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On the Efficiency of the γ-Ray Counter

By Hideki Yukawa and Shoichi Sakata

Abstract

The efficiency, i.e. the probability per one quantum of the electric discharge, of the γ-ray counter was computed by making various simplifying assumptions for several energies between 0.2 and 5MEV in the case of Al and between 1 and 10MEV in the case of Pb. The results are shown in Fig. 1 and 2.

§1. Introduction

In order to infer the absolute intensity of the γ-ray from the measurement by means of a counter, it is always necessary to have a reliable knowledge of the efficiency of the counter, i.e. the probability of its discharge due to the secondary electron per one quantum. Whereas it can not easily be determined experimentally, the theoretical evaluation of it is possible at least in principle, since it is approximately the same as the probability that, when a quantum falls on the counter, a secondary electron is produced in the wall and is emitted into the inner space.

We can not expect, however, to obtain a result, which is at once valid for wide range, because the above probability may depend on the material, the thickness, and the shape of the wall as well as the energy of the γ-ray quantum in a complicated manner. Further, the effect of multiple scaatering of the secondary electron in the wall prevent us from the rigorous calculation. Hence, in this paper approximate numerial calculations will be performed for several energies in the cases of Al- and Pb-counters on various simplifying assumptions.[1)]

§2. The Aluminium Counter

~~We first assume the direction~~ consider first a counter of rectangular shape and as shown in Fig 1, of incidence of the γ-ray to be normal to a pair of its faces, ~~to the wall of the counter~~, and the thickness of which ~~the latter~~ is ~~to be~~ small compared with the reciprocal of the absorption coefficient of the γ-ray and larger than the upper limit of the range of secondary electrons, the thickness of other two pair of faces are considered to be infinitely small. The ~~correction~~ due to the oblique incidence, which is necessary, for example, for the counter with the cylindrical shape, will be considered later. ~~small enough to be neglected on account of various factors compensating one another.~~

that it

Now in the case of alminium, most of secondary electrons responsible for the discharge is produced by the Compton effect, at least for γ-ray of energy between 0.5 and 5MEV. As the energy

The efficiency ~~becomes~~ increases first with the thickness as far as the latter is ~~smaller than the~~ to the ~~maximum~~ upper limit of the range of the secondary electrons and then decrease, on account of the absorption of the γ-ray in the wall. The counter has usually a cylindrical shape and if the thickness becomes ~~approximately~~ equal

If we assume the counter to have a cylindrical shape and the thickness of the wall to be small compared with the range of the secondary electrons, $\frac{N}{\pi d}(\tau_c + \tau_p + 2\tau_M)$

If the thickness of the cylindrical wall of a counter, we can easily show that the fraction of the γ-ray quanta which falls on the counter causes discharge

$\frac{\pi}{2}$

1) A semiempirical determination of the efficiency of the brass counter was made by v. Droste, Zeits. f. Phys. 100, 529, 1936.

becomes smaller or larger, the contribution of the electrons produced by the photoelectric effect or the materialization becomes gradually important.

The energy and the angular distributions of the Compton recoil electrons can be calculated immediately from Klein-Nishina's formula, which shows the angular distribution of the scattered photons. Namely, the differential cross section per one electron that a photon of energy $h\nu = \gamma mc^2$ produces an electron making an angle between χ and $\chi + d\chi$ with the direction of incidence is given by

$$\phi(\gamma, x)\,dx = 8\pi r_0^2 \frac{(\gamma+1)^2 dx}{\{(\gamma^2+4\gamma+2)-\gamma^2 x\}^2}\left[1 + \frac{2(1+x)}{\{(\gamma^2+2\gamma+2)-(\gamma^2+2\gamma)x\}}\right.$$
$$\left. \times \left\{\frac{\gamma^2(1+x)}{(\gamma^2+4\gamma+2)-\gamma^2 x} - \frac{(\gamma+1)^2(1-x)}{(\gamma^2+2\gamma+2)-(\gamma^2+2\gamma)x}\right\}\right], \qquad (1)$$

where $x = \cos 2\chi$ and $r_0 = \frac{e^2}{mc^2}$. As the electron is projected always into the forward direction, χ is smaller than $\frac{\pi}{2}$, so that x takes a value between $+1$ and -1. The kinetic energy of the electron is given by

$$\varepsilon(x)\,mc^2 = \frac{2\gamma^2(1+x)}{2(\gamma+1)^2 - \gamma^2(1+x)}\,mc^2 \qquad (2)$$

