\}u_1 + \kappa^2 J e^{i\theta} u_2 = 0 \\ &\frac{d^2u_2}{dr^2} + \{\kappa^2(E - D + V_2) - \frac{l(l+1)}{r^2}\}u_2 + \kappa^2 J e^{-i\theta} u_1 = 0 \end{aligned}\right\} \qquad (6)$$

for $r < a$, and

$$\left.\begin{aligned} &\frac{d^2u_1}{dr^2} + \{\kappa^2 E - \frac{l(l+1)}{r^2}\}u_1 = 0 \\ &\frac{d^2u_2}{dr^2} + \{\kappa^2(E - D) - \frac{(Z-1)e^2}{r} - \frac{l(l+1)}{r^2}\}u_2 = 0 \end{aligned}\right\} \qquad (7)$$

for $r > a$, ~~where~~ ~~If we put~~

$$r = \frac{h}{4\pi\mu c}\, x \qquad E = 2\mu c^2 \cdot E_0$$

~~where μ is the~~ mass of

~~that is~~ i.e. If we express the length in the unit of $\frac{h}{4\pi\mu c} = 2 \times 10^{-11}$ cm (10^{-12} cm, $\frac{h}{mc}$, 10^{-14})

and the energy in the unit of ~~$2\mu c^2$~~ $= 10^6$ eV, ~~( ) reduces to the constant κ^2~~

~~and~~ e^2 we obtain the numerical values and ( )

$$\kappa^2 \cong 5 \qquad \cancel{\kappa^2} e^2 \cong 0.9 \qquad e^2 \cong 0.14$$

for r a and κ

$5^2 \cdot (4.77 \times 10^{-10})^2$

$e^2 =$ energy · length.

$= (4.77 \times 10^{-10})^2$ erg. cm.

$= (4.77 \times 10^{-10})^2 \cdot 0.6285 \times 10^6 \cdot 10^{12}$

$= 2 \times 10^{-1} \times 0.63 = 0.13$

4.77
4.77
33.39
3339
1908
22.7529

6 28
22.8
50 24
1256
1256
143184

for r a.

i) The solution for $r < a$. If we put

~~$u_1 = \lambda_{11} f_1 + \lambda_{12} f_2$~~ ~~$u_2 = \lambda_{21} f_1 + \lambda_{22} f_2$~~,

$$u_1 = K f_1 + J f_2 \qquad u_2 = J f_1 - K f_2$$

where

$$K = \frac{1}{2}\left\{V_1 - V_2 + D + \sqrt{(V_1 - V_2 + D)^2 + 4J^2}\right\}$$

the equations (6) are separated into the form

6

$$\left.\begin{aligned} K\frac{d^2 f_1}{dr^2} + \left\{\kappa^2(E+V_1)K - \frac{l(l+1)}{r^2}K + \kappa J^2\right\} f_1 &= 0 \\ K\frac{d^2 f_2}{dr^2} + \left\{\kappa^2(E-D+V_2)K - \frac{l(l+1)}{r^2}K - \kappa^2 J^2\right\} f_2 &= 0 \end{aligned}\right\} (8)$$

~~where~~ or

$$\left.\begin{aligned} \frac{d^2 f_1}{dr^2} + \left\{k_1^2 - \frac{l(l+1)}{r^2}\right\} f_1 &= 0 \\ \frac{d^2 f_2}{dr^2} + \left\{-k_2^2 - \frac{l(l+1)}{r^2}\right\} f_2 &= 0 \end{aligned}\right\} (9)$$

where $k_1^2 = \frac{\kappa^2}{2}\left(2E + \frac{V_1+V_2-D}{2} + \frac{1}{2}\sqrt{\quad}\right)$

$k_2^2 = (\cdots - \frac{1}{4}\sqrt{\quad ''})$,

~~which are~~ k_1 and k_2 become both real, if V_1, V_2-D are both positive.

