10(+4)

DEPARTMENT OF PHYSICS
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	Cu	Zn	
Brass	80	20	8.6
	70	30	8.9
	66	34	8.4

DATE Dec 1, 1936
NO.

On the Efficiency of the γ-Ray Counter

G.F. von Droste (Zeits. f. Phys. 100, 529, 1936) は brass counter (length 4 cm, diameter 2 cm, thickness 1 mm) に於いて γ-ray の efficiency を estimate し図の如き結果を得た。

但し

$$2k\,(R_p\tau_p + \beta R_c\tau_c + 2R_M\tau_M)$$

が theoretical な式にして k, β なる constant を 4.7XE 及び 51.2XE の γ-ray に対する known efficiency と一致する様に定めた。

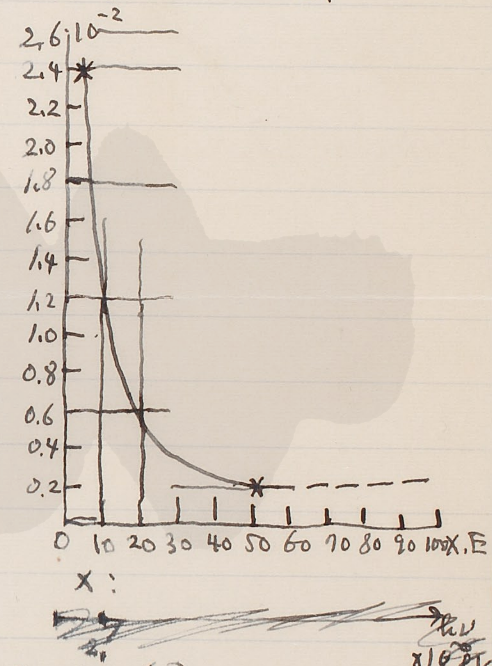
$\beta=0.6$　　$k=0.4$

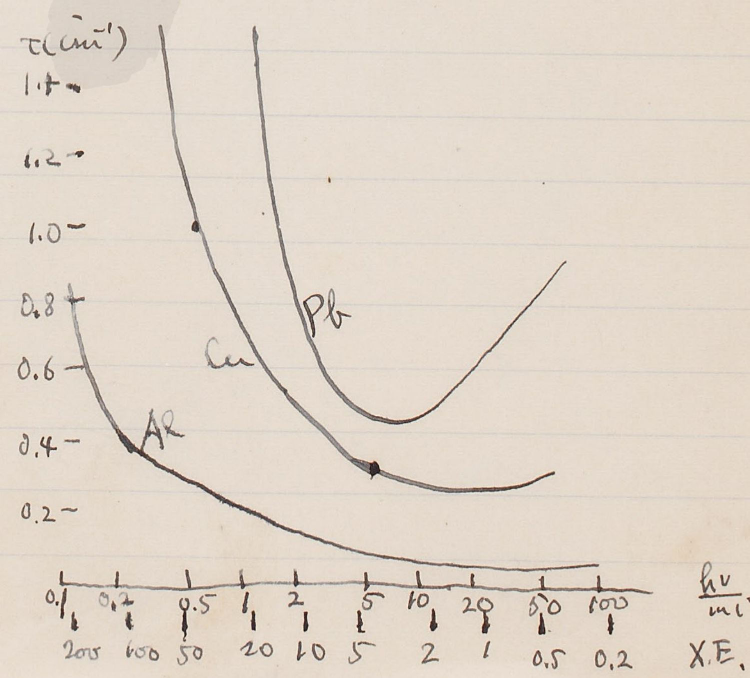
τに就いては、

τ_p: Hulme (Proc. 149, 131, 1935) の理論値 ~~…~~ より hν の高い値を参照にして決定。

τ_c: Klein-Nishina の式を用ひた。

τ_M: Jaeger and Hulme (Proc. 153, 443, 1936)





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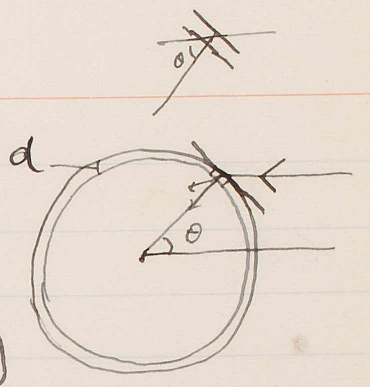
$$\frac{\cos\theta \cdot R \cdot d\theta}{\cos\theta} = R\,d\theta$$

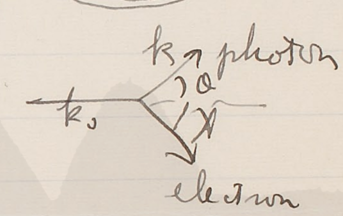
$R < d$



Compton Scattering.

