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On the Efficiency of the γ-Ray Counter.

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Abstract.

The efficiency, i. e. the probability of the electric discharge per one quantum, of the γ-ray counter with thick wall was computed by making various simplifying assumptions for several energies between 0·2 and 5 MEV in the case of Al and between 1 and 10 MEV in the case of Pb. The efficiency of the thin-walled counter was also discussed.

1. Introduction

In order to infer the absolute intensity of the γ-ray from the measurement by means of a counter, it is always necessary to have a reliable knowledge of the efficiency of the counter. Whereas it can not easily be determined experimentally, the theoretical evaluation of it is possible at least in principle, since, under favourable conditions, it is approximately equal to the probability that, when a γ-ray quantum falls on the counter, a secondary electron or a pair of a positron and a negatron is produced in the wall and is emitted into the inner space.

We can not expect, however, to obtain a result, which is valid at once for wide range, because the above probability depends on the material, the shape and the thickness of the wall as well as the energy of the γ-ray quantum in a complicated manner. Further, the effect of multiple scattering of the secondary electron in the wall prevent us from the rigorous calculation for the thick counter. Hence, approximate numerical computations were performed for several energies in the cases of Al and Pb on various simplifying assumptions[1].

On the contrary, an approximate expression of the efficiency can be deduced at once from elementary considerations for the counter

(1) A semi-empirical formula of the efficiency of the brass counter was obtained by v. DROSTE: *Z. Phys.*, **100** (1936), 529.

with thin wall.

2. The Aluminium Counter

In general, the efficiency increases at first with the thickness as far as the latter becomes roughly the same ~~which~~ with the upper limit of the range of the secondary electrons and then decreases on account of the futile absorption of the γ-ray. Hence, we want to consider the case in detail, in which the thickness is small compared with the reciprocal of the absorption coefficient of the γ-ray and larger than the range of most on the secondary electrons.

In order to simplify the calculation, the cylindrical counter may be replaced by a rectangular one with ~~the same~~ an equal cross section. The direction of incidence of the γ-ray is assumed to be normal to a pair of planes with the same thickness as that of the cylindrical counter and the remaining planes are assumed to be infinitely thin, as shown in Fig. 1. The necessary correction due to the oblique incidence will be considered later.

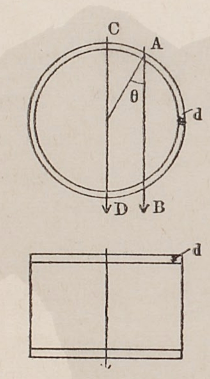

Fig. 1

Now, in the case of aluminium, most of the secondary electrons responsible for the discharge is produced by the Compton effect, at least for γ-ray of energy between 0·5 and 5 MEV. The contribution of the electrons produced by the photoelectric effect or the materialization becomes important gradually as the energy becomes smaller or larger.

The angular distribution of the Compton recoil electrons can be inferred immediately from Klein-Nishina's formula, which shows the angular distribution of the scattered photons. Thus, the differential cross section per one electron that a photon of energy $h\nu = \gamma\, mc^2$ produces an electron making an angle between χ and $\chi + d\chi$ with the direction of incidence is given by

$$\phi(x)dx = \frac{8\pi r_0^2(\gamma+1)^2 dx}{\{(\gamma^2+4\gamma+2)-\gamma^2 x\}^2}\left[1+\frac{2(1+x)}{(\gamma^2+2\gamma+2)-(\gamma^2+2\gamma)x}\right.$$
$$\left.\times\left\{\frac{\gamma^2(1+x)}{(\gamma^2+4\gamma+2)-\gamma^2 x}-\frac{(\gamma+1)^2(1-x)}{(\gamma^2+2\gamma+2)-(\gamma^2+2\gamma)x}\right\}\right], \qquad (1)$$

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where $x = \cos 2\chi$ and $r_0 = \dfrac{e^2}{mc^2}$. As the electron is projected forward, χ is always smaller than $\dfrac{\pi}{2}$, so that x takes a value between $+1$ and -1. The kinetic energy of the electron is given by

$$\varepsilon(x)\, mc^2 = \frac{2\gamma^2(1+x)\, mc^2}{2(\gamma+1)^2-\gamma^2(1+x)}\,. \tag{2}$$