The solutions of (9), which are cotinuous for r=0, are

$$f_1 = A_1\sqrt{k_1 r}\, J_{l+\frac{1}{2}}(k_1 r)$$
$$f_2 = A_2\sqrt{k_2 r}\, J_{l+\frac{1}{2}}(ik_2 r)$$

where A_1 and A_2 are arbitrary constants. Thus, the functions u_1 and u_2 ~~are~~ ~~become~~ takes the forms

$$u_1 = A_1 K\sqrt{k_1 r}\, J_{l+\frac{1}{2}}(k_1 r) + A_2 J\sqrt{k_2 r}\, J_{l+\frac{1}{2}}(ik_2 r)$$
$$u_2 = A_1 J\sqrt{k_1 r}\, J_{l+\frac{1}{2}}(k_1 r) - A_2 K\sqrt{k_2 r}\, J_{l+\frac{1}{2}}(ik_2 r)$$

k_1, k_2 become real or imag. ac. as relative magnitudes of E, V_1, V_2, D and J change.

ii) The solution for $r > a$. The first of the equations (7) has the general solution

$$u_1 = B_1\sqrt{kr}\, H^{(1)}_{l+\frac{1}{2}}(kr) + B_2\sqrt{kr}\, H^{(2)}_{l+\frac{1}{2}}(kr)$$

where B_1 and B_2 are arbitrary constants. The second ~~of~~ equation is nothing but the Schrödinger equation in the Coulomb field, so that theexact solution can be written down at once. It is more convenient, however, to use the approximate solution of W.K.B. type, ~~Namely,~~ which take different forms in the following different cases ~~according as the the value of E~~.

a) The case $\frac{(Z-1)e^2}{a} \gg (E-D) > 0$, i.e. the energy of the incident neutron is such ~~so large~~ that the proton the nucleus can be disintegrated with theemission of the proton, but the energy of the latter ~~which is, however~~ is, far smaller than the maximum of the potential barrier. In this case, the equation

$$F(r) = \kappa^2(E-D) - \frac{\kappa^2(Z-1)e^2}{r} - \frac{l(l+1)}{r^2} = 0$$

for r has a positive solution

$$r_0 = \frac{(Z-1)e^2 + \sqrt{(Z-1)^2e^4 + 4l(l+1)(E-D)\kappa^{-2}}}{2(E-D)}$$

which is large compared with a. We obtain approximately

$$u_2(r) \cong \{-F(r)\}^{-\frac{1}{4}}\left\{C_1\exp\left(\int_r^{r_0}\sqrt{-F(r)}\,dr\right) + C_2\exp\left(-\int_r^{r_0}\sqrt{-F(r)}\,dr\right)\right\}$$

for $a < r \ll r_0$, and

7

$$\cong \cancel{(r-r_0)^{\frac{1}{2}}}\, C_1' \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n} \left(\frac{\alpha}{3}\right)^{2n}}{n!\, \prod_{m=1}^{n}(m-\frac{1}{3})} + C_2' \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1} \left(\frac{\alpha}{3}\right)^{2n}}{n!\, \prod_{m=1}^{n}(m+\frac{1}{3})}$$

where $x = r - r_0$, $\alpha^2 = \frac{1}{r_0^2}\frac{\sqrt{(Z-1)^2e^4 + 4\kappa^4 l(l+1)(E-D)}}{\kappa^2(E-D)}$,

for $r \cong r_0$ and

$$\cong \{F(r)\}^{-\frac{1}{4}} \left\{ C_1'' \exp\left(i\int_{r_0}^{r} \sqrt{F(r)}\,dr\right) + C_2'' \exp\left(-i\int_{r_0}^{r} \sqrt{F(r)}\,dr\right)\right\}$$

for $r \gg r_0$.

b) The case $\frac{(Z-1)e^2}{a} \cong E-D > 0$, i.e. the energy of the ~~proton~~ neutron is so large ~~as~~ that it ~~can emitt~~ from the proton with the energy comparable with the maximum potential, can be emitted by disintegration. In this case r_0 is not ~~small~~ large compared with a, so that the region $a < r \ll r_0$ in the case a) should be excluded.