$$d\Phi = \frac{r_0^2}{4}\, d\Omega\, \frac{k^2}{k_0^2}\left[\frac{k_0}{k} + \frac{k}{k_0} - 2 + 4\cos^2\Theta\right]$$

$\sin^2\theta$



$$k = \frac{k_0\mu}{\mu + k_0(1-\cos\theta)}$$

$$\vec{p} = \vec{k}_0 - \vec{k}$$

$$p^2 = k_0^2 + k^2 - 2k_0 k\cos\theta = k_0^2\,\frac{\{\mu + k_0(1-\cos\theta)\}^2 + \mu^2 - 2\mu\{\mu + k_0(1-\cos\theta)\}}{\{\mu + k_0(1-\cos\theta)\}^2}$$

$$= k_0^2\,\frac{2\mu^2 + 2\mu k_0(1-\cos\theta) + k_0^2(1-\cos\theta)^2 - 2\mu k_0 - 2\mu^2(1-\cos\theta)}{\{\quad\}^2}$$

$$= k_0^2\,\frac{\cancel{2\mu^2} - 2\mu k_0\cos\theta - k_0^2(1-\cos^2\theta)}{\{\quad\}^2} = \frac{\quad}{\{\quad\}^2}$$

$k_0^2(1-\cos\theta)^2 - 2\mu k_0\cos\theta + 2\mu^2\cos\theta$

$$\vec{p}\,\vec{k}_0 = k_0^2 - k_0^2\cos\theta\,\frac{\mu}{\mu + k_0(1-\cos\theta)}$$

$$\cancel{\frac{\vec{p}\,\vec{k}_0}{p k_0} = \frac{k_0\left\{1 - \frac{\mu\cos\theta}{\mu + k_0(1-\cos\theta)}\right\}\{\mu + k_0(1-\cos\theta)\}}{\frac{\mu + k_0(1-\cos\theta)}{\sqrt{2\mu^2 - 2\mu k_0\cos\theta - k_0^2(1-\cos^2\theta)}}}}$$

$$= k_0^2\,\frac{\mu + k_0(1-\cos\theta) - \mu\cos\theta}{\mu + k_0(1-\cos\theta)} = \frac{k_0^2(\mu + k_0)(1-\cos\theta)}{\mu + k_0(1-\cos\theta)}$$

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$$p = \frac{k_0\sqrt{k_0(1-\cos\theta)^2 - 2\mu k_0\cos\theta + 2\mu^2\cos\theta}}{\mu + k_0(1-\cos\theta)}$$

$$\cos\chi = \frac{\vec{p}\vec{k_0}}{p k_0} = \frac{(\mu+k_0)(1-\cos\theta)}{\sqrt{k_0(1-\cos\theta)^2 - 2\mu k_0\cos\theta + 2\mu^2\cos\theta}}$$

$$\left.\begin{aligned} &k\cos\theta + p\cos\chi = k_0 \\ &k\sin\theta = p\sin\chi. \\ &k_0 + \mu = k + E, \qquad E^2 - p^2 = \mu^2. \end{aligned}\right\} k, p, \theta, \chi. \quad 3$$

$$(k_0 - p\cos\chi)^2 + p^2\sin^2\chi = k^2 = (k_0 - E)^2 = \left(k_0 + \mu - \sqrt{\mu^2+p^2}\right)^2$$

$$\cancel{k_0^2} - 2k_0 p\cos\chi + \cancel{p^2} = \cancel{k_0^2} - 2\sqrt{\mu^2+p^2}\,k_0 + \mu^2 + \cancel{p^2}$$

$$2\sqrt{\mu^2+p^2}\,k_0 = 2k_0\mu^2 + 2k_0 p\cos\chi$$

$$4k_0^2\mu^2 + 4k_0^2p^2 = \mu^4 + 4\mu^2 k_0 p\cos\chi + 4k_0^2p^2\cos^2\chi.$$

$$4k_0^2\sin^2\chi\cdot p^2 - 4\mu^2 k_0\cos\chi\, p + (4k_0^2 - \mu^2)\mu^2 = 0$$

$$p = \frac{2\mu^2 k_0\cos\chi \pm \sqrt{4\mu^4k_0^2\cos^2\chi - 16k_0^4\mu^2 + 4\mu^4k_0^2\sin^2\chi}}{4k_0^2\sin^2\chi}$$

$$= \frac{2\mu^2 k_0\cos\chi \pm 2\mu k_0\sqrt{\mu^2 - 4k_0^2}}{4k_0^2\sin^2\chi}$$

$$= \frac{\mu^2\cos\chi}{2k_0\sin^2\chi}$$

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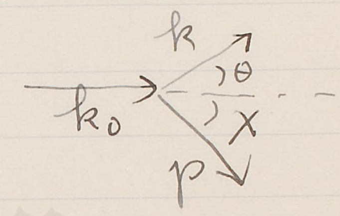
i) Compton Effect.