The probability that such an electron is produced at a depth between z and $z+dz$ from the internal surface of the wall is approximately equal to

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$$NZ dz \phi(x) dx\,, \tag{3}$$

where Z is the atomic number and N is the number of atoms per unit volume of the wall. Hence, the required probability that, when a photon falls on the wall, a recoil electron is emitted into the inner space without being intercepted in the wall by the Bremsung and the scattering is given by

$$NZ \int_{z=0}^{d} \int_{x=-1}^{+1} \{p_1(z, x, \varepsilon) + p_2(z, x, \varepsilon)\} dz \phi(x) dx\,, \tag{4}$$

where $p_1(z, x, \varepsilon)$ is the probability that the electron produced at the depth z in the upper wall with energy ε and direction x reaches the inner space and $p_2(z, x, \varepsilon)$ is that the similar electron produced in the lower wall is scattered back. The theoretical determination of p_1 and p_2 is very difficult owing to the complicated effect of the multiple scattering, so that we want to infer the effective thickness $z_0(x, \varepsilon)$ defined by

$$z_0(x,\varepsilon) = \int_0^d \{p_1(z, x, \varepsilon) + p_2(z, x, \varepsilon)\} dz\,, \tag{5}$$

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from the considerations of the following two cases.

(i) The effect of the multiple scattering becomes larger, as the energy of the γ-ray decreases and in the limit of complete diffusion, $z_0(x, \varepsilon)$ will be equal to the practical range $R(\varepsilon)$, which has been determined quite accurately by experiment[(2)], multiplied by a factor between $\frac{1}{2}$ and 1 nearly independent of the initial direction x.

(ii) The effect of the scattering becomes smaller, as the energy increases, until z_0 will be equal to $R(\varepsilon)\cos\chi$ multiplied by a factor a little smaller than 1.

Further, we have to take into account the effect due to the cylindrical shape of the counter instead of the rectangular shape. In the case (i), if the photon falls on the wall with the angle θ as indicated by the straight line AB in Fig. 1, the effective thickness z_0 becomes larger by a factor about $\frac{1}{\cos\theta}$ compared with the photon falling normally as indicated by CD. Hence, the efficiency will be larger by a factor about

$$\frac{1}{2}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{\cos\theta\, d\theta}{\cos\theta} = \frac{\pi}{2} \tag{6}$$

than that of the rectangular counter above considered.

In the case (ii), most of secondary electrons are emitted into nearly the same direction as the incident photon, so that the form facter will not differ appreciably from 1.

Thus, the required efficiency due to the Compton effect takes the form

$$\text{Eff. (Compton i)} = \delta NZ\int_{-1}^{+1} R(\varepsilon)\phi(x)dx \tag{7}$$

or

$$\text{Eff. (Compton ii)} = \delta NZ\int_{-1}^{+1} R(\varepsilon)\sqrt{\frac{1+x}{2}}\,\phi(x)dx\,, \tag{8}$$

according as the assumption (i) or (ii) is employed, where δ is a number a little smaller than 1. The evaluations of these integrals were made numerically for several values of the γ-ray energy, the results being shown in Table I. As the corresponding values for two cases do not

(2) VARDER: *Phil. Mag.*, **29** (1925), 725; SHONLAND: *Proc. Roy. Soc.*, (A) **104** (1923), 235; **108** (1923), 187. See, further, a summary report of Bothe, *Handb. d. Phys.* Bd. **XXII/2** (1933), 1.

differ appreciably from each other, the simple mean was taken as the probable value. (Fourth line in Table I).

TABLE I. The Efficiency of the Al-Counter.