c) The case $E-D < 0$, i.e. the neutron energy is smaller than the mass differe[nce] D, so that the proton emission is impossible. Inthis case

approximately for any value of r larger than a, where

iii) The boundary conditions. ~~Above solutions for different should~~ ~~These be connected small~~ The constants A's, B's and C's should be ~~connected smoothly~~ so chosen ~~that~~ as to form a ~~continuous~~ smooth function for the whole interval of r and ~~to~~ that it is small, converg to

~~General Solution~~ General ~~should~~ solution is the superposition of the above ~~solutions~~ for $l = 0, 1, 2, \cdots$, in which the constants A's, B's and C's functions n, m are so chosen as to form a smooth functions for the whole interval of r. For very large value of r, it should converge to the superposition of the plane neutron wave and the ~~scattered~~ diverging neutron ~~wave and the outgoing~~ and proton waves.

We want to consider in detail only the case a), ~~in which i.e.~~ which the neutron ~~In case a)~~, the scattering and the disintegration are only the ~~the scattering and the disintegration are important only for l=0, so that~~ energy E should satisfy the condition

$$0.7\, Z^{\frac{2}{3}} \gg E - D > 0$$

in the unit of 10^6 eV, if we assume ~~Z+1 ≫~~ $Z \gg 1$, and

$$a = \cancel{0.44}\ \frac{Z^{\frac{1}{3}}}{5}$$

in the unit of 10^{-12}cm. For example, the energy of the emitted

45
45
225
180
2025
45
90 . . .

iii) The boundary conditions. ~~The constant~~ ~~The~~ The most general solutions of ~~the problem~~ can be constructed by superposing the above functions u_1 and u_2 respectively for $l = 0, 1, 2, \cdots$, in which the constants A's, B's and C's are so chosen as to ~~become~~ form smooth functions for the whole interval of r. ~~For very large value of r, it should converge to~~ ~~The function is~~ The required solution uniquely determined by the condition that it should converge to the superposition of the plane neutron wave and the diverging neutron and proton waves for very large value of r.

Especially for $l = 0$, the solution should converge to the form

$$u_1 = \frac{\sin\sqrt{\kappa^2 E}\cdot r}{\sqrt{\kappa^2 E}} + C\, e^{i\sqrt{\kappa^2 E}\cdot r}$$

$$u_2 = D\, e^{i\sqrt{\kappa^2 (E-D)}\cdot r} \quad \text{or } 0$$

according as $E \gtrless D$.

The cross sections of s scattering and of s disintegration being ~~thus~~ thus expressed by

$$I_{scat.} = 4\pi|C|^2 \quad \text{and} \quad I_{dis} = 4\pi\sqrt{\frac{E-D}{E}}\,|D|^2 \quad \text{or } 0$$

~~respectively~~ according as $E \gtrless D$.

For ~~The~~ case a), ~~In this case~~ ~~i.e.~~, in which the neutron energy should satisfy the condition

$$0.7\,Z^{\frac{2}{3}} \gg E - D > 0$$

in the unit of 10^6 eV, if we assume $Z \gg 1$ and

$$a = \frac{Z^{\frac{1}{3}}}{5}$$

in the unit of 10^{-12} cm*. In this case, the ~~disintegration~~ ~~emitted~~ ~~proton~~ emitted proton ~~can be emitted~~ with has the energy small compared with $0.7Z^{\frac{2}{3}}$, which ~~equals to~~ is about 6×10^6 eV for $Z = 27$ for instance,

Letter to the Editor

Reduction of Nuclear Collision Problem

Among various processes caused by the collision of the heavy particle with the nucleus, only two cases, namely the elastic scattering and the capture with -ray emission, have been solved by straightforward application of the quantum mechanical method. Other processes such as the disintegration with the emission of another heavy particle or the inelastic scattering with the excitation of the nucleus have been considered to be so complicated that we should be satisfied only with rough estimations of their cross sections.

These cases, however, can also be reduced to one body problems so as to make it possible to apply the exact theory of collision in the following way.

