E 23 081 P 08

On the Efficiency of γ-Ray Counters

By Hideki Yukawa and Shoichi Sakata

12 + 2F

Abstract

The efficiency ~~of the γ-ray~~ counter, i.e. the probability per one quantum of the electric charge of the γ-ray counter was computed ~~calculated~~ in for various simplifying assumptions ~~the case of Al~~ for the energy ~~range~~ between 0,2 and 5 MEV in the case of Al and ~~for~~ between 1 and 10 MEV in the case of Pb. The results are shown in Fig. 1. and 2.

It is always

§ 1 Introduction

It is always ~~often~~ necessary for the ~~determination~~ from the rate of cou results to determination of the intensity of the ~~absolute~~ γ-ray

The efficiency of a γ-ray counter defined as the number of discharge per one quantum ~~will be~~ can be considered ~~the~~ is approximately the same as the probability that the secondary electron is produced in the wall of the counter, when a γ-ray quantum falls on it, and is emitted into the inner space. ~~Thus~~ The dependence of the efficiency on the material, thickness and shape of the wall as well as ~~the~~ on the energy of the γ-ray quantum, ~~which is important for the~~ ~~inter~~ ~~interpretation of the experimental~~ ~~results~~,

The efficiency of such a thin walled counter is very small in general and will not be considered further.

§1. Introduction

In order to infer the absolute intensity of γ-ray from its ~~the~~ measurement by mean of a counter, it is always necessary to have a reliable knowledge of the efficiency of the counter, i.e. the number of discharge due to the secondary electron per one quantum. Whereas it can not easily be determined ~~by~~ experimentally, ~~its~~ ~~con~~ the theoretical evaluation of it is possible at least in principle, since it is approximately the same as the probability that, when a quantum falls on the counter, a secondary electron is produced in the wall and is emitted into the inner space. A result, ~~which is one at once valid for wide range, can not be obtained difficult can not be obtain~~

We can not expect, however, to obtain a result, which is at once valid for wide range, because the above probability may depend~~s~~ on the material, the thickness and the shape of the wall as well as the energy of the γ-ray quantum in a complicated manner. ~~The~~ The ~~rigourous calculation is further prevent~~ further effects of multiple scattering of the secondary electron in the wall prevents us ~~further~~ from the rigorous calculation. Hence, in this paper ~~a~~ approximate numerical calculations ~~of the efficiency~~ ~~are made~~ were performed ~~for~~ for several energies of γ-ray ~~the cases~~ in the cases of Al- and Pb-counters on various simplifying assumptions[1].

1) A semiempirical determination of ~~An estimation of~~ the efficiency of the brass counter was made by von Droste, Zeits. f. Phys, 100, 529, 1936.

§2. The Aluminium Counter~~s~~

~~The~~ We first ~~assume~~ consider that the γ-ray falls normally on the wall of the counter ~~of thickness~~, whose thickness is small ~~compared~~ on the hand, compared with the reciprocal of the absorption coefficient of the γ-ray and; on the other hand, is larger than the ~~maximum~~ upper limit of the ranges of the secondary electrons. ~~As usually~~ A small correction ~~is needed~~ will be needed. The correction due to the oblique incidence, which is necessary, for example, for the ~~usual~~ cylindrical counter~~s~~ with cylindrical shape, will be small~~er than~~ and will not be considered in detail.

In the case of aluminium, we should consider first the ~~secondary~~ Compton effect responsible for the discharge, since most of the secondary electrons is produced by ~~Compton effect~~ this process at least for ~~energy~~ γ-rays of energy between $1\,mc^2$ and $10\,mc^2$. As the energy becomes smaller and smaller or ~~larger~~ ~~energies~~ larger, ~~the phot~~ those electrons which are produced by the photoelectric ~~effect~~ ~~and~~ or the pair materialization ~~become respectively~~ becomes ~~more or less~~ gradually important. The energy distribution and the angular of the Compton recoil electron ~~a~~ can be ~~obtained easily~~ easily calculated from ~~the~~ Klein-Nishina's formula, ~~Name~~ which expresses the angular ~~dif~~ distribution of ~~the~~ scattered ~~electron photons~~ γ-ray photons. Namely, the differential cross section per one electron for the ~~scattering~~ γ-ray of energy γmc^2 ~~of the emitting~~ producing ~~the~~ an electron ~~into the direction with angle~~ which makes an the angle between ~~ϕ,~~ χ, and $\chi + d\chi$ with the direction of incidence, is given by

