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Theory of Disintegration of the Nucleus

by Neutron Impact

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Abstract β dis

The cross sections of the elastic scattering of the ^{nuclear} disintegration by neutron impact with the emission of the proton was calculated on the assumption that the emitted proton can be identified with the impinging neutron. Also the cross sections of the elastic scattering and the capture of the neutron was calculated on the same assumption altered by taking the above process into account. Especially, the ratio $\beta_{\text{elastic}}/\beta_{\text{capture}}$ can be either larger or smaller than 1, but the anomalous absorption of the neutron without inelastic scattering, for Cd etc for example, can not be explained by the collision of the heavy particle ^{only}.

General considerations in the case of β disintegration, among various processes caused by the collision of the heavy particle with the nucleus, there are two cases, in which the cross sections can be obtained by the exact method of the quantum mechanics. They are, namely, (1) the exact method of the quantum mechanics (2) the exact method of the quantum mechanics

the elastic scattering and the capture of the heavy particle with the emission of the γ ray. Other processes such as the disintegration with the emission of another heavy particle or the ~~elastic~~ inelastic scattering with the excitation of the nucleus have been considered to be so complicated that ~~only~~ rough estimations we should be satisfied with of their cross sections.

In this paper, the authors want to work out a simple method, though rather phenomenological, of treating some of them, for example, the disintegration by neutron impact with proton emission.

Now, according to the present theory of Heisenberg, Fermi and others, the atomic nucleus consists of ~~the~~ neutrons and ~~the~~ protons, β -disintegration being considered to be ^{caused} accompanied by the transformation of a neutron in the nucleus into a proton. On this point of view, it is possible that the neutron colliding with the nucleus of atomic number Z changes itself into the proton,

the atomic number of the nucleus being reduced to $Z-1$ simultaneously. Consequently, if the neutron and the proton are considered to be two states of the so called heavy particle, a part, at least of the neutron produced disintegration with proton emission, can be attributed to simple scattering of the ~~so called~~ heavy particle by the nucleus. *(a sort of reduced)*

In order to formulate such an idea, we deal with a system consisting of a heavy particle and a nucleus, between which following types of forces act.

- i) Ordinary short range force between the heavy particles and the nucleus.
- ii) Coulomb force between the proton and the nucleus.
- iii) The force which causes the transition of the heavy particle from the neutron to the proton state at the same time with the reduction of the atomic number of the nucleus by one.

iv) The force which causes the transition inverse to the above.

We consider the case, in which a neutron and a nucleus of the atomic number Z existed initially transfer into a proton and a nucleus of the atomic number $Z-1$ *changes*

If we denote the state of the system, in which the former the heavy particle is neutron and the nucleus has the atomic number Z , by 1 , and the state, in

which the former is proton and the latter has the atomic number $Z-1$, by 2 , these two states are linked together *(should have)* by the forces iii), iv), so that the eigenfunction of the system ~~has~~ two components, which will be written as (ψ_1, ψ_2) . Corresponding to this, the interaction energy becomes *they*

matrix of the form

$$\begin{pmatrix} J & J \\ J & J \end{pmatrix}
 \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}
 \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

where $V_{11} = V_{11}$ ~~is the ordinary force~~ *is the ordinary force* ~~of the ordinary forces~~ *of the ordinary forces* $i)$, $ii)$, while V_{21} and V_{12} mean the ordinary forces $iii)$, $iv)$, *by the forces iii), iv), so that* ~~the potentials~~ *the potentials*. V_{12} and V_{21} mean the exchange forces $iv)$ and $iii)$ respectively, ~~both~~ *both*. In general V_{11}, V_{22} will depend on the coordinates, the momenta and the spins V_{12}, V_{21}

of all the particles constituting the system. We want, however, to deal with the problem as simply and as phenomenologically as possible. For this reason, we assume J 's to be functions only of the distance r between the heavy particle *from* V 's

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for which where V_{12} and $V_{1'2'}$ mean the ordinary forces i), ii), while $V_{12'}$ and

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} V_{12} & V_{12'} \\ V_{1'2'} & V_{1'2} \end{pmatrix} \quad \begin{pmatrix} V_{12} & V_{12'} \\ V_{1'2'} & V_{1'2} \end{pmatrix}$$

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and the centre of mass of the nucleus.

Next we denote the masses of the neutron, the proton, and the nuclei of atomic numbers Z and $Z-1$ by m_n , m_p , M_2 , M_{Z-1} respectively. The differences $m_n - m_p$ and $M_2 - M_{Z-1}$ being so small that the kinetic energy of the system can be written approximately as

$$H = \frac{1}{2m} \vec{p}'^2 + \frac{1}{2M'} \vec{P}'^2$$

where \vec{p}' and \vec{P}' are the momentum vectors of the heavy particle and the nucleus respectively and

$$M' = \frac{m_n + m_p}{2}, \quad M' = \frac{M_2 + M_{Z-1}}{2}$$

The proper energy of the system has the form

$$\begin{pmatrix} (m_n + M_2) c^2 & 0 \\ 0 & (m_p + M_{Z-1}) c^2 \end{pmatrix}$$

or simply

$$\begin{pmatrix} 0 & 0 \\ 0 & +D \end{pmatrix},$$

where $D = (m_n + M_2 - m_p - M_{Z-1}) c^2$, as the difference of the energies of two states is only important.

Thus the Hamiltonian of the system becomes

$$H = \frac{1}{2m} \vec{p}'^2 + \frac{1}{2M'} \vec{P}'^2 + \begin{pmatrix} V_1(r) & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & +D \end{pmatrix} \quad (1)$$

Which can be transformed into the form

$$H = \frac{1}{2m} \vec{p}^2 + \frac{1}{2M} \vec{P}^2 + \begin{pmatrix} V_1(r) & \vec{V}_1 \\ \vec{V}_2 & V_2(r) \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & +D \end{pmatrix} \quad (2)$$

where \vec{p} and \vec{P} are the relative momentum and the momentum of the centre of mass respectively and m and M are the reduced mass $\frac{mM}{m+M}$ (and $m+M$) respectively.

In the Schrödinger equation formed from this Hamiltonian, the relative coordinates \vec{r} and the coordinates of the centre of mass are separated, already, so that the eigenfunction $\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ can be written in the form $\chi(\vec{R}) \begin{pmatrix} \psi_1(\vec{r}) \\ \psi_2(\vec{r}) \end{pmatrix}$, whereas the energy E of the system can be decomposed into the kinetic energy E_r and the energy E_M of the relative motion.

The functions ψ_1, ψ_2 and χ satisfy the differential equations where in which u_1 and u_2 satisfy the differential equations

$$\frac{\hbar^2}{8\pi^2 M} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \chi + E_M \chi = 0, \quad (3)$$

$$\left. \begin{aligned} \frac{\hbar^2}{8\pi^2 m} \Delta \psi_1 + (E - V_{11}) \psi_1 - V_{12} \psi_2 &= 0 \\ \frac{\hbar^2}{8\pi^2 m} \Delta \psi_2 + (E - V_{22}) \psi_2 - V_{21} \psi_1 &= 0 \end{aligned} \right\} \quad (4)$$

As V 's are assumed to be functions of r only, ψ_1 and ψ_2 can take the forms

$$\left. \begin{aligned} \psi_1 &= \frac{u_1(r)}{r} Y_{lm}(\theta, \phi) \\ \psi_2 &= \frac{u_2(r)}{r} Y_{l'm'}(\theta, \phi) \end{aligned} \right\}$$

where Y_{lm} is the spherical harmonic, while u_1 and u_2 should satisfy the simultaneous equations

$$\left. \begin{aligned} \frac{d^2 u_1}{dr^2} + \left\{ k^2 (E - V_{11}) - \frac{l(l+1)}{r^2} \right\} u_1 - k^2 V_{12} u_2 &= 0 \\ \frac{d^2 u_2}{dr^2} + \left\{ k^2 (E - V_{22}) - \frac{l'(l'+1)}{r^2} \right\} u_2 - k^2 V_{21} u_1 &= 0 \end{aligned} \right\} \quad (5)$$

where $k^2 = \frac{8\pi^2 m E}{\hbar^2}$.