We consider a nucleus of atomic number Z and mass number M, which can be in one of the states

$(q\,q \quad q), \quad (q\,q \quad q), \quad (q\,q \quad q)$

with energies W , W , W , respectively, where q , q , q ~~are~~ represent coordinates and spins of M heavy particles constituting the nucleus. ~~Now, if a heavy particle, for instance a neutron for example, approaching~~ Now, if itis collided by a heavy particle, a neutron for example, ~~but~~ remains ~~still~~ always in the normal state, the eigenfunction of the system consisting of the nucleus and the neutron has the form u (q) (q q q) approximately, where q represent the coordinates and spin of the latter. ~~Frome the Hamiltonian~~

The ~~wave equation~~

~~H +H +V~~

If we denote the Hamiltonians of the nucleus and the neutron by H and H respectively ~~and~~, the interaction potential by V, and the total energy by E, the ~~wave~~ Schrödinger equation becomes

$$(H + H + V - E)\,u = 0, \qquad (1)$$

which can be reduced to the form

The cross section of disintegration ~~takes the form~~ becomes

$$\Phi_{dis} \cong \frac{4\pi\Lambda_1}{k} \, \frac{\exp\left(\cancel{-\pi} \dfrac{-\pi \kappa^2 (Z-1)e^2}{\sqrt{\kappa^2(E-D)}} + 4\sqrt{\kappa^2(Z-1)e^2 a}\right)}{\cancel{\sqrt{\kappa^2 E}}}$$

approximately, where

$$\Lambda_1 = \frac{(k_1 \tanh k_2 a - k_2 \tan k_1 a)^2}{(\cancel{\lambda_1}\Gamma_1 k_2 + \cancel{\lambda_2}\Gamma_2 k_1)^2 + k^2(\cancel{\lambda_1}\Gamma_1 \tanh k_2 a + \cancel{\lambda_2}\Gamma_2 \tan k_1 a)^2}$$

$$\lambda_1 = \frac{J}{K} \qquad \lambda_2 = -\frac{K}{J},$$

$$= \frac{-(V_1 - V_2 + D) + \sqrt{\quad}}{2J} \qquad \left(= \frac{-(\quad) - \sqrt{\quad}}{2J}\right)$$

$$\Gamma_1 = \frac{J}{K}\left(G^{\frac{1}{4}} \tan k_1 a + k_1 G^{-\frac{1}{4}}\right)$$

$$\Gamma_2 = \frac{K}{J}\left(G^{\frac{1}{4}} \tanh k_2 a + k_2 G^{-\frac{1}{4}}\right)$$

$$G = \kappa^2(E-D) - \frac{\kappa^2(Z-1)e^2}{a} \quad 2\kappa^2(E-D) - $$

$$-3.5\, Z^{\frac{2}{3}}$$

$$k_1 = $$

$$\Lambda_1 = \frac{J^2 K^2}{(J^2\Gamma_1 k_2 + K^2\Gamma_2 k_1)^2 + k^2(J^2\Gamma_1 \tanh k_2 a + K^2 \tan k_1 a\, \Gamma_2)^2}$$

$$\Gamma_1 = \frac{J}{K}\left(G^{\frac{1}{4}} \tan k_1 a + k_1 G^{-\frac{1}{4}}\right)$$

$$\Gamma_2 = $$

Letter to the Editor

Reduction of Nuclear Collision Problems

Among various processes caused by the collision of the heavy particle with the nucleus, only two cases, namely the elastic scattering and the capture with -ray emission, have been solved hitherto by straightforward application of the quantum mechanical method. Now it will be shown that other processes can also be reduced to one body problems in the following way so as to make it possible to apply the exact theory of collision. to them.

First, if only the possibility of the elastic scattering is considered, the stationary state of the whole system will be expressed in the form u (q) (q q ....q), where u is the superposition of plane and scattered waves for the impinging particle, in the neutron state for instance, and represents the normal state of the nucleus with atomic number Z and mass number N. Next, if the possibilities of the excitation of the nucleus to other states, ... with energies W, ... say, and of the disintegration with the emission of a heavy particle, in the proton state for instance, are taken into account, the stationary solution should have the general form

(q q q ....q) = u (q) (q q ....q) + u (q) (q q ....q) + ...