will be small on account of various ~~comp~~ factors ~~counterbalancing~~ compensating ~~to~~ to one another

† Hence, the ~~partial~~ efficiency of the counter due to the Compton effect alone ~~only~~ is given by

$$\frac{e_c}{NZ} = \int_{-1}^{+1} R_0(\varepsilon)\,\phi(x)\sqrt{\frac{1+x}{2}}\,dx .$$

Since, ~~in~~ in reality, the effect of scattering is considerable and can not ~~be predicted~~ hand be predicted theoretically, it is more convenient to introduce the ~~mean range depth~~ effective thickness $Z_d(\varepsilon, x)$ as a function the initial energy ε and the ~~depth~~ direction x, ~~which~~ so that the ~~above prob is given by~~ efficiency is takes the form

$$e_c = NZ\int_{-1}^{+1} Z_d(\varepsilon, x)\,\phi(x)\,dx .$$

Further we should ~~calculate~~ determine the probability ~~that~~ the electron reaches the internal surface without being ~~kept away by~~ interrupted by the scattering and ~~energy loss, which depends~~ the Bremsung; it depends

If we ~~altogether~~ neglect the effect of scattering, take into account ~~considered~~ the Bremsung, all the electrons with ~~the same~~ a given initial energy ε have approximately the same range $R_0(\varepsilon)$, which can be evaluated ~~theoretically according to the theory~~ by Bloch's theory, for example, so that the above ~~efficiency~~ probability becomes 1 or 0 according as

$$Z \lessgtr R_0(\varepsilon)\cos\chi = R_0(\varepsilon)\sqrt{\frac{1+x}{2}} . \dagger$$

$R_0(\varepsilon)$

4

$$\phi(\gamma, x)\,dx = 8\pi r_0^2 \frac{(\gamma+1)^2\,dx}{\{(\gamma^2+4\gamma+2)-\gamma^2 x\}^2}\Big[1+\frac{2(1+x)}{\{(\gamma^2+2\gamma+2)-(\gamma^2+2\gamma)x\}}$$

$$\times\Big\{\frac{\gamma^2(1+x)}{(\gamma^2+4\gamma+2)-\gamma^2 x}-\frac{(\gamma+1)^2(1-x)}{\{(\gamma^2+2\gamma+2)-(\gamma^2+2\gamma)x}\Big\}\Big] \qquad (1)$$

where $x = \cos 2\chi$ and r_0 is the electron radius $\frac{e^2}{m_0c^2}$. As the electron is always projected into the forward direction, χ is smaller than $\frac{\pi}{2}$ and x takes a value between +1 and −1.

The energy of the electron is given by

$$\varepsilon(x) = \frac{2(\gamma+1)^2+\gamma^2(1+x)}{2(\gamma+1)^2-\gamma^2(1+x)}$$

in unit of m_0c^2.

$$\frac{2\gamma^2(1+x)}{2(\gamma+1)^2-\gamma^2(1+x)} \qquad (2)$$

~~The probability that such an electron is produced will be emitted into the inner space only if the~~

The probability that such an electron is produced at a depth between z and $z+dz$ from the internal surface of the wall is approximately

$$NZ\,d\alpha\,d\phi\;\phi(\gamma, x)\,dx\,dz. \qquad (3)$$

where Z and N are the atomic number and the number of atoms per unit volume of the wall respectively.