a) Angular Distribution of ~~Scattering~~ Scattered Electrons

$$\left.\begin{aligned} k\cos\theta + p\cos\chi &= k_0 \\ k\sin\theta &= p\sin\chi \\ k_0+\mu &= k + \sqrt{\mu^2+p^2} \end{aligned}\right\}$$



$$(k_0 - p\cos\chi)^2 + p^2\sin^2\chi = \left(k_0+\mu-\sqrt{\mu^2+p^2}\right)^2$$

$$k_0^2 - 2k_0 p\cos\chi + p^2 = (k_0+\mu)^2 - 2\sqrt{\mu^2+p^2}\,(k_0+\mu) + \mu^2+p^2$$

$$2(k_0+\mu)\sqrt{\mu^2+p^2} = 2k_0 p\cos\chi + 2k_0\mu + 2\mu^2$$

$$(k_0+\mu)^2(\mu^2+p^2) = \left(\mu(k_0+\mu) + k_0 p\cos\chi\right)^2$$

$$(k_0+\mu)^2\mu^2 + (k_0+\mu)^2 p^2 = \mu^2(k_0+\mu)^2 + 2k_0\mu\, p\,(k_0+\mu)\cos\chi + k_0^2 p^2\cos^2\chi .$$

$\cancel{(2k_0+\mu)\mu\ p =}$

$$p = \frac{2k_0\mu(k_0+\mu)\cos\chi}{(k_0+\mu)^2 - k_0^2\cos^2\chi}$$

$$\sqrt{\mu^2+p^2} = \sqrt{\mu^2 + \frac{4k_0^2\mu^2(k_0+\mu)^2\cos^2\chi}{\{(k_0+\mu)^2 - k_0^2\cos^2\chi\}^2}}$$

$$= \frac{\sqrt{\mu^2(k_0+\mu)^2\{(k_0+\mu)^2 + 2k_0^2\cos^2\chi\} + k_0^4\mu^2\cos^4\chi}}{(k_0+\mu)^2 - k_0^2\cos^2\chi}$$

~~cross sectio~~

~~$\tan\theta =$~~

$$= \frac{\mu\{(k_0+\mu)^2 + k_0^2\cos^2\chi\}}{(k_0+\mu)^2 - k_0^2\cos^2\chi}$$

$$\cancel{k = \frac{\mu\{(k_0+\mu)^4 + 2k_0^2(k_0+\mu)^2\cos^2\chi + k_0^4\cos^4\chi\}^{\frac12}}{(k_0+\mu)^2 - k_0^2\cos^2\chi}}$$

$$k = -\mu\frac{(k_0+\mu)^2 + k_0^2\cos^2\chi}{(k_0+\mu)^2 - k_0^2\cos^2\chi} + \mu + k_0 = \frac{k_0(k_0+\mu)^2 - k_0^2(k_0+2\mu)\cos^2\chi}{(k_0+\mu)^2 - k_0^2\cos^2\chi}$$

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$$k = \frac{k_0\{(k_0+\mu)^2 - (\cancel{k_0^2} + 2k_0\mu)\cos^2\chi\}}{(k_0+\mu)^2 - k_0^2\cos^2\chi} \qquad k_0^2 + 2k_0\mu + 2\mu^2$$

$$= \frac{k_0\{}{} $$

$$= \frac{2\mu(k_0+\mu)\sin\chi\cos\chi}{(k_0+\mu)^2 - (k_0^2 + 2k_0\mu + 2\mu^2)\cos^2\chi}$$

$$\tan\theta = \frac{p\sin\chi}{k_0 - p\cos\chi} = \frac{2k_0\mu(k_0+\mu)\sin\chi\cos\chi}{k_0(k_0+\mu)^2 - k_0^3\cos^2\chi - 2k_0\mu(k_0+\mu)\cos^2\chi}$$

$$\frac{d\theta}{\cos^2\theta} = \frac{2k_0\mu(k_0+\mu)(2\cos^2\chi - 1)\,d\chi}{k_0(k_0+\mu)^2 - k_0^3\cos^2\chi - 2k_0\mu(k_0+\mu)\cos}$$

$$= \frac{2\cancel{k_0}\mu(k_0+\mu)\,\cancel{\sin\chi\cos\chi}\,(2\cos^2\chi - 1)\,d\chi}{(k_0+\mu)^2 - (k_0^2 + 2k_0\mu + 2\mu^2)\cos^2\chi}$$

$$= \cancel{2\mu(k_0+\mu)(2\cos^2\chi - 1)\,d\chi}$$

$$\cancel{=} \frac{4\mu(k_0+\mu)\sin^2\chi\cos^2\chi\,d\chi\,(k_0^2 + 2k_0\mu + 2\mu^2)}{\{\qquad\}^2}$$