+ v (q) (q q ....q) + ... , (1)

where u ,.... and v ,.... represent inelastically scattered neutron waves and outgoing proton waves respectively and ,...., represent stationary

1) The recoil of the nucleus will be neglected throughout, which will be permitted for majority of cases.

The factor Λ_1 ~~is does not depends on~~ changes rather slowly as the neutron energy E changes, & In general, and has the order of magnitude 1 in the unit above used, of ~~10^{-24}~~ cm^2 so that

$$\Sigma_{dis} \cong 10^{-24} \exp\left(\frac{-\pi\kappa^2(Z-1)e^2}{\sqrt{\kappa^2(E-D)}} + 4\sqrt{\kappa^2(Z-1)e^2 a}\right) \Big/ \sqrt{\kappa^2 E} \quad cm^2$$

D=1. 0.65 9 exp.(−6.30)

$3\,2=27$, 5.85, 0.45 2.9, 12.15 / 6.30, 3.15, 9.0, 12.15

$$= \frac{0.5\cdot 10^{-24} \exp\left(-0.45\frac{Z}{\sqrt{E-D}} + 0.65\, Z^{\frac{2}{3}}\right)}{\sqrt{E}} \quad cm^2$$

1.6×10^{-4}

where E and D are expressed in the unit of 10^6 eV, Thus the cross section ~~In The resonance of Λ_1 becomes, however~~ ~~becomes~~ is very small ~~when~~ as the energy E − D of the emitted proton becomes ~~becomes small compared with 0.45 Z~~
~~in general~~ as the energy E − D of the emitted proton becomes small compared

$$\frac{0.45\,Z}{\sqrt{E-D}} \gg 0.65\, Z^{\frac{2}{3}}$$

with ~~0.9~~ $0.5\, Z^{\frac{2}{3}}$ ~~E−D~~

1.35, 6.3, 4.05, 8.10

$kT = 1.38\times10^{-16}$ erg
$\times 300$
$= 0.6285\times 10^{-6}\times \cdot \times 10^6$ eV
2.5×10^{-2}

0.8.5 / 2.5

~~In general,~~ The mass defect D is positive in the majority of cases, so that 45435

If the mass defect D is negative, ~~which is seldom~~ a rare case ~~realized~~, the cross section for slow neutron ~~will~~ becomes ~~very large or as can~~ be expected, of course, very large,
~~The exceptional case~~

Except such a rare occasion, the ~~factor~~ cross section becomes abnormally large ~~when~~ the ~~numer denominator~~ factor of Λ_1, ~~is~~ becomes very ~~small~~ large ~~i.e.~~ only for slow neutron ~~i.e.~~

$\Gamma_1 k_2 + \Gamma_2 k_1 \cong 0,$

$\Gamma_1 \cong$ ~~$\tan k_1 a$~~ ( )

$$\frac{J}{K}\frac{\tan k_1 a}{k_1} + \frac{K}{J}\frac{\tan k_2 a}{k_2} = -\left(\cancel{A_0}\frac{J}{K} + \frac{K}{J}\right) G^{-\frac{1}{2}}$$

Letter to the Editor

Reduction of Nuclear Collision Problems

Among various processes caused by the collision of the heavy particle with the nucleus, only two cases, namely the elastic scattering and the capture with -ray emission, have been solved hitherto by straightforward application of the quantum mechanical method. Now ~~it~~ it will be shown that other processes can also be reduced to one body problems in the following way so as to make it possible to apply the exact theory of collision to them.