Only a part of these electrons will be emitted into the inner space, the remaining part being scattering. All except a fraction of these electrons will be kept away from reaching the inner space by the scattering and energy loss during the passage of the remaining thickness. This fraction, which will depend on the depth z, the angle χ or x and on the initial energy $\varepsilon(x)$ of the electron, ~~can not be determined theoretically~~ hardly.

5

it is more convenient to determine ~~Instead~~ ~~I want~~ ~~introduce~~ define instead defined in the following way with the following ~~the~~ definition

~~we have~~ to be satisfied with ~~the~~ ~~following~~ estimation of ~~Namely, an elect~~

the mean ~~range~~ depth Z ~~which is a function~~, ~~depending (x, ε)~~, which as the function of x and ε $^{(x)}$ ~~the total probability of emission of a certain~~ Namely, the ~~total~~ ~~probability of the~~ ~~number~~ fraction per one quantum that electrons ~~produced~~ which are produced at any depth in the wall with initial direction $(\chi, \chi+d\chi)$ and initial energy $(\varepsilon, \varepsilon+d\varepsilon)$ and reach the inner space, is given by.

$$N Z\, z(x)\, d\phi \qquad (4)$$

~~Integrating this with respect to x, we~~ Hence ~~If we can know Z, the total fraction~~ the efficiency of the counter due to the Compton effect, i.e. the total fraction of the recoil electron per one quantum which reach the inner space is obtained by integrating (4) with respect to x in the interval $(-1, 1)$, if we ~~Unfortunately, Z can only~~ ~~can determine~~ find the expression of $z(x)$.

Now, if we altogether ~~neglect~~ neglect the effect of ~~scattering~~, the range of the electron assume the electron ~~to undergo only Bremsung~~ ~~loses its energy~~ to undergo ~~only~~ Bremsung only, ~~z can be calculated~~ ~~become~~ the relation

$$z = R_0 \cos \chi_{\prime\prime} = r_0 \sqrt{\frac{1+x}{2}} \qquad (5)$$

will hold good, where r_0 is the range which can be ~~det~~ ~~determined~~ calculated theoretically as the function of the energy ~~by using~~ according ~~for example~~, to the ~~formula~~ result ~~of obtain~~ theory of Bloch[1], for example. ~~In reality~~, The true

[1]) Bloch, Zeits. f. Phys. **81**, 363, 1933,

6

value of z however, will be considerably smaller than (5) owing to the frequent occurrence of scattering and it will be more accurate to ~~determine~~

Now ~~the mean depth~~ the effective thickness $z_0(x)$, in the case of Al, the maximum range ~~experimental~~ experimental of the electron in aluminium is known experimentally rather accurately as the function of the energy ε and $z_0(\varepsilon, x)$ ~~will be~~ ~~can be taken~~ ~~it will be permitted~~ to as will be ~~smaller than this range~~ which is by a factor equal to this range multiplied by a factor smaller than 1 ~~and~~ is approximately independent of ε the energy ~~as well as~~ ~~and the direction χ~~. This factor ~~Conclusion factor~~ will depend ~~also~~ on the ~~direction x angle~~ direction χ so that the calculation of the efficiency was made for ~~the~~ assumption more or less ~~that either~~ for the limiting cases in which

and i) the factor $z_0\delta$ is ~~independent of~~ a constant δ for χ ~~as~~

ii) $z\delta$ has the form, ~~is proportional to~~ $\delta \cos\chi = \delta\sqrt{\frac{1+x}{2}}$, The results being shown in Table I.

~~as shown The efficiencies in these cases and the average $\cos\chi$ are shown in Table 1~~ ~~the results being shown in Table 1. and the dotted line in Fig 1.~~ The required efficiency ~~will be~~ should lie somewhere ~~near~~ between these two limits ~~and~~ so that the average is also ~~tabulated~~ shown in Table 1 (and by the dotted ~~line~~ curve in Fig. 1.)