$$= \frac{2\mu(k_0+\mu)\{(k_0+\mu)^2(2\cos^2\chi - 1) - (k_0^2 + 2k_0\mu + 2\mu^2)\cos^4\chi}{\{\qquad\}^2} \quad (2\cos^2\chi - 1) -$$

$$- 2(k_0^2 + 2k_0\mu + 2\mu^2)(1 - \cos^2\chi)\cos^2\chi$$

$$= \frac{2\mu(k_0+\mu)\{(k_0+\mu)^2(2\cos^2\chi - 1) - (k_0^2 + 2k_0\mu + 2\mu^2)\cos^2\chi}{\{\qquad\}^2}\,d\chi$$

$$= \frac{2\mu(k_0+\mu)\{(k_0^2 + 2k_0\mu)\cos^2\chi - (k_0+\mu)^2\}\,d\chi}{\{(k_0+\mu)^2 - (k_0^2 + 2k_0\mu + 2\mu^2)\cos^2\chi\}^2}$$

$$\cos^2\theta = \frac{1}{1 + \tan^2\theta} = \frac{\{(k_0+\mu)^2 - (k_0^2 + 2k_0\mu + 2\mu^2)\cos^2\chi\}^2}{\{(k_0+\mu)^2 - (k_0^2 + 2k_0\mu + 2\mu^2)\cos^2\chi\}^2 + 4\mu^2(k_0+\mu)^2\,\cancel{(\cos^2\chi - 1)}\,\sin^2\chi\cos^2\chi}.$$

$$\cos^2\chi = \frac{\cos 2\chi + 1}{2}$$

$$(k_0+\mu)^2 - \frac{k_0^2}{2} = $$

$$(k_0+\mu)^2 - \frac{k_0^2+2k_0\mu}{2} = \frac{k_0^2+k_0\mu+2\mu^2}{2}$$

$$=$$

$$\sin\theta = \frac{2\mu(k_0+\mu)\sin(2\chi)}{\{k_0^2+k_0\mu+2\mu^2-(k_0^2+2k_0\mu)\cos 2\chi\}}$$

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$\frac{(k_0+\mu)^2}{(k_0+\mu)^2} \times \frac{k_0^2}{(k_0+2\mu)^2}$

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$$= \frac{\{\quad\}^2}{(k_0+\mu)^4 - 2(k_0+\mu)^2(k_0^2+2k_0\mu+2\mu^2)\cos^2\chi + (k_0^2+2k_0\mu+2\mu^2)^2\cos^4\chi + 4\mu^2(k_0+\mu)^2\cos^2\chi - 4\mu^2(k_0+\mu)^2\cos^4\chi}$$

$$= \frac{\{\quad\}^2}{(k_0+\mu)^4 - 2(k_0+\mu)^2(k_0^2+2k_0\mu)\cos^2\chi + (k_0^2+2k_0\mu+2\mu^2 - 2k_0\mu + 2\mu^2)(k_0^2 + \cancel{2k_0\mu})\cos^4\chi}$$

$$= \frac{\{\quad\}^2}{(k_0+\mu)^4 - 2(k_0+\mu)^2(k_0^2+2k_0\mu)\cos^2\chi + (k_0+2\mu)^2 k_0^2\cos^4\chi}$$

$$d\theta = \frac{-2\mu(k_0+\mu)\{(k_0+\mu)^2 - (k_0^2+2k_0\mu)\cos^2\chi\}\,d\chi}{(k_0+\mu)^4 - 2(k_0+\mu)^2(k_0^2+2k_0\mu)\cos^2\chi + (k_0+2\mu)^2 k_0^2\cos^4\chi}$$

$$= \frac{-2\mu(k_0+\mu)\{(k_0+\mu)^2 - (k_0^2+2k_0\mu)\cos^2\chi\}\,d\chi}{\{(k_0+\mu)^2 - \cancel{k_0^2}\,k_0(k_0+2\mu)\cos^2\chi\}\{\cancel{(k_0+\mu)^2 - (k_0+2\mu)^2\cos^2\chi}\}}$$

$$\sin\theta = \frac{p}{k}\sin\chi = \frac{\cancel{2k_0}\;2\mu(k_0+\mu)\cos\chi}{\{(k_0+\mu)^2 - (k_0^2+2k_0\mu)\cos^2\chi\}}\sin\chi$$

$$\sin\theta\, d\theta = \frac{-4\mu^2(k_0+\mu)^2\cos\chi\sin\chi\, d\chi}{\{\cancel{(k_0+\mu)^2 - k_0^2\cos^2\chi}\}\{(k_0+\mu)^2 - (k_0+2\mu)^2\cos^2\chi\}^2}$$

$$d\phi = \frac{r_0^2}{2}\,d\Omega\,\frac{k^2}{k_0^2}\left[\frac{k_0}{k} + \frac{k}{k_0} - \sin^2\theta\right]$$