Thus the problem of the interaction of the heavy particle with the nucleus is reduced to that of solving these simultaneous equations under suitable conditions.

§ 2. General Expressions of the Cross Sections of Scattering and Disintegration by Neutron Impact.

Now we want to apply the above general method to the calculation of the cross section of the nuclear disintegration by neutron impact with proton of the nucleus with the atomic number Z .

As the simplest form of the interaction potential, we take

$$\left. \begin{aligned} V_{12} &= -J \\ V_{21} &= V J e^{i\theta} \\ V_{11} &= 0 \end{aligned} \right\} \quad (6)$$

for $r < a$ and for $r > a$, where J is a positive quantity constant, respectively and a is the nuclear radius. As the phase θ can be reduced to zero without further restriction, we write simply $V_{12} = V_{21} = -J$ hereafter.

Similarly, we take $V_{11} = -V_1$ and $V_{22} = -V_2$ for $r < a$ and

for $r < a$ and

$$V_{12} = -V_1 \text{ and } V_{22} = -V_2$$

Similarly, we take $V_{12} = -V_1$ and $V_{22} = -V_2$ for $r > a$, where Δ is a positive quantity comparable to the nuclear radius.

for $r < a$ and

$$V_{12} = 0$$

$$V_{22} = -V_2 e^{-\Delta r}$$

$$V_{12} = -V_1 e^{-\Delta r} \quad V_{22} = -V_2 e^{-\Delta r} \quad (1)$$

As the simplest form of the interaction potential, we take

cross section of the nuclear disintegration by neutron impact with proton

Now we want to apply the above general method to the calculation of the

conditions:

is reduced to that of solving these simultaneous equations (2), (3), (4) and (5)

Thus the problem of the interaction of the heavy particle with the nucleus

$$\text{where } K = \frac{h}{2\pi m v}$$

$$\frac{\partial \psi}{\partial t} + \left\{ K_2 (E \pm D - \Delta^2 r^2) - \frac{V(r)}{h} \right\} \psi - K_1 \Delta^2 \psi = 0 \quad (2)$$

simultaneous equations

where Y_{lm} is the spherical harmonic, while N_1 and N_2 should satisfy the

$$\Delta^2 \psi = \frac{1}{N_1} \frac{\partial \psi}{\partial r} Y_{lm}(\theta, \phi)$$

$$\Delta^2 \psi = \frac{1}{N_2} \frac{\partial \psi}{\partial r} Y_{lm}(\theta, \phi)$$

As N_1 and N_2 are assumed to be functions of r only, $\Delta^2 \psi$ and $\Delta^2 \psi$ can take the forms

$$\frac{\partial \psi}{\partial r} = \frac{1}{N_1} \Delta^2 \psi + (E - \Delta^2 r^2) \psi - \Delta^2 \psi = 0 \quad (3)$$

$$\frac{\partial \psi}{\partial r} = \frac{1}{N_2} \Delta^2 \psi + (E - \Delta^2 r^2) \psi - \Delta^2 \psi = 0 \quad (4)$$

where $\Delta^2 \psi$ and $\Delta^2 \psi$ satisfy the differential equations

$$\Delta^2 \psi = 0$$

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$$V_{11} = 0 \text{ and } V_{22} = \frac{(Z-1)e^2}{r}$$

for $r > a$, where V_1 and V_2 are positive constants.

In this case, the simultaneous equations become

$$\left. \begin{aligned} \frac{d^2 u_1}{dr^2} + k^2(E + V_1) - \frac{l(l+1)}{r^2} u_1 + i\sigma \alpha_1 e^{i\sigma} u_2 &= 0 \\ \frac{d^2 u_2}{dr^2} + k^2(E + V_2) - \frac{l(l+1)}{r^2} u_2 + i\sigma \alpha_2 e^{i\sigma} u_1 &= 0 \end{aligned} \right\} \quad (6)$$

for $r < a$, and

$$\left. \begin{aligned} \frac{d^2 u_1}{dr^2} + k^2 E - \frac{l(l+1)}{r^2} u_1 &= 0 \\ \frac{d^2 u_2}{dr^2} + k^2(E - D) - \frac{l(l+1)}{r^2} u_2 &= 0 \end{aligned} \right\}$$

for $r > a$, where

$$k = \frac{h}{\hbar} \sqrt{2m(E - V)}$$

$$E = 2mc^2 \cdot E_0$$

where \hbar is the mass of

that is i.e. If we express the length in the unit of $\frac{\hbar}{4\pi mc}$

and the energy in the unit of 10^6 eV , α reduces to the constant χ^2

$$\chi^2 \approx 5$$

$$\alpha e^2 \approx 0.19$$

$$e^2 \approx 0.14$$

for $r < a$ and χ

$$5^2 \cdot (4.77 \times 10^{-10})^2$$

$$\begin{array}{r} 4.77 \\ 4.77 \\ \hline 33.39 \\ 33.39 \\ \hline 1408 \\ 1408 \\ \hline 2275.29 \end{array}$$

$e^2 = \text{energy} \cdot \text{length}$

$$= (4.77 \times 10^{-10}) \cdot \text{erg} \cdot \text{cm}$$

$$= (4.77 \times 10^{-10})^2 \cdot 0.6285 \times 10^6 \cdot 10^{-12}$$

$$= 2 \times 10^{-11} \times 0.63 = 0.13$$

$$\begin{array}{r} 628 \\ 22.78 \\ \hline 5024 \\ 1256 \\ \hline 143184 \end{array}$$

for $r > a$, 2275.29

i) The solution for $r < a$. If we put

$$\left. \begin{aligned} u_1 &= A_1 f_1 + \lambda_{12} f_2 \\ u_2 &= B_1 f_1 + \lambda_{22} f_2 \\ A_{11} &= K \\ A_{21} &= \frac{1}{2} (V_1 - V_2 + D + \sqrt{(V_1 - V_2 + D)^2 + 4J^2}) \end{aligned} \right\}$$

the equations (6) are separated into the form

for $\tau > a$, where V_1 and V_2 are positive constants.

$$V_{II} = 0 \text{ and } V_{II} = \frac{(S-I)e^{-\tau}}{I}$$

In this case the homogeneous equations become

$$\begin{cases} \frac{d^2 N}{d\tau^2} + k(E+V_1)N = 0 \\ \frac{d^2 N}{d\tau^2} + k(E+V_2)N = 0 \end{cases} \quad (P)$$

for $\tau < a$, and

$$\frac{d^2 N}{d\tau^2} + k(E+V_1)N = 0$$

for $\tau > a$, where I is the current

$$E = 5hmc \cdot E_0$$

To find the length in the unit of λ , we substitute $V_1 = 10^6$ eV and the energy in the unit of mc^2 is 10^6 eV.