Table 1. Efficiency of Al-Counter ~~due to Compton Effect~~

γ-Ray Energy ~~in MEV~~	0.2	0.5	1.0	2.0	5.0 MEV
$\gamma = \frac{h\nu}{mc^2}$	0.4	1	2	4	10
Efficiency (Compton) (i)	0.068	0.30	1.0 ~~0.9~~	2.1	3.8 × δ%
Efficiency (Compton) (ii)	0.058	0.34	~~0.92~~ 0.9	2.1	3.8 × δ%
Efficiency (average)	0.063	0.32	0.95	2.1	3.8 × δ%

Next, ~~the (b)~~ ~~The correct~~ The efficiency due to the photoelectric effect, which is only appreciable for small energy of the γ-ray, can be calculated in the similar manner as above. ~~For γ = 0.4,~~

7

$\gamma^{-7/2}$ $2\pi \sin\chi \cdot d\chi$ $\qquad \frac{\mu}{\sqrt{1-\beta^2}} - \mu \simeq \gamma$,

$Z/137 < \beta$, $\qquad \frac{1}{\sqrt{1-\beta^2}} = 1+\gamma \qquad 1-\beta^2 = \frac{1}{(1+\gamma)^2}$

$\sqrt{1-(Z\alpha)^2}$ $\qquad \beta = \sqrt{1-\frac{1}{(1+\gamma)^2}}$

The ~~theoretical formula~~ differential cross section per one atom of emitting the electrons ~~in the~~ with ~~the~~ an angle between χ ~~or χ + and~~ $\chi + d\chi$ with an angle from the K-shell between χ and $\chi + d\chi$ is given approximately by

$$\phi_p(\gamma\chi)\,d\chi = \frac{2^5 r_0^2}{137} Z^5 \alpha^4 \gamma^{-7/2} \frac{8\pi\sqrt{2}\sin^3\chi\, d\chi}{(1-\beta\cos\chi)^4}$$

for $1 \gg \gamma \gg \frac{1}{\sqrt{1-(Z\alpha)^2}} - 1$, $\beta \gg Z\alpha$, where α is the fine structure ~~$\alpha = \frac{e^2}{\hbar c}$~~ constant $\frac{e^2}{\hbar c}$ and β is the velocity of the photo electron divided by c. ~~From these~~ Assuming the above equation is valid for ~~Al~~ $\gamma = 0.4$ ~~and~~ in the case of Al ~~and the~~ efficiency was ~~can be~~ ~~can~~ be calculated by ~~averaging~~ ~~in~~ the limiting cases i) and ii) ~~at~~ as above, and their average was taken. ~~the result being tabulated was found to~~ The efficiency becomes negligible already ~~very small~~ for $\gamma = 1$. These results are shown in the sixth line of Table 1.

On the other hand, the efficiency due to the pair creation ~~photoelectric~~ is not zero for $\gamma > 2$ ~~and~~, but is always small for the energy range above considered in the case of Al compared with that due to the Compton effect. Rough estimations was made for $\gamma = 4$ and 10 as shown in the seventh line of ~~Table 2.~~ by using the result of Bethe and Heitler.

~~Table 2. Total Efficiency of Al-Counter~~

~~γ-Ray Energy~~	~~0.2~~	~~0.5~~	1.0	2.0	5.0	MEV
$\gamma = \frac{h\nu}{mc^2}$	0.4	1	2	4	10	
Efficiency (Photo)						
Efficiency (Pair)						

By summing the above three magnitude, ~~values~~ Thus we obtain finally the total eff. as shown in the last line of Table 1, and by the full curve of Fig. 1.

Table 1.