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$$d\phi = \frac{r_0^2}{2}\sin\chi\, d\chi\cdot d\varphi\cdot\frac{\{(k_0+\mu)^2-(k_0^2+2k_0\mu)\cos^2\chi\}^2}{\{(k_0+\mu)^2-k_0^2\cos^2\chi\}^2}$$

$$\times\left[\frac{(k_0+\mu)^2-k_0^2\cos^2\chi}{(k_0+\mu)^2-(k_0^2+2k_0\mu)\cos^2\chi}+\frac{(k_0+\mu)^2-(k_0^2+2k_0\mu)\cos^2\chi}{(k_0+\mu)^2-k_0^2\cos^2\chi}\right.$$
$$\left.-\frac{4\mu^2(k_0+\mu)^2(\cos^2\chi-\cos^4\chi)}{\{(k_0+\mu)^2-(k_0^2+2k_0\mu)\cos^2\chi\}^2}\right]$$

$$=\frac{r_0^2}{2}\sin\chi\, d\chi\cdot d\varphi\,\frac{\{(k_0+\mu)^2-(k_0^2+2k_0\mu)\cos^2\chi\}^2}{\{(k_0+\mu)^2-k_0^2\cos^2\chi\}^2}$$

$$\times\left[\frac{\cdots}{\{(k_0+\mu)^2-(k_0^2+2k_0\mu)\cos^2\chi\}^2\{(k_0+\mu)^2-k_0^2\cos^2\chi\}}\right.$$

$$\lambda + \frac{4\gamma^2\cos^4\chi \cdot \cancel{4\gamma^2\cos^4}}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}\{(1+\gamma)^2-\gamma^2\cos^2\chi\}}$$

$$\Vert$$

$$1+\frac{2\gamma\cos^2\chi}{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi} + 1 + \frac{2\gamma\cos^2\chi}{(1+\gamma)^2-\gamma^2\cos^2\chi} -$$

$$\cos^2\chi = \frac{1+\cos 2\chi}{2} = \frac{1+x}{2}$$

$$2(1+\gamma)^2-\gamma^2-\gamma^2 x = \gamma^2+2\gamma+2-\gamma^2 x$$

$$-\gamma^2-\gamma^2 x - 2\gamma - 2\gamma x = (\gamma^2+2)-(\gamma^2+2\gamma)x$$

$$d\phi = -\frac{\nu_0^2}{2}\,d\varphi\; d(\underset{\overset{\Vert}{x}}{\cos(2\chi)})\,\frac{4(1+\gamma)^2}{\{(\gamma^2+2\gamma+2)-\gamma^2 x\}^2}$$

$$\times\left[\frac{(\gamma^2+2\gamma+2)-\gamma^2 x}{(\gamma^2+2)-(\gamma^2+2\gamma)x} + \frac{(\gamma^2+2)-(\gamma^2+2\gamma)x}{(\gamma^2+2\gamma+2)-\gamma^2 x} - \frac{4(\gamma+1)^2(1-x^2)}{\{(\gamma^2+2)-(\gamma^2+2\gamma)x\}^2}\right]$$

$$\downarrow$$

$$\times\;\frac{}{(\gamma^2+2)-(\gamma^2+2\gamma)}$$

$$2-4\,\frac{1+\cos 2\chi}{2}$$

$$= -2\cos 2\chi.$$

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$$\frac{k}{\mu} = \cancel{\gamma}(\gamma+1) - \frac{(1+\gamma)^2+\gamma^2\cos^2\chi}{(1+\gamma)^2-\gamma^2\cos^2\chi}$$

$$= \frac{\gamma\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}}{(1+\gamma)^2-\gamma^2\cos^2\chi}$$

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$$\frac{k_0}{\mu} = \gamma$$

$$\frac{p}{\mu} = \frac{2\gamma(\gamma+1)\cos\chi}{(1+\gamma)^2-\gamma^2\cos^2\chi}$$

$$\frac{\sqrt{\mu^2+p^2}}{\mu} = \frac{(1+\gamma)^2+\gamma^2\cos^2\chi}{(1+\gamma)^2-\gamma^2\cos^2\chi}$$

$$\frac{k_1}{\mu} = \frac{\gamma\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}}{(1+\gamma)^2-\gamma^2\cos^2\chi}$$

$$\tan\theta = \frac{\frac{p}{\mu}\sin\chi}{\gamma-\frac{p}{\mu}\cos\chi} = \frac{2(1+\gamma)\sin\chi\cos\chi}{(1+\gamma)^2-(\gamma^2+2\gamma+2)\cos^2\chi}$$

$$\frac{d\theta}{\cos^2\theta} = \frac{2(1+\gamma)\{(\gamma^2+2\gamma)\cos^2\chi-(1+\gamma)^2\}d\chi}{\{(1+\gamma)^2-(\gamma^2+2\gamma+2)\cos^2\chi\}^2}$$