we obtain the numerical values

$$k \approx 2, \quad \lambda \approx 0.14 \mu$$

for $\tau < a$ and $\tau > a$

$$N = A e^{i(k\tau - \omega\tau)} + B e^{-i(k\tau - \omega\tau)}$$

for $\tau < a$, $N = C e^{i(k\tau - \omega\tau)} + D e^{-i(k\tau - \omega\tau)}$

$$= 5 \times 10^{-1} \times 10^6 = 0.13$$

1) The solution for $\tau < a$. If we put

where

$$N = \frac{1}{2} (N_1 - N_1 + D + \sqrt{N_1^2 - N_1 + D}) e^{i(k\tau - \omega\tau)}$$

the equations (P) are separated into the form

$$K \frac{d^2 f_1}{dr^2} + \left\{ \kappa^2 (E + V_1) K - \frac{l(l+1)}{r^2} K + \kappa T^2 \right\} f_1 = 0 \quad (8)$$

or where

$$K \frac{d^2 f_2}{dr^2} + \left\{ \kappa^2 (E - D + V_2) K - \frac{l(l+1)}{r^2} K - \kappa T^2 \right\} f_2 = 0$$

$$\frac{d^2 f_1}{dr^2} + \left\{ k_1^2 - \frac{l(l+1)}{r^2} \right\} f_1 = 0 \quad (9)$$

$$\frac{d^2 f_2}{dr^2} + \left\{ -k_2^2 - \frac{l(l+1)}{r^2} \right\} f_2 = 0$$

The solutions of (8), which are continuous for $r=0$, are
 $f_1 = A_1 \sqrt{k_1 r} J_{l+\frac{1}{2}}(k_1 r)$
 $f_2 = A_2 \sqrt{k_2 r} J_{l+\frac{1}{2}}(ik_2 r)$
 where A_1 and A_2 are arbitrary constants. Thus, the functions u_1 and u_2 become takes the forms

$$u_1 = A_1 K \sqrt{k_1 r} J_{l+\frac{1}{2}}(k_1 r) + B_1 A_2 J \sqrt{k_2 r} J_{l+\frac{1}{2}}(ik_2 r)$$

$$u_2 = A_1 J \sqrt{k_1 r} J_{l+\frac{1}{2}}(k_1 r) + B_2 A_2 K \sqrt{k_2 r} J_{l+\frac{1}{2}}(ik_2 r)$$

ii) The solution for $r > a$. The first of the equations (7) has the general solution

$$u_1 = B_1 \sqrt{k r} H_{l+\frac{1}{2}}^{(1)}(k r) + B_2 \sqrt{k r} H_{l+\frac{1}{2}}^{(2)}(k r)$$

where B_1 and B_2 are arbitrary constants. The second of equations is nothing but the Schrödinger equation in the Coulomb field, so that the exact solution can be written down at once. It is more convenient, however, to use the approximate solution of W.K.B. type, which take different forms in different regions.

A) The case $(Z-1)e^2 \gg (E-D) a$, i.e. the energy of the incident neutron is much larger than the energy of the proton, but the latter being in the potential barrier. In this case, the equation for r has a positive solution

$$F(r) = \frac{(Z-1)e^2}{r} e^{-\frac{r}{a}} \frac{r}{a} = 0$$

$$r_0 = \frac{(Z-1)e^2}{2(E-D)}$$

which is large compared with a . We obtain approximately

$$u_2(r) \approx \left\{ F(r) \right\}^{-\frac{1}{2}} \left\{ C \exp \left(\int_a^{r_0} \sqrt{-F(r)} dr \right) + C_2 \exp \left(- \int_a^{r_0} \sqrt{-F(r)} dr \right) \right\}$$

for $a \ll r \ll r_0$ and

for $a \ll r \ll r_0$ and

$$M_2(r) \approx \left\{ F(r) \right\} \int_{r_0}^{\infty} \sqrt{V(r)-E} dr + C_2 \exp(-\int_{r_0}^{\infty} \sqrt{V(r)-E} dr)$$

which is large compared with $e^{-\dots}$ we obtain approximately

$$M_2(r) \approx \frac{5\pi(E-D)}{2k} \left(\frac{1}{\sqrt{5-1}} \sqrt{5-1} + \frac{1}{\sqrt{6+1}} \sqrt{6+1} \right) (E-D)$$

for r has a positive solution

$$k(r) = \sqrt{V(r)-E} = \sqrt{B-D} - \frac{1}{r} = 0 \Rightarrow r = \frac{1}{B-D}$$

the potential barrier. In this case, the equation

the proton, ψ , the energy E is smaller than the maximum of

re-~~fer~~ence that the proton the nucleus can be disintegrated with the emission of

α a). The case $\frac{1}{B} \gg \frac{1}{B-D} > 0$, i.e. the energy of the incident neutron is

approximate solution of W.K.B. type, ψ is $\exp(-\int \sqrt{V(r)-E} dr)$ and

can be written down at once. It is more convenient, however, to use the

but the Schrödinger equation in the Coulomb field, so that present solution

where B_+ and B_- are arbitrary constants. The second eq. equation is nothing

solution $M_1 = B_+ \sqrt{k_+} H_0^{(1)}(k_+ r) + B_- \sqrt{k_-} H_0^{(2)}(k_- r)$

ii) The solution for $r > a$. The first of the equations (3) has the general

become take the forms $M_2 = A_1 \sqrt{k_1} J_{l+1/2}(k_1 r) + A_2 \sqrt{k_2} J_{l+1/2}(k_2 r)$

where A_1 and A_2 are arbitrary constants. Thus, the functions M_1 and M_2 are

$M_1 = A_1 \sqrt{k_1} J_{l+1/2}(k_1 r) + A_2 \sqrt{k_2} J_{l+1/2}(k_2 r)$

$M_2 = A_3 \sqrt{k_3} J_{l+1/2}(k_3 r) + A_4 \sqrt{k_4} J_{l+1/2}(k_4 r)$

where A_3 and A_4 are arbitrary constants. Thus, the functions M_1 and M_2 are

$M_1 = A_1 \sqrt{k_1} J_{l+1/2}(k_1 r) + A_2 \sqrt{k_2} J_{l+1/2}(k_2 r)$

where A_1 and A_2 are arbitrary constants. Thus, the functions M_1 and M_2 are

$M_1 = A_1 \sqrt{k_1} J_{l+1/2}(k_1 r) + A_2 \sqrt{k_2} J_{l+1/2}(k_2 r)$

where A_1 and A_2 are arbitrary constants. Thus, the functions M_1 and M_2 are

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where A_1 and A_2 are arbitrary constants. Thus, the functions M_1 and M_2 are

$M_1 = A_1 \sqrt{k_1} J_{l+1/2}(k_1 r) + A_2 \sqrt{k_2} J_{l+1/2}(k_2 r)$

$$\begin{aligned} & \approx \sum_{n=0}^{\infty} C_n' \frac{(-1)^n x^{3n} \left(\frac{\alpha}{3}\right)^{2n}}{n! \prod_{m=1}^n (m - \frac{1}{3})} + C_2' \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1} \left(\frac{\alpha}{3}\right)^{2n}}{n! \prod_{m=1}^n (m + \frac{1}{3})} \\ \text{where } x &= r - r_0, \quad \alpha^2 = \frac{1}{r_0^2} \frac{\sqrt{(Z-1)^2 e^4 + 4\mu^2} (l+1)(E-D)}{\kappa^2 (E-D)}, \\ & \approx e_n \sqrt{F(r)} \Big|_{r_0}^{\frac{1}{2}} \left\{ C_n'' \exp(i\int_{r_0}^r \sqrt{F(r)} dr) + C_n' \exp(-i\int_{r_0}^r \sqrt{F(r)} dr) \right\} \end{aligned}$$

for $r \gg r_0$.

b) The case $\frac{(Z-1)e^2}{a} \approx E-D > 0$, i.e. the energy of the proton neutron is so large that it can emit from the proton with the energy comparable with the maximum potential. In this case, r_0 is not small compared with a , so that the region should be excluded.

c) The case $E-D < 0$, i.e. the neutron energy is smaller than the mass difference, so that the proton emission is impossible. In this case

approximately for any value of r larger than a , where

iii) The boundary conditions.

~~The constants should be connected smoothly to chosen r_0 so as to form a continuous smooth function for the whole interval of r and r_0 converge to~~

General solution should include the superposition of the above solutions for $l=0, 1, 2, \dots$, in which the constants A 's, B 's and C 's functions n, m

are so chosen as to form a smooth functions

for the whole interval of r . For very large value of r ,

it should converge to the superposition of the plane neutron waves

The scattered neutron wave and the outgoing and proton waves!