First, if only the possibility of the elastic scattering is considered, the stationary state of the whole system will be expressed in the form u (q) (q q ....q), where u is the superposition of plane and scattered waves for the impinging particle, in the neutron state for instance, and represents the normal state of the nucleus with atomic number Z and mass number N. Next, if the possibilities of the excitation of the nucleus to other states, ... say, and of the disintegration with the emission of a heavy particle, in the proton state for instance, are taken into account, the stationary solution should have the general form

$$(q\, q\, q \ldots q) = u\,(q)\,(q\, q \ldots q) + u\,(q)\,(q\, q \ldots q) + \ldots\ldots$$
$$+\, v\,(q)\,(q\, q \ldots q) + \ldots\ldots, \qquad (1)$$

where u ,.... and v ,.... represent inelastically scattered neutron waves and outgoing proton waves respectively and ,.... represent stationary

1) The recoil of the nucleus will be neglected throughout, which will be permitted for majority of cases.

~~which is the condition~~
only when the resonance condition

$$\cancel{\Gamma_1 k_2 + \Gamma_2 k_1 \cong 0}$$

is satisfied,

~~Further~~, the factor $\exp\left(-0.45\,\frac{2}{\sqrt{E-D}} + 0.65\right.$

The cross section of elastic scattering takes the form

$$\Sigma_{scat} = \frac{4\pi}{k^2}\,\frac{\left[\left(\lambda_1\frac{\Gamma_1}{\Gamma_1}k_2 - \lambda_2\frac{\Gamma_2}{\Gamma_2}k_1\right)\sin ka - (\Gamma_1\tanh k_2 a - \Gamma_2\tanh k_1 a)k\cos ka\right]^2}{\cancel{\lambda_1\Gamma_1 k_2 - \lambda_2\Gamma}\;(\Gamma_1 k_2 - \Gamma_2 k_1)^2 + k^2(\Gamma_1\tanh k_2 a - \Gamma_2\tanh k_1 a)^2}$$

which becomes ~~so~~ large only when the mass defect D is positive. The occurrence of ~~The case~~ b) is very rare, so that it need not ~~so frequent as to~~ be considered in detail. ~~sure that it will be altogether~~

The case b), ~~in which~~ ~~$E-D<0$, i.e.~~. The proton emission is impossible because of the condition $E-D<0$. The cross section of the elastic scattering the same as ( ).

For slow neutrons, it ~~becomes~~ takes the form

$$\Sigma_{scat.} = 4\pi a^2\left(1 - \frac{\lambda\Gamma_1\tanh k_2 a - \Gamma_2\tanh k_1 a}{\Gamma_1 k_2 a - \Gamma_2 k_1 a}\right)^2, \quad (\ )$$

which becomes ~~a~~ ~~very large if the~~

$$= 0.5\cdot 2^{\frac{2}{3}}\left(1 - \frac{\Gamma_1\tanh k_2 a - \Gamma_2\tanh k_1 a}{\Gamma_1 k_2 a - \Gamma_2 k_1 a}\right)^2 \quad (\ )$$

$\times 10^{-24}$

resonance condition

$$\Gamma_1 k_2 - \Gamma_2 k_1 = 0 \quad (\ )$$

is satisfied.

~~If the~~ The exchange force is neglected, the expression ( ).

~~become~~ becomes

3

In the above deduction, it was assumed that the scattered or the emitted particle can be identified with the impinging particle. In more complicated case, in which it is possible that the latter is captured and one of the particles in the nucleus is emitted as a neutron or a proton, similar reduction as above can be performed, only if the range of forces (except Coulomb force, of course,) between heavy particles are so small that they can be replaced by δ-functions.

Namely, inserting the stationary solution of the form

$$\psi = u_0(q_0)\chi_0(q_1 q_2 \cdots q_N) + \cdots\cdots + v_1(q_1)\varphi_1(q_0 q_2 \cdots q_N) + \cdots\cdots$$

in the equation (2) and integrating with respect $q_1 q_2 \cdots q_N$ after having multiplied both sides by $\tilde{\chi}_0, \cdots, \tilde{\varphi}_1, \cdots$ in turn, we obtain the equations

$$H_1 u_0 + V_{01} u_1 + \cdots + K_{01} v_1 + \cdots = (E - W_0) u_0 \qquad (4)$$

$$\cdots\cdots$$

~~for~~ involving q_0 only, where

$$K_{01}(q_0) v_1(q_0) = \int \cdots \int \tilde{\chi}_0(q_1 q_2 \cdots q_N)\, V^* v_1(q_1)\varphi_1(q_0 q_2 \cdots q_N)\, dq_1 dq_2 \cdots dq_N, \cdots\cdots$$

on account of the term in V^* involving $\delta(q_1, q_0)$.3)