$$\cos^2\theta = \frac{1}{1+\tan^2\theta} = \frac{\{(1+\gamma)^2-(\gamma^2+2\gamma+2)\cos^2\chi\}^2}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}^2}$$

$$\sin\theta = \frac{p}{k}\sin\chi = \frac{2(\gamma+1)\cos\chi\sin\chi}{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi}$$

$$\sin\theta\, d\theta = \frac{-4(1+\gamma)^2\cos\chi\sin\chi\, d\chi}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}^2}$$

$$d\phi = \frac{-r_0^2}{2}d\varphi\sin\chi\, d\chi\cdot\frac{4(1+\gamma)^2\cos\chi}{\{(1+\gamma)^2-\gamma^2\cos^2\chi\}^2}$$

$$\times\left[\frac{(1+\gamma)^2-\gamma^2\cos^2\chi}{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi}+\frac{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi}{(1+\gamma)^2-\gamma^2\cos^2\chi}-\frac{4(\gamma+1)^2\cos^2\chi\sin^2\chi}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}^2}\right]$$

$$\{(1+\gamma)^4+2\gamma^2(1+\gamma)^2\cos^2\chi-\gamma^2(\gamma^2+2\gamma)\cos^4\chi\}^2$$
$$+\{(1+\gamma)^4-(\gamma^2+2\gamma)^2\cos^4\chi\}^2$$
$$-4(\gamma+1)^2(\cos^2\chi-\cos^4\chi)\{(1+\gamma)^4-2(\gamma^2+\gamma)(1+\gamma)^2\cos^2\chi$$
$$+\gamma^2(\gamma^2+2\gamma)\cos^4\chi\}$$

$$=2(1+\gamma)^8+\{4\gamma(1+\gamma)^6-4(1+\gamma)^6\}\cos^2\chi$$
$$+\{4\gamma^2(1+\gamma)^4-2\gamma^2(\gamma^2+2\gamma)(1+\gamma)^4-2(\gamma^2+2\gamma)^2(1+\gamma)^4$$
$$+4(\gamma+1)^6+8(\gamma+1)^4(\gamma^2+\gamma)\}\cos^4\chi$$
$$-\{4\gamma^3(1+\gamma)^2(\gamma^2+2\gamma)+8(1+\gamma)^5\cancel{(\gamma^2+\gamma)}\gamma$$
$$+4(1+\gamma)^2\cdot\gamma^2(\gamma^2+2\gamma)\}\cos^6\chi$$
$$+\{\gamma^4(\gamma^2+2\gamma)^2+(\gamma^2+2\gamma)^4+4(\gamma+1)^2\gamma^2(\gamma^2+2\gamma)\}\cos^8\chi$$

$$=2(1+\gamma)^4\pm4(1+\gamma)^6\cos^2\chi$$
$$+2(1+\gamma)^4\{2\gamma^2-\gamma^2(\gamma^2+2\gamma)-(\gamma^2+2\gamma)^2$$
$$+2(\gamma+1)^2+4(\gamma^2+\gamma)\}\cos^4\chi$$
$$-4(1+\gamma)^4\{\gamma^4+3\gamma^3-2\gamma^2+4\gamma+1\}\cos^4\chi$$
$$-4(1+\gamma)^2\gamma\{\gamma^2(\gamma^2+2\gamma)+2(1+\gamma)^3$$
$$+\gamma(\gamma^2+2\gamma)\}\cos^6\chi$$
$$-4(1+\gamma)^3\gamma\{\gamma^4+4\gamma^2+4\gamma+2\}\cos^6\chi$$
$$+2\gamma^2(\gamma^2+2\gamma)^2\{3\gamma^2+6\gamma+4\}\cos^8\chi$$

$\cancel{2\gamma^2}+4\gamma^2+8\gamma$

$\cancel{3\gamma^4}+10\gamma^3$

$-6\gamma^2$

$2\gamma^2-\gamma^4-2\gamma^3$

$-\gamma^4-4\gamma^3-4\gamma^2$

$+2\gamma^2+4\gamma+2$

$\cancel{+4\gamma^2}+4\gamma$

$=\cancel{-2\gamma^4}$

$-2\{\gamma^4+3\gamma^3-2\gamma^2+4\gamma+1\}$

$\gamma^4+2\gamma^3+2\gamma^3+3\gamma^2+6\gamma^2+2\gamma+\gamma^3+2\gamma^2$

$\cancel{3\gamma^4+9\gamma^3+8\gamma^2+2\gamma}$

$(\gamma+1)\,\gamma^4+5\gamma^3+8\gamma^2+6\gamma+2$

$\gamma^3+4\gamma^2+4\gamma+2$

$\gamma^2(\gamma^2+2\gamma)$

$(\gamma^2+2\gamma)^2\{\gamma^4+\gamma^4+4\gamma^3+4\gamma^2\}$

$\cancel{(\gamma^2+2\gamma)^2\gamma^2}$

$\cancel{\{4(\gamma+1)^2+2\gamma^2+4\gamma+4\}}$

$\cancel{4\gamma^2+8\gamma+4}$

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$$= -\frac{r_0^2}{\chi} d\varphi \sin\chi d\chi \frac{4(1+\gamma)^2\cos\chi}{\{(1+\gamma)^2-\gamma^2\cos^2\chi\}^2}\left\{1+\frac{2\gamma^2\cos^4\chi}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}\{(1+\gamma)^2-\gamma^2\cos^2\chi\}}\right.$$