We want to consider in detail (the case a), in which $E-D > 0$, which the neutron in case a), the scattering and the disintegration are only the

The scattering and the disintegration are important only for $l=0$, so that energy should satisfy the condition

$$0.7 \sqrt{2} \gg E-D > 0$$

in the unit of 10^6 eV, if we assume

$$2 \gg 1, \text{ and}$$

$$a = \frac{2.25}{4.5}$$

in the unit of 10^{-12} cm. For example, the energy of the emitted

$$\begin{array}{r} 45 \\ 45 \\ \hline 225 \\ 2025 \\ \hline 90 \dots \end{array}$$

in the limit of 10^{-15} cm. For example, the width of the emission

$$\alpha = \frac{0.1 \times 10^{-15} \text{ cm}}{10^{-15} \text{ cm}} = 10^{-1}$$

in the limit of 10^6 eV, if we assume $S \gg I$ and

$$0.1 \times 10^{-15} \gg E - D > 0$$

the scattering and the emission of neutrons are independent for

the case a), the scattering and the emission are only the

neutron energy in the case a), instead of E and $E - D$ in the case b).

It should be noted that the superposition of spherical waves in

the region $r < a$ is to form a standing wave in

the region $r > a$ is to form a spherical wave in

the region $r > a$ is to form a spherical wave in

III) The boundary conditions. The constants A_1, B_1, C_1 and C_2 should be determined by different boundary conditions.

approximately for any value of r larger than a , where

D, so that the proton emission is impossible. In this case

the case $E - D < 0$, i.e., the neutron energy is smaller than the mass difference Q , region should be excluded.

maximum potential. In this case Q is not small compared with a , so that the

large part of the neutron with the energy comparable with Q can be emitted from the proton with the energy comparable with Q .

p) The case $E - D > 0$, i.e., the energy of the neutron is so

$$\text{for } r \gg a, \quad \psi = e^{i\alpha r} \left\{ C_1' \sin(\alpha r) + C_2' \cos(\alpha r) \right\} + e^{-i\alpha r} \left\{ C_3' \sin(\alpha r) + C_4' \cos(\alpha r) \right\}$$

$$\text{where } \alpha = \frac{1}{\hbar} \sqrt{2m(E - D)}$$

$$= C_1' \frac{N_1! \Gamma(m - \frac{1}{2})}{\sum_{n=0}^{\infty} (-1)^n x^{2n} (\frac{\alpha}{2})^{2n}} + C_2' \frac{N_1! \Gamma(m + \frac{1}{2})}{\sum_{n=0}^{\infty} (-1)^n x^{2n+1} (\frac{\alpha}{2})^{2n+1}}$$

iii) The boundary conditions. ~~The constant~~ ^{The most} general solutions of the problem can be constructed by superposing the above functions u_1 and u_2 respectively for $l=0, 1, 2, \dots$, in which the constants A 's, B 's and C 's are so chosen as to become form smooth functions for the whole interval of r . For very large value of r , it should converge to ~~the function~~ ^{required solution} uniquely determined by the condition that it should converge to the superposition of the plane neutron wave and the diverging neutron and proton waves for very large value of r . Especially for $l=0$, the solution should converge to the form

$$u_1 = \frac{\sin \sqrt{k^2 E} \cdot r}{\sqrt{k^2 E}} + C e^{i\sqrt{k^2 E} \cdot r}$$

$$u_2 = D e^{i\sqrt{\pi^2(E-D)} \cdot r} \approx 0$$

according as $E \geq 0$. The cross sections of s scattering and of s disintegration being ~~thus~~ expressed by

$$I_{sig} = 4\pi |C|^2 \quad \text{and} \quad I_{dis} = 4\pi \sqrt{\frac{E-D}{E}} |D|^2 \approx 0$$

respectively accordingly as $E \geq 0$. In the case a), for this case $i.e.$, in which the neutron energy should satisfy the condition

$$0.7 Z^3 \gg E - D > 0$$

in the unit of $10^6 eV$, if we assume $Z \gg 1$ and $a = \frac{Z^3}{5}$. In this case, the proton emitted in the unit of $10^{12} cm^2$ can be emitted with the energy small compared with $0.7 Z^3$, which is approximately $6 \times 10^6 eV$ for $Z=27$ for instance,

Letter to the Editor

Reduction of Nuclear Collision Problem

Among various processes caused by the collision of the heavy particle with the nucleus, only two cases, namely the elastic scattering and the capture with γ -ray emission, have been solved by straightforward application of the quantum mechanical method. Other processes such as the disintegration with the emission of another heavy particle or the inelastic scattering with the excitation of the nucleus have been considered to be so complicated that we should be satisfied only with rough estimations of their cross sections.

These cases, however, can be reduced to one body problems so as to make it possible to apply the exact theory of collision in the following way.

We consider a nucleus of atomic number Z and mass number M , which can be in one of the states

$(q, q), (q, q, q), (q, q, q)$

with energies W, W, W , respectively, where q, q, q are coordinates and spins of M heavy particles constituting the nucleus. Now, if a heavy particle, for instance a neutron for example, approaching now, if it is collided by a heavy particle, a neutron for example, ~~it~~ remains still in the normal state, the eigenfunction of the system consisting of the nucleus and the neutron has the form $u(q), (q, q)$ approximately, where q

represent the coordinates and spin of the latter. From the Hamiltonian

The wave equation

$H + H + V$

If we denote the Hamiltonians of the nucleus and the neutron by H and H respectively and the interaction potential by V , the wave equation becomes

$(H + H + V - E)u = 0,$

which can be reduced to the form

(1)

The cross section of disintegration ^{beams} ~~is~~ ~~the~~ ~~form~~

$$I_{dis} \approx 4\pi \frac{\Lambda_1}{k} \exp\left(-\pi \frac{(k_2 - 1)^2 e^2}{k(E-D)} + 4\pi \sqrt{k^2(2-1)^2 e^2 a}\right)$$

approximately, where

$$\Lambda_1 = \frac{(k_1 \tan k_2 a - k \tan k_1 a)^2}{(k_1^2 I_1 + k_2^2 I_2) + k^2 (k_1^2 I_1 \tan k_1 a + k_2^2 I_2 \tan k_2 a)^2}$$

$$\Lambda_1 = \frac{J}{K} \quad \Lambda_2 = \frac{K}{J}$$

$$= \frac{-(V_1 - V_2 + D) + \sqrt{c^2 - (V_1 - V_2 + D)^2}}{2J}$$

$$I_1 = \frac{J}{K} \left(\frac{1}{4} \tan k_1 a + k_1 G \frac{1}{4} \right)$$

$$I_2 = \frac{K}{J} \left(\frac{1}{4} \tan k_2 a + k_2 G \frac{1}{4} \right)$$

$$G = k^2(E-D) - \frac{k^2 D - 1}{k a} = 2k^2(E-D)$$

where $k_1 = \dots$ the asymptotic of the elastic scattering $\rightarrow 2.5 \sqrt{2}$
 and so on to make it possible to obtain the exact formula of collision, to them.
 other processes can be neglected for our problems in the following

$$\Lambda_1 = \frac{I_1^2 k_1 + I_2^2 k_2}{(I_1^2 k_1 + I_2^2 k_2)^2 + k^2 (k_1^2 I_1 \tan k_1 a + k_2^2 I_2 \tan k_2 a)^2}$$

$$I_1 = \frac{J}{K} \left(G \frac{1}{4} \tan k_1 a + k_1 G \frac{1}{4} \right)$$

$$I_2 =$$

refer to the Editor

The factor A_1 is ~~not~~ ^{neutron} ~~depends on~~ the energy E changes & in general, χ rather slowly as above used, and has the order of magnitude 1 in the unit of 10^{-24} cm²