γ-Ray energy	0·2	0·5	1·0	2·0	5·0 MEV	
Eff. (Compton i)	0·068	0·34	1·0	2·1	3·8	$\times\delta\%$
Eff. (〃 ii)	0·058	0·30	0·9	2·1	3·8	
Eff. (〃 average)	0·063	0·32	0·95	2·1	3·8	
Eff. (Photo.)	0·18	0·02	0	0	0	
Eff. (Pair)	0	0	0	0·1	0·4	
Eff. (Total)	0·24	0·34	0·95	2·2	4·2	

Next, the efficiency due to the photoelectric effect, which is only appreciable for small energy of the γ-ray, can be calculated in the similar manner as above. For $1 \gg \gamma \gg \frac{1}{\sqrt{1-Z^2\alpha^2}}-1$, the differential cross section per one atom that the electron is emitted from the K-shell making an angle between χ and $\chi+d\chi$ with the direction of incidence is given approximately by

α^4

$$\phi'(\chi)d\chi = Z^5\alpha_4\gamma^{-7/2}r_0^2\frac{8\pi\sqrt{2}\sin^3\chi\, d\chi}{(1-\beta\cos\chi)^4}, \tag{9}$$

where α is the fine structure constant and β is the velocity of the photoelectron divided by c. By using this expression, the probable value of the efficiency was estimated for $\gamma = 0{\cdot}4$ (0·2 MEV). It was found to be negligibly small already for 0·5 MEV. (Fifth line in Table I).

On the other hand, the efficiency due to the pair creation is always small compared with that due to the Compton effect for the energy range above considered, so that only rough estimations were made. (Sixth line in Table I). By summing up the above three parts, we obtain finally the required efficiency as shown in the last line of Table I.

The theoretical determination of the common factor δ is very difficult, but if we take presumably $\delta = 0{\cdot}8$, the efficiency of the Al-counter for energy $h\nu = 2{\cdot}65$ MEV (of the hard γ-ray of ThC″) becomes about $2{\cdot}75\times 0{\cdot}8 = 2{\cdot}2\%$, which is a reasonable value, since it is a little smaller than the efficiency 2·4% for this energy in the case of the brass counter determined experimentally by Droste[3]. Hence, the efficiency was plotted in Fig. 2 as the function of the energy assuming $\delta = 0{\cdot}8$. The full curve shows the total efficiency and the ~~dotted~~ dashed curve shows that due to the Compton effect alone.

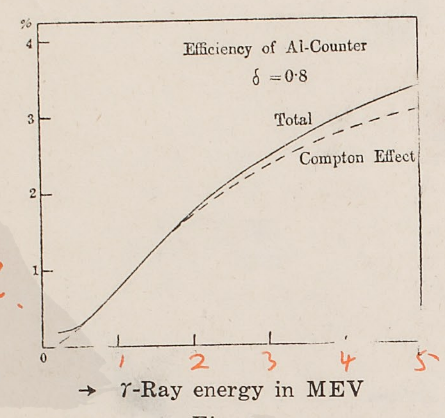

Fig. 2

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3. The Lead Counter

Next, we want to determine the efficiency of the lead counter for the energy between 1 and 10 MEV, assuming again the wall thickness to be small compared with the reciprocal of the absorption coefficients of the γ-ray and larger than the range of most of the secondary electrons. This condition is approximately satisfied, if we take the thickness about $0{\cdot}3$ cm, for instance. In this case, the evaluation becomes still more difficult than in the previous case for the following reasons.

(i) The photoelectric effect and the pair creation have the cross sections of the same order as the Compton effect and the energy and the angular distributions of the secondary particles in the former process are not known in detail.

(ii) On account of the simultaneous emission of two particles, the efficiency due to the pair creation should be multiplied by a certain factor between 1 and 2, which can hardly be determined accurately.

(iii) The range-energy relation of the electron in lead is not known in detail experimentally.

Therefore, we are to be contented with rough estimations which can be performed by using the results of Bethe-Heitler's theory of the absorption of the high energy γ-ray and the fast electron[4]. Only the final results are shown in Table II.

(3) Droste: *loc. cit.*
(4) See, for instance, Heitler: *Quantum Theory of Radiation*, Oxford, 1939.

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TABLE II. The Efficiency of the Pb-Counter.