	0.4	1	2	4	10	
Eff. (Photo)	0.18	0.02	0	0	0	$\times\,\delta$%
Eff. (Pair)	0	0	0	0.1	0.4	$\times\,\delta$%
Eff. (Total)	0.24	0.34	0.95	2.2	4.2	$\times\,\delta$%

§3. The Lead Counter

~~In this case~~ Next, we ~~want~~ to determine the efficiency of the lead counter for the ~~range~~ γ-ray energy ~~between~~ range between γ = 2 and 20, assuming the wall thickness to be ~~small~~ small compared with the reciprocal of the absorption coefficient of the γ-ray, ~~and~~ but larger than ~~the upper limit of~~ the range of ~~the elect~~ most of secondary electrons. This condition is approximately satisfied if we take the thickness about ~~3mm~~ 0.3 cm, for instance. In this case, the calculation become the photoelectric effect and the pair creation have the ~~cross sections of~~ same order of magnitude as that of the Compton effect, so that the evaluation of the efficiency becomes more difficult owing to the facts ~~that~~

i) ~~that~~ the photoelectric effect and the pair creation have the same order of magnitude as that of the Compton effect,

ii) that the range-energy relation of the electron in lead ~~is~~ not known ~~in detail~~ accurately, and

iii) that ~~the angular~~ the energy distr. distribution ~~the~~ the electron and ~~the~~ positron ~~to emitted~~ created simultaneously ~~for~~ by materialization ~~(the increase of~~ the increase of the efficiency ~~of the electron is more~~ ~~for due to the pair creation can not estimated easily, owing~~ ~~to the simultaneous emit~~ ~~The factor to be multiplied~~ ~~for~~ The efficiency due to the pair creation should be multiplied certainly by a factor ~~large~~ corresponding to the simultaneous emission of

~~iv) that the angular distribution~~

vii) that the efficiency due to the pair creation should ~~certainly~~ be multiplied by a certain factor between 1 and 2 ~~larger than 1 and~~ corresponding to the simultaneous ~~appearance~~ emission of an electron and a positron.

9

(≳ MEV, lower)

Thus, the efficiency is roughly proportion to the energy for the interval (3, 20) of γ. ($6mc^2 < h\nu < 20mc^2$)

Hence, we want to content ourselves with rough estimations which can be performed by using the results of Heitler's theory of the absorption of the high energy γ-ray and the fast electron. The results are given in the Table 2 and Fig. 2. The common factor δ, in this case, are likely to have approximately the same value as that for aluminium, since

$\gamma = 5$

Al 2.65 4.8

Pb 5.2

Table 2.

γ-Ray Energy	1	2.5	5	10
$\gamma = \frac{h\nu}{mc^2}$	2	5	10	20
Eff (Photo)	1.2	0.4	0.4	0.3
Eff (Compton)	2.2	3.9	5.6	6.0
Eff (Pair)	0	0.5	4.3	1.5
Eff (Total)	3.4	5.0	9.9	20.1
	1.7 2.9	4.8 4.7	9.	21.6 21.4 21.3