Further, by integrating (2) with respect to $q_1 q_2 \cdots q_N$ after having interchanged q_0 and q_1 in (2) and multiplied by $\tilde{\varphi}_1(q_1 q_2 \cdots q_N)$, …… in turn, we obtain the remaining equations

$$\left.\begin{array}{l} H_1 v_1 + K_{10} u_0 + \cdots + U_{11} v_1 + \cdots = (E - W_1') v_1 \\ \cdots\cdots, \end{array}\right\} \qquad (4)'$$

3) The symbol * means the interchange of q_0 and q_1 in V. Other terms in V^*, ~~a~~ invoving $\delta(q_1, q_2)$ for example, do not contribute to the integral on account of the exclusion principle for the particles in the nucleus.

$$\Phi_{scat.} = 4\pi a^2 \left(1 - \frac{\tan k_1 a}{k_1 a}\right)^2,$$

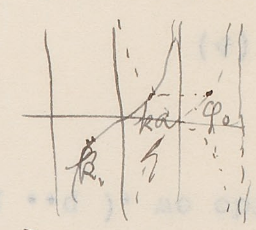
$\varphi(r) = \int^r y(\rho)\,d\rho$

where $k_1 = \sqrt{\kappa^2 V_1}$,

which ~~is equ~~ agrees with the expression (19) of Bethe[1],

~~and~~ if we put $a = r_0$, $k_1 = \cancel{k_0}\,\lambda_0^{-1}$ and $k_1 a = \varphi_0 \pm \frac{\pi}{2}$.

($\tan k_1 a = -\cot \varphi_0$) always ~~the phase~~ ~~the condition~~ deter



The ~~condition~~ of resonance ( ) reduces to the simple form $k_1 a = (n + \frac{1}{2})\pi$

where n is any integer, in the ~~extre~~ extreme limiting cases

$J = 0$ or $J \gg V_1, V_2, D, E$. In general case,

J ~~has no simple relation~~ means,

. . . .

means, in general, a rather complicated ~~cor~~relations between the ~~p~~ phases of the nuclear wave functions u_1 and u_2 at the boundary of the nucleus, and reduces

In the above deduction, it was assumed that the scattered or the emitted particle can be identified with the impinging particle. In more complicated case, in which it is possible that the latter is captured and one of the particles in the nucleus is emitted as a neutron or a proton, similar reduction as above can be performed, only if the range of forces (except Coulomb force, of course,) between heavy particles are so small that they can be replaced by -functions.

Namely, inserting the stationary solution of the form

$$= u(q)\ (q\ q \ldots q) + \ldots + v(q)\ (q\ q \ldots q) + \ldots$$

in the equation (1) and integrating with respect q q ...q after having multiplied both sides by in turn, we obtain the equations

$$H\,u + V\,u + \ldots + K\,v + \ldots = (E - W)u \qquad (4)$$

$$\ldots\ldots$$

for q only, where

$$K(q)v(q) = \quad (q\ q \ldots q)\ V\ v(q)\ (q\ q \ldots q)\ dq\ dq \ldots dq\ , \ \ldots$$

on account of the term in V involving (q ,q).

Further, by integrating (1) with respect to q q ...q after having interchanged q and q in (1) and multiplied by (q q ..q), we obtain the remaining equations

$$H\,v + K\,u + \ldots + U\,v + \ldots = (E - W)v \qquad (4)$$

$$\ldots\ldots,$$

3) The symbol * means the interchange of q and q in V. Other terms in V, involving (q ,q) for example, do not contribute to the integral on account of the exclusion principle for the particles in the nucleus.