so that

$$D_{dis} \cong 10^{-24} \exp\left(\frac{-\pi \kappa^2 (Z-1)e^2}{\sqrt{E}(E-D)}\right) + 4\sqrt{\kappa^2 (Z-1)e^2 a} \text{ cm}^2$$

$$D = 1.0659 \exp(-6.50)$$

$$D = \frac{2\pi \cdot 1.0659 \cdot 5.85}{3.15 \cdot 12.15} = 0.5 \cdot 10^{-24} \exp(-0.45 \frac{Z}{\sqrt{E-D}} + 0.65 Z^{\frac{2}{3}}) \text{ cm}^2$$

where E and D are expressed in the unit of 10^6 eV, ~~in the resonance of~~ A_1 becomes, however χ plus the cross section ~~is~~ ^{is} very small when the energy $E \sim D$ of the emitted proton becomes small compared with $0.45 \cdot Z$

in general as the energy $E-D$ of the emitted proton becomes small compared with $0.45 \cdot Z$ ^{1.35 / 6.3 = 0.21} ^{4.05 / 8.10 = 0.5} ^{1.35 / 4.05 = 0.33} ^{1.35 / 8.10 = 0.167} ^{1.35 / 12.15 = 0.111} ^{1.35 / 16.2 = 0.083} ^{1.35 / 20.25 = 0.067} ^{1.35 / 24.3 = 0.055} ^{1.35 / 28.35 = 0.048} ^{1.35 / 32.4 = 0.042} ^{1.35 / 36.45 = 0.037} ^{1.35 / 40.5 = 0.033} ^{1.35 / 44.55 = 0.030} ^{1.35 / 48.6 = 0.028} ^{1.35 / 52.65 = 0.026} ^{1.35 / 56.7 = 0.024} ^{1.35 / 60.75 = 0.022} ^{1.35 / 64.8 = 0.021} ^{1.35 / 68.85 = 0.020} ^{1.35 / 72.9 = 0.019} ^{1.35 / 77.0 = 0.018} ^{1.35 / 81.0 = 0.017} ^{1.35 / 85.05 = 0.016} ^{1.35 / 89.1 = 0.015} ^{1.35 / 93.15 = 0.015} ^{1.35 / 97.2 = 0.014} ^{1.35 / 101.25 = 0.014} ^{1.35 / 105.3 = 0.013} ^{1.35 / 109.35 = 0.013} ^{1.35 / 113.4 = 0.012} ^{1.35 / 117.45 = 0.012} ^{1.35 / 121.5 = 0.011} ^{1.35 / 125.55 = 0.011} ^{1.35 / 129.6 = 0.010} ^{1.35 / 133.65 = 0.010} ^{1.35 / 137.7 = 0.009} ^{1.35 / 141.75 = 0.009} ^{1.35 / 145.8 = 0.009} ^{1.35 / 149.85 = 0.008} ^{1.35 / 153.9 = 0.008} ^{1.35 / 157.95 = 0.008} ^{1.35 / 162.0 = 0.007} ^{1.35 / 166.05 = 0.007} ^{1.35 / 170.1 = 0.007} ^{1.35 / 174.15 = 0.007} ^{1.35 / 178.2 = 0.006} ^{1.35 / 182.25 = 0.006} ^{1.35 / 186.3 = 0.006} ^{1.35 / 190.35 = 0.006} ^{1.35 / 194.4 = 0.005} ^{1.35 / 198.45 = 0.005} ^{1.35 / 202.5 = 0.005} ^{1.35 / 206.55 = 0.005} ^{1.35 / 210.6 = 0.004} ^{1.35 / 214.65 = 0.004} ^{1.35 / 218.7 = 0.004} ^{1.35 / 222.75 = 0.004} ^{1.35 / 226.8 = 0.004} ^{1.35 / 230.85 = 0.003} ^{1.35 / 234.9 = 0.003} ^{1.35 / 238.95 = 0.003} ^{1.35 / 243.0 = 0.003} ^{1.35 / 247.05 = 0.003} ^{1.35 / 251.1 = 0.002} ^{1.35 / 255.15 = 0.002} ^{1.35 / 259.2 = 0.002} ^{1.35 / 263.25 = 0.002} ^{1.35 / 267.3 = 0.002} ^{1.35 / 271.35 = 0.002} ^{1.35 / 275.4 = 0.001} ^{1.35 / 279.45 = 0.001} ^{1.35 / 283.5 = 0.001} ^{1.35 / 287.55 = 0.001} ^{1.35 / 291.6 = 0.001} ^{1.35 / 295.65 = 0.001} ^{1.35 / 299.7 = 0.001} ^{1.35 / 303.75 = 0.001} ^{1.35 / 307.8 = 0.001} ^{1.35 / 311.85 = 0.001} ^{1.35 / 315.9 = 0.001} ^{1.35 / 319.95 = 0.001} ^{1.35 / 324.0 = 0.001} ^{1.35 / 328.05 = 0.001} ^{1.35 / 332.1 = 0.001} ^{1.35 / 336.15 = 0.001} ^{1.35 / 340.2 = 0.001} ^{1.35 / 344.25 = 0.001} ^{1.35 / 348.3 = 0.001} ^{1.35 / 352.35 = 0.001} ^{1.35 / 356.4 = 0.001} ^{1.35 / 360.45 = 0.001} ^{1.35 / 364.5 = 0.001} ^{1.35 / 368.55 = 0.001} ^{1.35 / 372.6 = 0.001} ^{1.35 / 376.65 = 0.001} ^{1.35 / 380.7 = 0.001} ^{1.35 / 384.75 = 0.001} ^{1.35 / 388.8 = 0.001} ^{1.35 / 392.85 = 0.001} ^{1.35 / 396.9 = 0.001} ^{1.35 / 400.95 = 0.001} ^{1.35 / 405.0 = 0.001} ^{1.35 / 409.05 = 0.001} ^{1.35 / 413.1 = 0.001} ^{1.35 / 417.15 = 0.001} ^{1.35 / 421.2 = 0.001} ^{1.35 / 425.25 = 0.001} ^{1.35 / 429.3 = 0.001} ^{1.35 / 433.35 = 0.001} ^{1.35 / 437.4 = 0.001} ^{1.35 / 441.45 = 0.001} ^{1.35 / 445.5 = 0.001} ^{1.35 / 449.55 = 0.001} ^{1.35 / 453.6 = 0.001} ^{1.35 / 457.65 = 0.001} ^{1.35 / 461.7 = 0.001} ^{1.35 / 465.75 = 0.001} ^{1.35 / 469.8 = 0.001} ^{1.35 / 473.85 = 0.001} ^{1.35 / 477.9 = 0.001} ^{1.35 / 481.95 = 0.001} ^{1.35 / 486.0 = 0.001} ^{1.35 / 490.05 = 0.001} ^{1.35 / 494.1 = 0.001} ^{1.35 / 498.15 = 0.001} ^{1.35 / 502.2 = 0.001} ^{1.35 / 506.25 = 0.001} ^{1.35 / 510.3 = 0.001} ^{1.35 / 514.35 = 0.001} ^{1.35 / 518.4 = 0.001} ^{1.35 / 522.45 = 0.001} ^{1.35 / 526.5 = 0.001} ^{1.35 / 530.55 = 0.001} ^{1.35 / 534.6 = 0.001} ^{1.35 / 538.65 = 0.001} ^{1.35 / 542.7 = 0.001} ^{1.35 / 546.75 = 0.001} ^{1.35 / 550.8 = 0.001} ^{1.35 / 554.85 = 0.001} ^{1.35 / 558.9 = 0.001} ^{1.35 / 562.95 = 0.001} ^{1.35 / 567.0 = 0.001} ^{1.35 / 571.05 = 0.001} ^{1.35 / 575.1 = 0.001} ^{1.35 / 579.15 = 0.001} ^{1.35 / 583.2 = 0.001} ^{1.35 / 587.25 = 0.001} ^{1.35 / 591.3 = 0.001} ^{1.35 / 595.35 = 0.001} ^{1.35 / 599.4 = 0.001} ^{1.35 / 603.45 = 0.001} ^{1.35 / 607.5 = 0.001} ^{1.35 / 611.55 = 0.001} ^{1.35 / 615.6 = 0.001} ^{1.35 / 619.65 = 0.001} ^{1.35 / 623.7 = 0.001} ^{1.35 / 627.75 = 0.001} ^{1.35 / 631.8 = 0.001} ^{1.35 / 635.85 = 0.001} ^{1.35 / 639.9 = 0.001} ^{1.35 / 643.95 = 0.001} ^{1.35 / 648.0 = 0.001} ^{1.35 / 652.05 = 0.001} ^{1.35 / 656.1 = 0.001} ^{1.35 / 660.15 = 0.001} ^{1.35 / 664.2 = 0.001} ^{1.35 / 668.25 = 0.001} ^{1.35 / 672.3 = 0.001} ^{1.35 / 676.35 = 0.001} ^{1.35 / 680.4 = 0.001} ^{1.35 / 684.45 = 0.001} ^{1.35 / 688.5 = 0.001} ^{1.35 / 692.55 = 0.001} ^{1.35 / 696.6 = 0.001} ^{1.35 / 700.65 = 0.001} ^{1.35 / 704.7 = 0.001} ^{1.35 / 708.75 = 0.001} ^{1.35 / 712.8 = 0.001} ^{1.35 / 716.85 = 0.001} ^{1.35 / 720.9 = 0.001} ^{1.35 / 724.95 = 0.001} ^{1.35 / 729.0 = 0.001} ^{1.35 / 733.05 = 0.001} ^{1.35 / 737.1 = 0.001} ^{1.35 / 741.15 = 0.001} ^{1.35 / 745.2 = 0.001} ^{1.35 / 749.25 = 0.001} ^{1.35 / 753.3 = 0.001} ^{1.35 / 757.35 = 0.001} ^{1.35 / 761.4 = 0.001} ^{1.35 / 765.45 = 0.001} ^{1.35 / 769.5 = 0.001} ^{1.35 / 773.55 = 0.001} ^{1.35 / 777.6 = 0.001} ^{1.35 / 781.65 = 0.001} ^{1.35 / 785.7 = 0.001} ^{1.35 / 789.75 = 0.001} ^{1.35 / 793.8 = 0.001} ^{1.35 / 797.85 = 0.001} ^{1.35 / 801.9 = 0.001} ^{1.35 / 805.95 = 0.001} ^{1.35 / 810.0 = 0.001} ^{1.35 / 814.05 = 0.001} ^{1.35 / 818.1 = 0.001} ^{1.35 / 822.15 = 0.001} ^{1.35 / 826.2 = 0.001} ^{1.35 / 830.25 = 0.001} ^{1.35 / 834.3 = 0.001} ^{1.35 / 838.35 = 0.001} ^{1.35 / 842.4 = 0.001} ^{1.35 / 846.45 = 0.001} ^{1.35 / 850.5 = 0.001} ^{1.35 / 854.55 = 0.001} ^{1.35 / 858.6 = 0.001} ^{1.35 / 862.65 = 0.001} ^{1.35 / 866.7 = 0.001} ^{1.35 / 870.75 = 0.001} ^{1.35 / 874.8 = 0.001} ^{1.35 / 878.85 = 0.001} ^{1.35 / 882.9 = 0.001} ^{1.35 / 886.95 = 0.001} ^{1.35 / 891.0 = 0.001} ^{1.35 / 895.05 = 0.001} ^{1.35 / 899.1 = 0.001} ^{1.35 / 903.15 = 0.001} ^{1.35 / 907.2 = 0.001} ^{1.35 / 911.25 = 0.001} ^{1.35 / 915.3 = 0.001} ^{1.35 / 919.35 = 0.001} ^{1.35 / 923.4 = 0.001} ^{1.35 / 927.45 = 0.001} ^{1.35 / 931.5 = 0.001} ^{1.35 / 935.55 = 0.001} ^{1.35 / 939.6 = 0.001} ^{1.35 / 943.65 = 0.001} ^{1.35 / 947.7 = 0.001} ^{1.35 / 951.75 = 0.001} ^{1.35 / 955.8 = 0.001} ^{1.35 / 959.85 = 0.001} ^{1.35 / 963.9 = 0.001} ^{1.35 / 967.95 = 0.001} ^{1.35 / 972.0 = 0.001} ^{1.35 / 976.05 = 0.001} ^{1.35 / 980.1 = 0.001} ^{1.35 / 984.15 = 0.001} ^{1.35 / 988.2 = 0.001} ^{1.35 / 992.25 = 0.001} ^{1.35 / 996.3 = 0.001} ^{1.35 / 1000.35 = 0.001} ^{1.35 / 1004.4 = 0.001} ^{1.35 / 1008.45 = 0.001} ^{1.35 / 1012.5 = 0.001} ^{1.35 / 1016.55 = 0.001} ^{1.35 / 1020.6 = 0.001} ^{1.35 / 1024.65 = 0.001} ^{1.35 / 1028.7 = 0.001} ^{1.35 / 1032.75 = 0.001} ^{1.35 / 1036.8 = 0.001} ^{1.35 / 1040.85 = 0.001} ^{1.35 / 1044.9 = 0.001} ^{1.35 / 1048.95 = 0.001} ^{1.35 / 1053.0 = 0.001} ^{1.35 / 1057.05 = 0.001} ^{1.35 / 1061.1 = 0.001} ^{1.35 / 1065.15 = 0.001} ^{1.35 / 1069.2 = 0.001} ^{1.35 / 1073.25 = 0.001} ^{1.35 / 1077.3 = 0.001} ^{1.35 / 1081.35 = 0.001} ^{1.35 / 1085.4 = 0.001} ^{1.35 / 1089.45 = 0.001} ^{1.35 / 1093.5 = 0.001} ^{1.35 / 1097.55 = 0.001} ^{1.35 / 1101.6 = 0.001} ^{1.35 / 1105.65 = 0.001} ^{1.35 / 1109.7 = 0.001} ^{1.35 / 1113.75 = 0.001} ^{1.35 / 1117.8 = 0.001} ^{1.35 / 1121.85 = 0.001} ^{1.35 / 1125.9 = 0.001} ^{1.35 / 1129.95 = 0.001} ^{1.35 / 1134.0 = 0.001} ^{1.35 / 1138.05 = 0.001} ^{1.35 / 1142.1 = 0.001} ^{1.35 / 1146.15 = 0.001} ^{1.35 / 1150.2 = 0.001} ^{1.35 / 1154.25 = 0.001} ^{1.35 / 1158.3 = 0.001} ^{1.35 / 1162.35 = 0.001} ^{1.35 / 1166.4 = 0.001} ^{1.35 / 1170.45 = 0.001} ^{1.35 / 1174.5 = 0.001} ^{1.35 / 1178.55 = 0.001} ^{1.35 / 1182.6 = 0.001} ^{1.35 / 1186.65 = 0.001} ^{1.35 / 1190.7 = 0.001} ^{1.35 / 1194.75 = 0.001} ^{1.35 / 1198.8 = 0.001} ^{1.35 / 1202.85 = 0.001} ^{1.35 / 1206.9 = 0.001} ^{1.35 / 1210.95 = 0.001} ^{1.35 / 1215.0 = 0.001} ^{1.35 / 1219.05 = 0.001} ^{1.35 / 1223.1 = 0.001} ^{1.35 / 1227.15 = 0.001} ^{1.35 / 1231.2 = 0.001} ^{1.35 / 