The probability that such an electron is produced at a depth between z and $z + dz$ from the internal surface of the wall is approximately

$$\cancel{NZdz}\ NZ\,dz\,\phi(\gamma, x)\,dx \qquad NZ\int_{x=-1}^{+1}\int_{z=0}^{d}\{p_1(z,x,\varepsilon) + p_2(z,x,\varepsilon)\}\,dz\,\phi(x)\,dx \qquad (3)$$

where Z is the atomic number and N is the number of atoms in unit

$NZdz$
$\int p_1(z,x,\varepsilon)$
$+ \int p_2(z,x,\varepsilon)$

$p_1(z,x,\varepsilon)$ is the prob. that the electron with the energy ε and the direction x produced in the upper wall at the depth z reaches the internal surface, and $p_2(z,x,\varepsilon)$ is that produced in the lower wall

$NZ\,dz\,\phi(x,\varepsilon)\,dx$

$NZ\iint \phi(x)\,dx\,dz$

p is the probability that the electron reaches the internal surface without being intercepted

$$Z_0(x,\varepsilon) = \int_{z=0}^{d} \{ p_1(z,x,\varepsilon) + p_2(z,x,\varepsilon) \} dz$$

volume of the wall.

Further, we have to determine the probability that ~~the~~ this electron reaches the internal surface of without being interrupted by the Bremsung and the scattering. If we take only the former into account, all the electrons with a given initial energy ε have approximately the same range $R_0(\varepsilon)$, which can be evaluated, for example, by Bloch's theory,[1] so that the above probability becomes 1 or 0 according as

$$z \lessgtr R_0(\varepsilon)\cos\chi = R_0(\varepsilon)\sqrt{\frac{1+x}{2}}\,, \qquad (4)$$

Consequently, the efficiency of the counter due to the Compton effect alone is given by

$$\mathrm{Eff}_c = NZ\int_{-1}^{+1} R_0(\varepsilon)\sqrt{\frac{1+x}{2}}\,\phi(\gamma,x)\,dx. \qquad (5)$$

In reality, however, the effect of scattering is considerable and can hardly be predicted theoretically. Therefore, it is more convenient to introduce the effective thickness $z_0(\varepsilon,x)$ as a function of ε and x, so that the efficiency takes the form

$$\mathrm{Eff}_c = NZ\int_{-1}^{+1} Z_0(\varepsilon,x)\,\phi(\gamma,x)\,dx \qquad (6)$$

In the case of aluminium, the practical ~~maximum~~ range of the electron is known experimentally fairly well as the function of the energy[2] and the above $z_0(\varepsilon,x)$ will be smaller than this range by a factor, which ~~is can beassumed to be nearly independent of the energy . This factor~~ will depend

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1) Bloch, Zeits. f. Phys. 81, 363, 1933 and further Heitler, Quantum Theory of Radiation, Oxford, 1936.

2) See, for example, the summary report of Bothe, Handb. d. Phys. Bd. XXII/2, 1, 1933.
Vauder, Phil. Mag. 29, 725, 1925; Schonland, Proc. Roy. Soc. (A) 104, 235, 1923; 108, 187, 1923 ~~and~~ further, ~~information will be found~~ see! ~~A~~

more or less on ~~the direction x~~ ~~the energy~~ ε and x, so that the calculation was made for the two limiting cases of

i) complete diffusion, ~~i.e.~~ in which the above factor ~~is~~ will be a constant between about $\frac{1}{2}$ and 1 independent of the initial direction x, and

ii) absence of scattering, in which it ~~has~~ will have the form $\cos\chi = \sqrt{\frac{1+x}{2}}$.