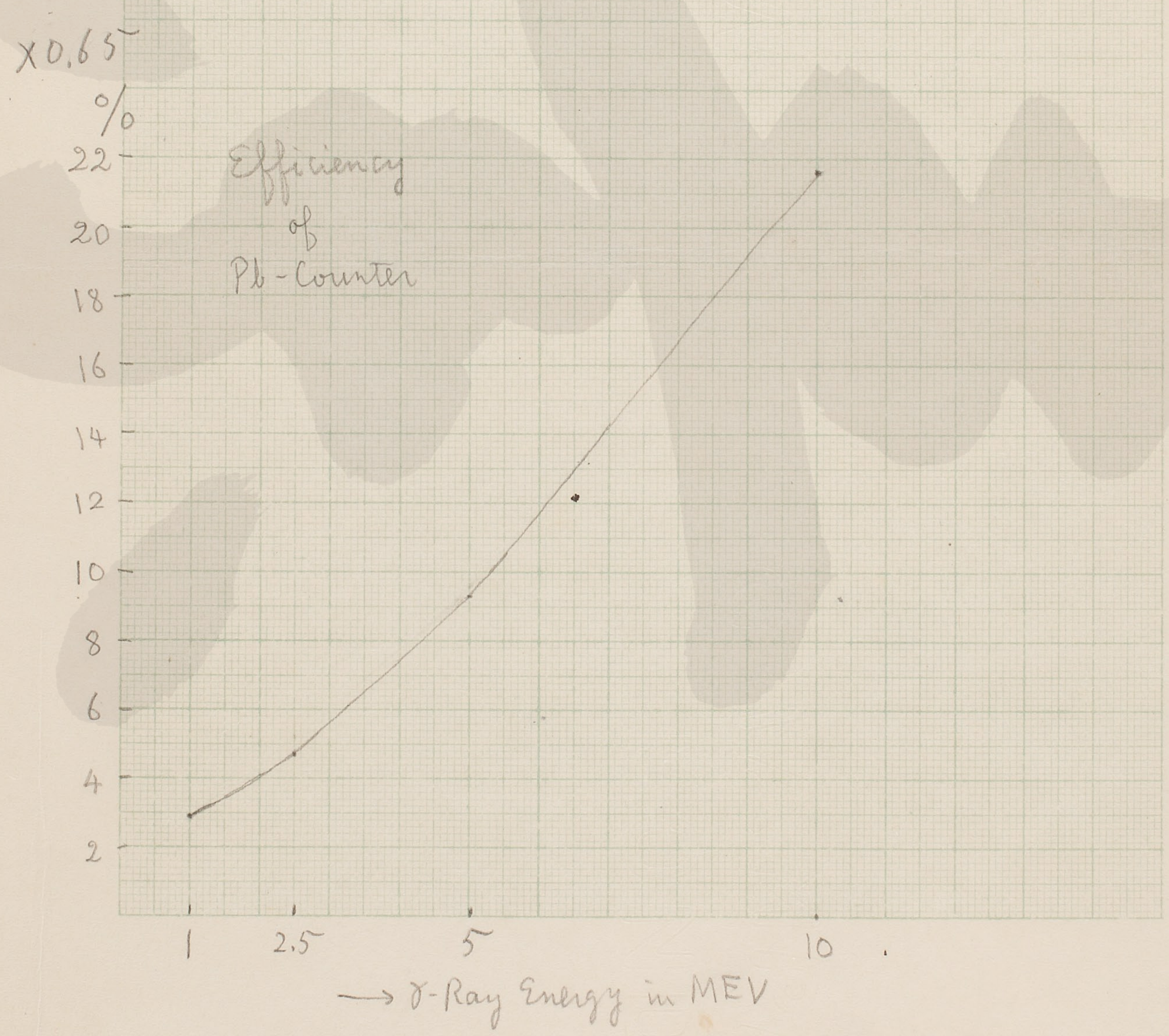
In conclusion, the authors have the pleasure of expressing their thanks to Prof. S. Kikuchi and H. Aoki for helpful discussions of the subject.

~~Institute of Physics,~~

~~Osaka Imperial Uni~~

2.9 4.7 9.3 21.3

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY



11

$Z_0(\varepsilon^{2})$ will be ~~smaller~~ equal to the practical range $R_0(\varepsilon)$, which ~~was~~ have been determined ~~experimentally~~ rather accurately by experiment, multiplied by a factor ~~little~~ between $\frac{1}{2}$ and 1 ~~larger than~~ $\frac{1}{2}$ independent of the direction x.

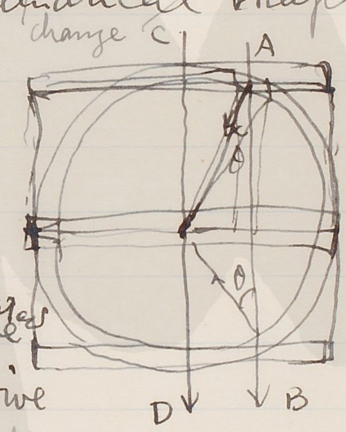
The calculations ~~was~~ were made for the following limiting cases.

i) As the energy of the γ-ray ~~becomes small~~ decreases, the effect of scattering of the secondary electron becomes large and, in the limit ~~case~~ of the complete diffusion, the above factor will tend to a constant between $\frac{1}{2}$ and 1 ~~nearly~~ independent of the initial direction x.

ii) As the energy increases, the effect of scattering becomes smaller and Z_0 will tend to the value $R_0(\varepsilon)\cos\chi$ ~~approximately equal~~ in the absence of scattering, the above factor will tend to a $\cos\chi = \sqrt{\frac{1}{1+x}}$ multiplied by a factor a little smaller than 1, as shown in (5').