1235.25 = 0.001} ^{1.35 / 1239.3 = 0.001} ^{1.35 / 1243.35 = 0.001} ^{1.35 / 1247.4 = 0.001} ^{1.35 / 1251.45 = 0.001} ^{1.35 / 1255.5 = 0.001} ^{1.35 / 1259.55 = 0.001} ^{1.35 / 1263.6 = 0.001} ^{1.35 / 1267.65 = 0.001} ^{1.35 / 1271.7 = 0.001} ^{1.35 / 1275.75 = 0.001} ^{1.35 / 1279.8 = 0.001} ^{1.35 / 1283.85 = 0.001} ^{1.35 / 1287.9 = 0.001} ^{1.35 / 1291.95 = 0.001} ^{1.35 / 1296.0 = 0.001} ^{1.35 / 1300.05 = 0.001} ^{1.35 / 1304.1 = 0.001} ^{1.35 / 1308.15 = 0.001} ^{1.35 / 1312.2 = 0.001} ^{1.35 / 1316.25 = 0.001} ^{1.35 / 1320.3 = 0.001} ^{1.35 / 1324.35 = 0.001} ^{1.35 / 1328.4 = 0.001} ^{1.35 / 1332.45 = 0.001} ^{1.35 / 1336.5 = 0.001} ^{1.35 / 1340.55 = 0.001} ^{1.35 / 1344.6 = 0.001} ^{1.35 / 1348.65 = 0.001} ^{1.35 / 1352.7 = 0.001} ^{1.35 / 1356.75 = 0.001} ^{1.35 / 1360.8 = 0.001} ^{1.35 / 1364.85 = 0.001} ^{1.35 / 1368.9 = 0.001} ^{1.35 / 1372.95 = 0.001} ^{1.35 / 1377.0 = 0.001} ^{1.35 / 1381.05 = 0.001} ^{1.35 / 1385.1 = 0.001} ^{1.35 / 1389.15 = 0.001} ^{1.35 / 1393.2 = 0.001} ^{1.35 / 1397.25 = 0.001} ^{1.35 / 1401.3 = 0.001} ^{1.35 / 1405.35 = 0.001} ^{1.35 / 1409.4 = 0.001} ^{1.35 / 1413.45 = 0.001} ^{1.35 / 1417.5 = 0.001} ^{1.35 / 1421.55 = 0.001} ^{1.35 / 1425.6 = 0.001} ^{1.35 / 1429.65 = 0.001} ^{1.35 / 1433.7 = 0.001} ^{1.35 / 1437.75 = 0.001} ^{1.35 / 1441.8 = 0.001} ^{1.35 / 1445.85 = 0.001} ^{1.35 / 1449.9 = 0.001} ^{1.35 / 1453.95 = 0.001} ^{1.35 / 1458.0 = 0.001} ^{1.35 / 1462.05 = 0.001} ^{1.35 / 1466.1 = 0.001} ^{1.35 / 1470.15 = 0.001} ^{1.35 / 1474.2 = 0.001} ^{1.35 / 1478.25 = 0.001} ^{1.35 / 1482.3 = 0.001} ^{1.35 / 1486.35 = 0.001} ^{1.35 / 1490.4 = 0.001} ^{1.35 / 1494.45 = 0.001} ^{1.35 / 1498.5 = 0.001} ^{1.35 / 1502.55 = 0.001} ^{1.35 / 1506.6 = 0.001} ^{1.35 / 1510.65 = 0.001} ^{1.35 / 1514.7 = 0.001} ^{1.35 / 1518.75 = 0.001} ^{1.35 / 1522.8 = 0.001} ^{1.35 / 1526.85 = 0.001} ^{1.35 / 1530.9 = 0.001} ^{1.35 / 1534.95 = 0.001} ^{1.35 / 1539.0 = 0.001} ^{1.35 / 1543.05 = 0.001} ^{1.35 / 1547.1 = 0.001} ^{1.35 / 1551.15 = 0.001} ^{1.35 / 1555.2 = 0.001} ^{1.35 / 1559.25 = 0.001} ^{1.35 / 1563.3 = 0.001} ^{1.35 / 1567.35 = 0.001} ^{1.35 / 1571.4 = 0.001} ^{1.35 / 1575.45 = 0.001} ^{1.35 / 1579.5 = 0.001} ^{1.35 / 1583.55 = 0.001} ^{1.35 / 1587.6 = 0.001} ^{1.35 / 1591.65 = 0.001} ^{1.35 / 1595.7 = 0.001} ^{1.35 / 1599.75 = 0.001} ^{1.35 / 1603.8 = 0.001} ^{1.35 / 1607.85 = 0.001} ^{1.35 / 1611.9 = 0.001} ^{1.35 / 1615.95 = 0.001} ^{1.35 / 1620.0 = 0.001} ^{1.35 / 1624.05 = 0.001} ^{1.35 / 1628.1 = 0.001} ^{1.35 / 1632.15 = 0.001} ^{1.35 / 1636.2 = 0.001} ^{1.35 / 1640.25 = 0.001} ^{1.35 / 1644.3 = 0.001} ^{1.35 / 1648.35 = 0.001} ^{1.35 / 1652.4 = 0.001} ^{1.35 / 1656.45 = 0.001} ^{1.35 / 1660.5 = 0.001} ^{1.35 / 1664.55 = 0.001} ^{1.35 / 1668.6 = 0.001} ^{1.35 / 1672.65 = 0.001} ^{1.35 / 1676.7 = 0.001} ^{1.35 / 1680.75 = 0.001} ^{1.35 / 1684.8 = 0.001} ^{1.35 / 1688.85 = 0.001} ^{1.35 / 1692.9 = 0.001} ^{1.35 / 1696.95 = 0.001} ^{1.35 / 1701.0 = 0.001} ^{1.35 / 1705.05 = 0.001} ^{1.35 / 1709.1 = 0.001} ^{1.35 / 1713.15 = 0.001} ^{1.35 / 1717.2 = 0.001} ^{1.35 / 1721.25 = 0.001} ^{1.35 / 1725.3 = 0.001} ^{1.35 / 1729.35 = 0.001} ^{1.35 / 1733.4 = 0.001} ^{1.35 / 1737.45 = 0.001} ^{1.35 / 1741.5 = 0.001} ^{1.35 / 1745.55 = 0.001} ^{1.35 / 1749.6 = 0.001} ^{1.35 / 1753.65 = 0.001} ^{1.35 / 1757.7 = 0.001} ^{1.35 / 1761.75 = 0.001} ^{1.35 / 1765.8 = 0.001} ^{1.35 / 1769.85 = 0.001} ^{1.35 / 1773.9 = 0.001} ^{1.35 / 1777.95 = 0.001} ^{1.35 / 1782.0 = 0.001} ^{1.35 / 1786.05 = 0.001} ^{1.35 / 1790.1 = 0.001} ^{1.35 / 1794.15 = 0.001} ^{1.35 / 1798.2 = 0.001} ^{1.35 / 1802.25 = 0.001} ^{1.35 / 1806.3 = 0.001} ^{1.35 / 1810.35 = 0.001} ^{1.35 / 1814.4 = 0.001} ^{1.35 / 1818.45 = 0.001} ^{1.35 / 1822.5 = 0.001} ^{1.35 / 1826.55 = 0.001} ^{1.35 / 1830.6 = 0.001} ^{1.35 / 1834.65 = 0.001} ^{1.35 / 1838.7 = 0.001} ^{1.35 / 1842.75 = 0.001} ^{1.35 / 1846.8 = 0.001} ^{1.35 / 1850.85 = 0.001} ^{1.35 / 1854.9 = 0.001} ^{1.35 / 1858.95 = 0.001} ^{1.35 / 1863.0 = 0.001} ^{1.35 / 1867.05 = 0.001} ^{1.35 / 1871.1 = 0.001} ^{1.35 / 1875.15 = 0.001} ^{1.35 / 1879.2 = 0.001} ^{1.35 / 1883.25 = 0}