The former corresponds to the ~~to~~ low energy limit of the secondary electron and consequently to that of the γ-ray and the latter to the high energy limit. In these former cases the ~~effe~~ form factor f, which ~~should be multiplied~~ necessary on account of the oblique incidence in the case of cylindrical counter ~~becomes~~ tends to ~~[illegible]~~ $\frac{\pi}{2}$ and in the latter case f ~~becomes~~ to 1. Thus the product of these two factors ~~has the~~ ~~for~~ ~~tends~~ to ~~tends~~ about 1 or cos χ to the value ~~in according~~ as as the γ-ray energy ~~becomes~~ increases or decreases. We computed the efficiency ~~for~~ on these two ~~cas~~ limiting assumptions and ~~simp~~ took their ~~simple~~ average, although it ~~is not~~ will not ~~as further~~ precision can ~~not the~~ not be expected for the sake of simplicity.

and their average was taken. these results are shown in the third, fourth and fifth lines of Table 1. The absolute value of is certainly smaller than 1 but can not determined accurately owing to the additional complication caused by the backward scattering in the wall opposite to the direction of incidence. legitimate.

Table 1. Efficiency of Al-Counter

γ-Ray Energy	0.2	0.5	1.0	2.0	5.0 MEV
	0.4	1	2	4	10
Eff. (Compton ε i)	0.068	0.30	1.0	2.1	3.8
Eff. (" ii)	0.058	0.34	0.9	2.1	3.8
Eff. (" average)	0.063	0.32	0.95	2.1	3.8
Eff. (Photo)	0.18	0.02	0	0	0
Eff. (Pair)	0	0	0	0.1	0.4
Eff. (Total)	0.24	0.34	0.95	2.2	4.2

The efficiency due to the photoelectric effect, which is only appreciable for small energy of the γ-ray, can be calculated in the similar manner as above. For 1 the differential cross section per one atom of emitting the electron from the K-shell with an angle between   and   is given approximately by

where   is the fine structure constant and   is the velocity of the photoelectron divided by c. This equation is valid for   in the case of Al, and the efficiency was calculated ~~(for the limiting case~~ i). ~~as the average of two limiting cases as above~~. It becoemes negligiblt small already for   These resultts are shown in the sixth line of Table 1.

On the other hand, the efficiency due to the pair production is always small compared with that due to the Compton effect for the energy range above considered in the case of Al. Resultes of rough estimation are shown in the seventh line of Table 1. By summing the above three magnitudes, we obtain finally the required efficiency as shown in the seventh line of Table 1 and by the full curve of Fig. 1, the dotted curve showing the efficiency due to the Compton effect alone.

§3. The Lead Counter

Next, we want to determine the efficiency of the lead counter for the energy range between 2 and 20, assuming the wall thickness to be small compared with the reciprocal of the absorption coefficient of the -ray and larger than most the range of most of secondary secondary electrons. This condition is approximately satisfied if we take the thickness about 0.3 cm, for instance. In this case, the evaluation becomes more difficult than in the case of Al owing to the facts

i) ~~that the photoelectric effect and the pair creation have the same order of cross section as the Compton effect,~~ The energy and the angular distributions of the secondary particles ~~electron~~ in the former cases ~~and~~ are not known in detail,

ii) ~~that~~ the range-energy relation of the electron in lead is not known in detail experimentally,

iii) ~~that~~ the efficiency due to the pair creation should be multiplied by a certain factor between 1 and2 2, which can ~~not~~ be predicted theoretically hardly, on account of the simultaneous emission of an electron and a positron.

Hence, we want to be contented with rough estimations which can be performed by using the resultés of Bethe-Heitler's theory of the absorption of the high energy γ-ray and of the fast electron. Final resultés are given in Table 2 and Fig. 2.