Further, We ~~should~~ want to ~~take~~ consider the effect of oblique incidence ~~into account~~ for these limiting cases, assuming the counter to have the cylindrical shape, correction due to the assumption of the shape of the counter whose ~~radius~~ diameter is large compared with the thickness of the wall.

i) In the case i), if the photon ~~passes~~ falls ~~through the wall~~ on the wall making an angle θ with the surface normal, as indicated, the effective depth by the straight line AB, the effective thickness of Z_0 becomes larger by a factor about $\frac{1}{\cos\theta}$ compared with the photon falling normally, case of normal incidence, as indicated by CD. So that the ~~apparent~~ efficiency will be ~~multiplied~~ larger by the form factor



$$\frac{1}{2}\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{\cos\theta\, d\theta}{\cos\theta} = \frac{\pi}{2}$$

in comparison to the rectangular counter above considered with the same cross sectional area.

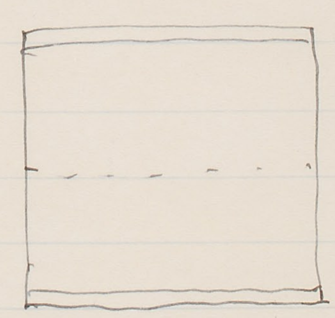


Fig.

ii) In the case ii), most of the recoil electrons are emitted into the forward direction, so that the form factor ~~does not differ appreciably~~ will be almost ~~from 1~~ equal to 1.

~~Thus~~ Consequently, the product of the two factors above becomes ~~about~~ a ~~constant~~ number a little smaller than 1 in the case i) and $\cos\chi$ in the case ii).

The results of the calculations in these two cases ~~does not are not~~ does not differ appreciably from each other, as shown in Table 1, so that we took simply the average of them as ~~the~~ approximate value of the efficiency of the cylindrical counter due to the Compton effect.

13

Table 1. The Efficiency of the Al-Counter

-Ray Energy	0.2	0.5	1.0	2.0	5.0	MEV
Eff. (Compton i)	0.068	0.34	1.0	2.1	3.8	
Eff. (ii)	0.0$8	0.30	0;9	2;1		

The theoretical ~~method~~ determination of ~~The unknown~~ the factor δ is very difficult, but if we put commonly ~~presumably~~ $\delta = 0.8$, ~~seems to be a reasonable value,~~ ~~since as~~ the efficiency ~~the energy~~ for energy 2.65 MEV becomes about $\mathrm{Eff} = 2.75 \times 0.8 =$ 2200 for the hard γ-Ray of ThC'' ($h\nu =$ 2.65 MEV) becomes about

$$2.75 \times 0.8 = 2.2\ \%,$$

which is a reasonable value, since it a little smaller than the efficiency ~~of~~ this γ-ray for in the case of the brass counter 2.4% determined ~~by the~~ experimentally by Droste.[1)] Hence, the ~~exact~~ efficiency was plotted as the function of the energy in Fig. 2 ~~by~~ taking $\delta = 0.8$. The full curve shows the total efficiency and the dotted curve shows that due to the Compton effect alone.

~~It~~ It will be ~~increase~~ ~~larger~~ increase further & ~~further~~ as the energy ~~increases~~ becomes larger ~~further~~ owing to the rapid increase of the cross section of the pair creation, ~~until the~~ whereas ~~above estimation assumption that~~ will ~~it will again increases~~

1) l.c.