Letter to the Editor
 Reduction of Nuclear Collision Problems

Among various processes caused by the collision of the heavy particle

with the nucleus, only two cases, namely the elastic scattering and the capture with γ -ray emission, have been solved, hitherto by straightforward application of the quantum mechanical method. Now it will be shown that other processes can also be reduced to one body problems in the following way so as to make it possible to apply the exact theory of collision to them.

First, if only the possibility of the elastic scattering is considered, the stationary state of the whole system will be expressed in the form $u(q, q, \dots, q)$, where u is the superposition of plane and scattered waves for the impinging particle, in the neutron state for instance, and represents the normal state of the nucleus with atomic number Z and mass number N . Next, if the possibilities of the excitation of the nucleus to other states, \dots say, and of the disintegration with the emission of a heavy particle, in the proton state for instance, are taken into account, the stationary solution should have the general form

$$\psi(q, q, q, \dots, q) = u(q, q, \dots, q) + v(q, q, \dots, q) + \dots \quad (1)$$

where u, \dots and v, \dots represent inelastically scattered neutron waves and outgoing proton waves respectively and \dots represent stationary

1) The recoil of the nucleus will be neglected throughout, which will be permitted for majority of cases.

Do you
 one for the case of inelasticity Γ in the limit of 10^{-5} cm
 the form V is \dots

which is the condition only when the resonance condition

$$I_1 k_2 + I_2 k_1 \approx 0$$

is satisfied.

Further, the factor $\exp(-0.45 \frac{D}{\sqrt{E-D}} + 0.65)$

the cross section of elastic scattering takes the form

$$I_{\text{scat}} = \frac{4\pi}{k} \left[\frac{A I_1 k_2 - A I_2 k_1}{(I_1 k_2 - I_2 k_1)^2 + k^2} \right]^2 \left(\frac{I_1 \tan k_2 a - I_2 \tan k_1 a}{I_1 k_2 a - I_2 k_1 a} \right)^2$$

which becomes ~~large~~ large only when the mass defect D is positive. ~~is not to be neglected~~ ~~it will be neglected~~ in detail.

The case (b) is ~~note that~~ ~~it will be neglected~~ ~~in detail~~ ~~in detail~~. The proton emission is impossible because of the condition $E - D < 0$. The cross section of the elastic scattering is the same as ().

For slow neutrons, it becomes takes the form

$$I_{\text{scat}} \approx 4\pi a^2 \left(1 - \frac{A I_1 \tan k_2 a - I_2 \tan k_1 a}{I_1 k_2 a - I_2 k_1 a} \right)^2$$

which becomes a very large if the

$$\left(\text{except } \cos \theta = 1 \right) \approx 0.5 \cdot D^{\frac{2}{3}} \left(1 - \frac{I_1 \tan k_2 a - I_2 \tan k_1 a}{I_1 k_2 a - I_2 k_1 a} \right)^2$$

resonance condition $I_1 k_2 - I_2 k_1 = 0$ is satisfied. In this case, the cross section of the elastic scattering is calculated and is given by ().

is particularly

if the exchange force is neglected, the expression becomes

becomes

In the above deduction, it was assumed that the scattered or the emitted particle can be identified with the impinging particle. In more complicated case, in which it is possible that the latter is captured and one of the particles in the nucleus is emitted as a neutron or a proton, similar reduction as above can be performed, only if the range of forces (except Coulomb force, of course,) between heavy particles are so small that they can be replaced by δ -functions.

Namely, inserting the stationary solution of the form

$$\psi = u_0(q_0) \chi_0(q_1, q_2, \dots, q_N) + \dots + v_1(q_1) \varphi_1(q_0, q_2, \dots, q_N) + \dots$$

in the equation (2) and integrating with respect to q_1, q_2, \dots, q_N after having multiplied both sides by $\tilde{\chi}_0, \dots, \tilde{\varphi}_1, \dots$ in turn, we obtain the equations

$$H_1 u_0 + V_0 u_1 + \dots + K_{01} v_1 + \dots = (E - W_0) u_0 \quad (4)$$

involving $\tilde{\chi}_0$ only, where $K_{01}(q_0, v_1) = \int \dots \int \tilde{\chi}_0(q_1, q_2, \dots, q_N) V^* v_1(q_1) \varphi_1(q_0, q_2, \dots, q_N) dq_1 dq_2 \dots dq_N, \dots$ on account of the term in V^* involving $\delta(q_1, q_0)$.

Further, by integrating (2) with respect to q_1, q_2, \dots, q_N after having interchanged q_0 and q_1 in (2) and multiplied by $\tilde{\varphi}_1(q_1, q_2, \dots, q_N)$, we obtain the remaining equations

$$H_1 v_1 + K_{10} u_0 + \dots + U_1 v_1 + \dots = (E - W_1) v_1 \quad (4')$$

3) The symbol * means the interchange of q_0 and q_1 in V . Other terms in V^* , δ involving $\delta(q_1, q_2)$ for example, do not contribute to the integral on account of the exclusion principle for the particles in the nucleus.

*with other in exchange symmetry
 appear in the equation*

$$f_{scat} = \tan^{-1} \left(1 - \frac{\tan k_0 a}{k_0 a} \right)$$

$$\text{where } k_0 = \sqrt{k^2 - V_0}$$

$$\varphi(r) = \int y(p) dp$$

which agrees with the expression (19) of Bethe's

and if we put $a = r_0$, $k_0 = \lambda_0^{-1}$ and $k_0 a = \varphi_0 + \frac{\pi}{2}$.

($\tan k_0 a = -\cot \varphi_0$) is always the phase of the wave function at the boundary



The condition of resonance (1) reduces to the simple form $k_0 a = (n + \frac{1}{2})\pi$ extreme limiting cases

where n is any integer, in the extreme limiting cases

$J=0$ or $J \gg 1$, V_0, V_2, D, E . In general case,

it is a simple relation means,

means, in general, a rather complicated relation between the phases of the nuclear wave functions u , and u_0 at the boundary of the nucleus, and reduces

$$\dots + \lambda (d \dots d) + \dots + \lambda (d \dots d) (d \dots d) + \dots$$

... (except for the case of course) ...

In the above deduction, it was assumed that the scattered or the emitted particle can be identified with the impinging particle. In more complicated case, in which it is possible that the latter is captured and one of the particles in the nucleus is emitted as a neutron or a proton, similar reduction as above can be performed, only if the range of forces (except Coulomb force, of course,) between heavy particles are so small that they can be replaced by δ -functions.

Namely, inserting the stationary solution of the form

$$= u(q) (q q \dots q) + \dots + v(q) (q q \dots q) + \dots$$

in the equation (1) and integrating with respect to $q q \dots q$ after having multiplied both sides by ψ^* in turn, we obtain the equations

$$H u + V u + \dots + K v + \dots = (E - W) u \quad (4)$$

for q only, where

$$K(q) v(q) = 2 (q q \dots q) V V (q) (q q \dots q) \int d q d q \dots d q \dots$$

on account of the term in V involving (q, q) . Further, by integrating (1) with respect to $q q \dots q$ after having interchanged q and q in (1) and multiplied by $(q q \dots q)$, we obtain the remaining equations

$$H v + K u + \dots + U v + \dots = (E - W) v \quad (4)$$

3) The symbol * means the interchange of q and q in V . Other terms in V , δ involving (q, q) for example, do not contribute to the integral on q , account of the exclusion principle for the particles in the nucleus.

$$\Phi_{2000} = \frac{K' v}{4\pi r^2}$$