$$\left.-\frac{2(\gamma+1)^2\cos^2\chi\sin^2\chi}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}^2}\right\}$$

$$-(1+\gamma)^2(1-\cos^2\chi)\{(1+\gamma)^2-\gamma^2\cos^2\chi\}$$

$$\{\quad\} = 1+\frac{2\cos^2\chi\{\gamma^2\cos^2\chi\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}^2\{(1+\gamma)^2-\gamma^2\cos^2\chi\}}$$

$$1+2\gamma+3\gamma^2$$

$$=1+\frac{2\cos^2\chi}{\{\ \}\{\ \}}\{-(1+\gamma)^4+(1+\gamma)^2(1+2\gamma+3\gamma^2)\cos^2\chi-\gamma^2(2\gamma^2+4\gamma+1)\cos^4\chi\}$$

$$(1+\gamma)^2 \qquad -(2\gamma^2+4\gamma+1)$$
$$(1+\gamma)^2 \qquad -\gamma^2$$

$$\cos^2\chi = \frac{(1+\gamma)^2(3\gamma^2+2\gamma+1)\pm\sqrt{(3\gamma^2+2\gamma+1)^2-4\gamma^2(2\gamma^2+4\gamma+1)}}{2\gamma^2(2\gamma^2+4\gamma+1)}$$

$$\sqrt{\ } = \sqrt{9\gamma^4+12\gamma^3+10\gamma^2+4\gamma+1-8\gamma^4-16\gamma^3-4\gamma^2}$$
$$=\sqrt{\gamma^4-4\gamma^3+6\gamma^2+4\gamma+1}$$

$$\frac{2\cos^2\chi}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}^2}\left\{\frac{\gamma^2\cos^2\chi}{\{(1+\gamma)^2-\gamma^2\cos^2\chi\}}-\frac{(\gamma+1)^2\sin^2\chi-(\gamma^2+1)^2\cos^2\gamma}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}}\right\}$$

$$= \frac{2\cos^2\chi}{\{\ \ \}^2}\left\{\frac{\gamma^2\cos^2\chi}{\{(1+\gamma)^2-\gamma^2\cos^2\chi\}}-\frac{\gamma\cos^2\chi}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}}-\frac{1}{(1+\gamma)^2-\gamma^2+2\gamma}\right.$$

$$= \frac{2\cos^2\chi}{\{\ \ \}^2}\left\{\frac{\cos^2\chi\{(\gamma^2-1)(\gamma+1)^2-\gamma^2(\gamma^2+2\gamma-1)\cos^2\chi\}}{\{(1+\gamma)^2-\gamma^2\cos^2\chi\}\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}}-1\right\}$$

$\frac{23}{35}$

$\frac{16}{49} \times \frac{152}{175}$

$\gamma = 1$ $\quad x = -1$: $\frac{1}{64}$

$x = 0$: $\frac{16}{49}\left(1 + \frac{1}{5}\left(\frac{1}{7} - \frac{4}{5}\right)\right)$

$= \frac{16 \times 152}{49 \times 175}$

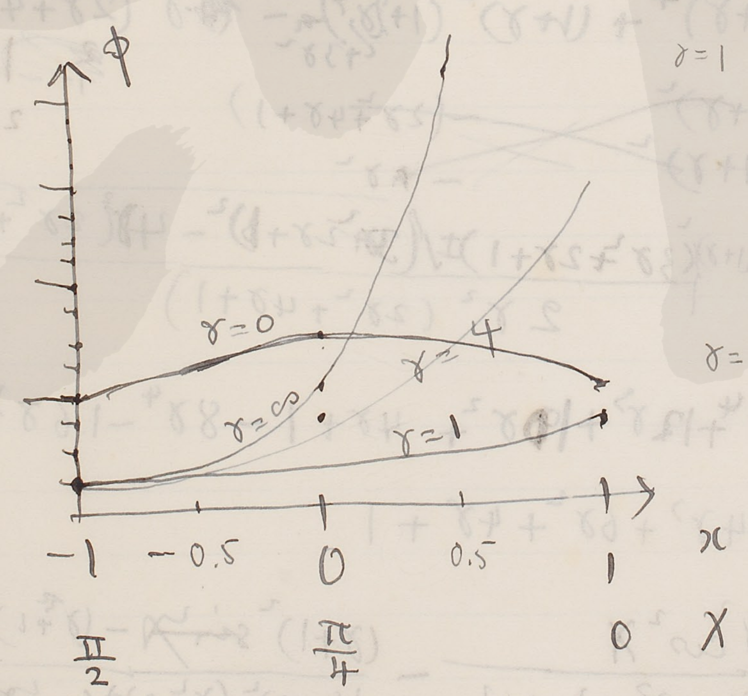
$x = 1$: $\frac{4}{9}\left(1 + \frac{1}{3}\right) = \frac{16}{27}$