γ-Ray Energy	1	2·5	5	10 MEV
Eff.	2·9	4·7	9·3	21·3 $\times\delta'\%$

i The numerical factor δ' appreciably smaller than 1 was necessary, as we used the theoretical range of the electron in Pb, which is surely too large on account of neglecting altogether the effect of scattering. According to the experimental results of Kikuchi, Aoki and Husimi[5], the ratio of the efficiencies of Al- and Pb-counters for
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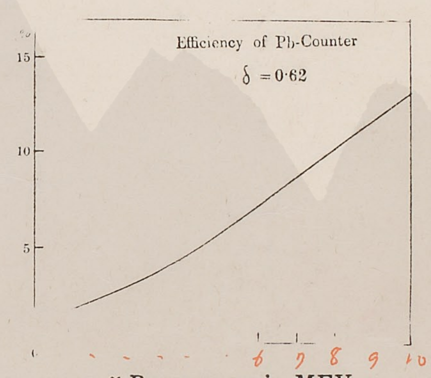
the hard ~~X~~-ray of ~~T~~horium is about $1:1\cdot4$, whereas it is $\delta : 1\cdot8\times\delta'$ according to the above results. Thus, $\delta' = 0\cdot62$ seems to be a reasonable value corresponding to $\delta = 0\cdot8$, so that the efficiency was plotted as the function of the energy in Fig. 3 by using ~~that~~ this value for δ'. The curve shows that it is roughly proportional to the energy for the whole range above considered. It will increase further with the energy owing to the rapid increase of the cross section of the pair creation.



→ γ-Ray energy in MEV

Fig. 3

4. THIN-WALLED COUNTERS

Next, we want to consider the cylindrical counter with the wall thin compared with the range of most of the secondary electrons. In this case, we can easily show that the number of its discharge per one quantum falling on it is approximately equal to

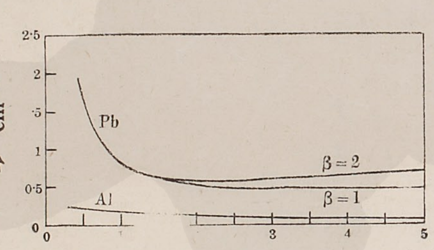
$$\frac{\pi d}{2}(\tau_C+\tau_P+\beta\tau_M)\,, \tag{10}$$

where d is the thickness of the wall and τ_C, τ_P, and τ_M are the co-

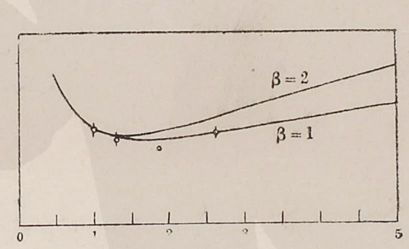
(5) KIKUCHI, AOKI and HUSIMI: *Proc. Phys-Math. Soc. Japan*, **18** (1936), 727.

efficients of absorption of the γ-ray due to the Compton effect, the photoelectric effect and the pair creation respectively. β is a number between 1 and 2, which is necessary owing to the simultaneous emission of a pair of particles. β will decrease gradually with the energy of the γ-ray, as the angular distributions of the pair incline the more to the forward direction the larger the energy.

The factor $\tau_C+\tau_P+\beta\tau_M$ was plotted for Al and Pb as the function of the energy in Fig. 4, by taking $\beta=1$ and 2. The ratio of the efficiencies in these cases is a function of the energy alone, except the arbitrary constant factor, as shown in Fig. 5. The experimental values of Chao and Kung[6] are indicated by the circles in Fig. 5, the arbitrary constant beinng determined so as to be in accord with the theoretical result for the energy 1 MEV. The agreement seems to be satisfactory, if we consider the presence of various factors affecting the results both in theory and experiment[7].



→ γ-Ray energy in MEV
Fig. 4



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→ γ-Ray energy in MEV
Fig. 5

In conclusion, the authors desire to express their gratitudes to Prof. S. Kikuchi and Mr. H. Aoki for helpful suggestions and valuable discussions.

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(6) Chao and Kung: *Chinese Jour. Phys.*, **1** (1934), 56.

(7) Especially, it is inevitable for the thin counter, that the secondary electrons produced in the substances outside the wall are responsible more or less for the discharge.