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY

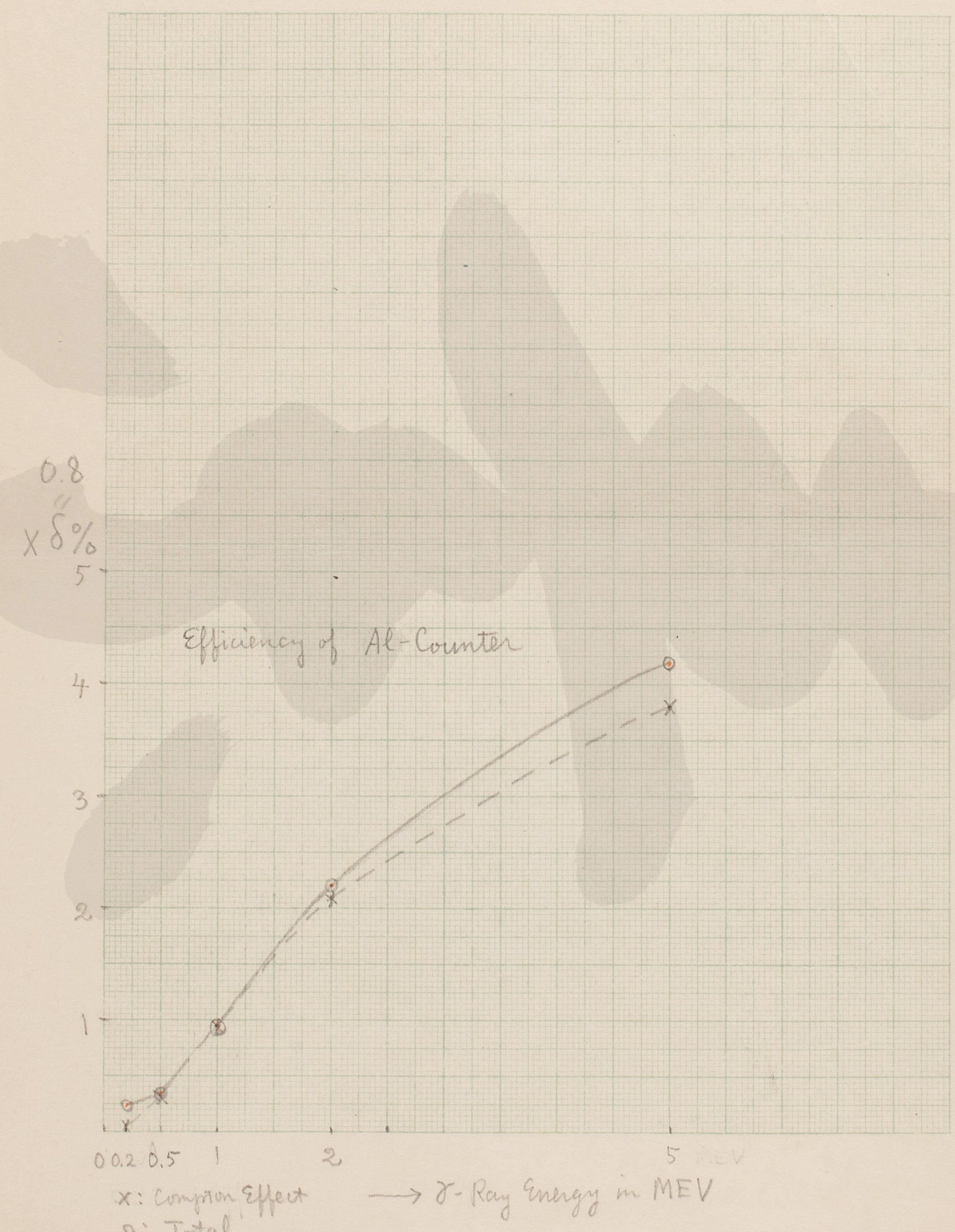


Fig. 1.