$\gamma = 4$ $\quad x = -1$: $\frac{1}{25}$

$x = 0$: $\frac{25}{289}\left(1 + \frac{1}{26}\left(\frac{4}{17} - \frac{25}{26}\right)\right)$

$=$

$x = 1$: $\frac{25}{81}\left(1 + \frac{8}{9}\right)$



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$$(1+\gamma)^2-\gamma^2\frac{1+x}{2}=\frac{1}{2}\{\gamma^2+4\gamma+2-\gamma^2 x\}$$

$$=\frac{1}{2}\{\gamma^2+2\gamma+2-(\gamma^2+2\gamma)x\}$$

$$d\phi=-\frac{r_0^2\,d\varphi\,\sin\chi\cos\chi\,d\chi\cdot 4(1+\gamma)^2}{\{(1+\gamma)^2-\gamma^2\cos^2\chi\}^2}\left[1+\frac{2\cos^2\chi}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}}\times\right.$$

$$\left.\left\{\frac{\gamma^2\cos^2\chi}{\{(1+\gamma)^2-\gamma^2\cos^2\chi\}}-\frac{(\gamma+1)^2\sin^2\chi}{\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}}\right\}\right]$$

$$\left\{\cancel{1}\frac{(1+\gamma)^2}{(1+\gamma)^2-\gamma^2\cos^2\chi}-1+\frac{\cos^2\chi}{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi}\right\}$$

$$\parallel$$

$$\left\{\frac{(1+\gamma)^4-(1+\gamma)^2(\gamma^2+2\gamma-1)\cos^2\chi-\gamma^2\cos^4\chi}{\{(1+\gamma)^2-\gamma^2\cos^2\chi\}\{(1+\gamma)^2-(\gamma^2+2\gamma)\cos^2\chi\}}\right\|-2\Big\}$$

$$\gamma^2+2\gamma$$

$$x=\cos 2\chi$$

$$d\phi=\frac{\cancel{4}r_0^2\,d\varphi\cdot dx\,(1+\gamma)^2}{\{\gamma^2+4\gamma+2-\gamma^2 x\}^2}\left[1+\frac{2(1+x)}{\{\gamma^2+2\gamma+2-(\gamma^2+2\gamma)x\}}\right.$$

$$\left.\times\left\{\frac{\gamma^2(1+x)}{\{\gamma^2+4\gamma+2-\gamma^2 x\}}-\frac{(\gamma+1)^2(1-x)}{\{\gamma^2+2\gamma+2-(\gamma^2+2\gamma)x\}}\right\}\right]$$

$$\gamma^2(\gamma^2+2\gamma+2)-(\gamma^2+2\gamma+1)(\gamma^2+4\gamma+2)=-4\gamma^3-9\gamma^2-8\gamma-2$$

$$\gamma=0:\ d\phi=\cancel{4}r_0^2\,d\varphi\,dx\left[\cancel{1+}\frac{1+x}{2}\right\{\left[1\pm\frac{\cancel{1+x}\,1-x^2}{4}\right]=r_0^2\,d\varphi\,dx\left(\frac{3}{4}\,\frac{1}{2}+\frac{x^2}{4}\right)$$

$$\gamma\simeq\infty:\ d\phi=\frac{4r_0^2\,d\varphi\,dx}{\gamma^2(1-x)^2}\left[1+\frac{1+x}{\gamma^2(1-x)}\left\{\frac{1+x}{1-x}-1\cancel{\pm x}\right\}\right]=\frac{4r_0^2\,d\varphi\,dx}{\gamma^2(1-x)^2}$$

$$\gamma=1:\ d\phi=\frac{16r_0^2\,d\varphi\,dx}{(7-x)^2}\left[1+\frac{1+x}{5-3x}\left\{\frac{1+x}{7-x}-\frac{4(1-x)}{5-3x}\right\}\right]$$

$$\gamma=4:\ d\phi=\frac{25r_0^2\,d\varphi\,dx}{(17-8x)^2}\left[1+\frac{1+x}{2(13-12x)}\left\{\frac{4(1+x)}{(17-8x)}-\frac{25(1-x)}{2(13-12x)}\right\}\right]$$