Table 2. Efficiency of Pb-Counter

-Ray Energy	1	2.5	5	10	MEV
Eff.(Compton)					
(Photo)					
(PAIR)					
(Total)					

$$\begin{array}{r} 2.65 \\ 4.7 \\ \hline 1855 \\ 1060 \\ \hline 12.455 \end{array} \qquad \begin{array}{r} 1.03 \\ 12.1\overline{)12.455} \\ 121 \\ \hline 355 \end{array}$$

$$\frac{2.65 \times 4.7}{12.1}$$

$$\begin{array}{r} 0.1 \\ 0.092 \\ \hline 0.0092 \end{array}$$

$\gamma = 5$

Al $\tau = 0.1$

Pb $\tau_a = 0.475$

$$\begin{array}{r} 0.475 \\ 0.093 \\ \hline 0.568 \\ 0.010 \\ \hline 0.58 \end{array} \qquad \begin{array}{r} 0.1274 \\ 0.073 \\ \hline 3822 \\ 8918 \\ \hline 0.093 \end{array} \qquad \begin{array}{r} 0.58 \\ 0.022 \\ \hline 116 \\ 116 \\ \hline 0.01276 \end{array}$$

$\gamma = 2$ Al $\tau = 0.16$

Pb $\tau = 0.8$

$$\begin{array}{r} 0.022 \\ 0.8 \\ \hline 0.0176 \end{array} \qquad \begin{array}{r} 0.16 \\ 0.092 \\ \hline 36 \\ 144 \\ \hline 0.01476 \end{array}

~~Since~~ Since we ~~has~~ used the theoretical range of the electron, which ~~was determined~~ had been calculated by Bethe and Heitler altogether neglecting the effect of scattering, ~~The numerical~~ a factor δ' ~~will~~ appreciably small than 1 ~~will be so~~ is necessary, as was surely too large on account of

we have used theoretical the range used in the above estimation is too large on account of the complete

The efficiency for the larger value the energy will increase

Thus, the efficiency is roughly proportional to the energy for the interval (3, 10) between 3 and 10 MEV. ~~As~~ the energy ~~be~~ increases. The common factor in this case ~~are is likely to have approximately~~ will be appreciably smaller than 1, since further owing to the ~~rapid~~ increase of the cross section of the ~~cross~~ pair production ~~the same value as that in the case of Al, since~~ somewhere

~~According to the experimental of Kikuchi and in the~~ results of Kikuchi laboratory, the ~~range of the secondary electrons~~ ~~on the other hand~~ The numerical value of it, however ~~becomes~~ ~~however, be determ~~ cannot ~~and~~ depends largely on the thickness of the wall, and the condition for the latter ~~that the it is large small compared with the~~ to be small compared with the reciprocal of absorption coefficient of the γ-ray and larger than the range of most of secondary electrons ~~will~~ can not be satisfied at once.

so that, we plotted the efficiency ~~in this~~ ~~from and~~ the ~~was plotted in~~ Fig. 3 by ~~with this~~ using the value for δ' ~~that δ~~

The factor δ' in this case ~~is will be~~ ~~much~~ is surely smaller than ~~1 and~~ that in the case of Al, as ~~the~~ theoretical range due to ~~the~~ ~~scattering~~ Bremsung alone was used in this case. According to the experimental results in Kikuchi's laboratory, the ~~relat~~ ratio of the efficiencies of the ~~other~~ ~~Pb~~ Al and ~~Al~~ Pb counters for the ~~soft~~ hard γ-ray of ThC″ (2.65 MEV) is about 1 : 1.4[1)], whereas it is ~~1.8~~ $\delta_{Al} : 1.8\,\delta'_{Pb}$, ~~so that~~ ~~accordi~~ according to the ~~theo~~ above ~~results~~ evaluation results, so that we ~~obtain the~~ want to assume approximate relation $\delta'_{Pb} \cong 0.8\,\delta_{Al}$. ~~The last line of Table in~~ ~~Hence,~~ ~~The efficiency was~~ The last line of the Table 2 ~~shows~~ ~~the and Fig 3.~~ rewritten by using this relation, and was ~~shown~~ plotted in ~~the~~ Fig. 3.

1) Similar ~~comparison~~ experiment was made by Chao and Kung, Chinese Jour. Phys. 1, 56, 1934, but their result can not be compared with ~~the above res~~ our result, ~~theo~~ as they used the counters ~~with~~ of which the walls were very thin.

2.25 | 5 | 1.8
2.25) 5 | 2.28 | 2250 | 2200
1.8 1.4 | 0.8
9) 7 | 6.2 | 5.6 | 5.4 | 20
1.8 | 0.8 | 4 | 2.8
δ' = 1.4 / 1.8 α