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Elementary Calculations on the Slowing Down of Neutrons by Thin Plate

By Hideki Yukawa

(Read July 4, 1936)

Abstract

The energy distribution of ~~a~~ neutrons slowed down by a thin plate containing hydrogen, whose thickness is small compared with the mean free path of the incident neutrons of definite energy, is calculated for following cases, taking only the single and the double scatterings into account.

i) Normal incidence: In this case, the distribution function~~s~~ for slow neutrons increases with decreasing energy E as -log E. (§ 2, §3, §4, §5.)

ii) A point source: In this case, the distribution function is nearly constant.(§ 6.)

iii) A point source and a small detector placed face to face on either side of the plate: In this case also, the distribution fuction of neutrons, which hit the detector after single scattering, is nearly constant. This result is compared with the experiment. (§ 7.)

§ 1. Introduction.

It is, of course, an important, but not an easy task to find the change of the energy distribution of neutrons due to the presence of a substance containing hydrogen in sufficiently general case. The wellknown probability distribution of the neutron energy after having scattered by protons a fixed number of times[1] can be applied to special problems, only if the probability of occurrence for each number of times of scattering is calculated in each case. On the other hand, we are met with mathematical complications, if we want to extend the homogeneous and isotropic equilibrium distribution found by Fermi[2] to more general cases.

Hence, it seems to be justified for the time being to confine our attention to an elementary problem of slowing down of neutrons by a plate, whose thickness is small compared with the mean free path of primary neutrons. In order to ~~To~~ simplify the calculation, it is necessary even to assume the thickness to be smaller ~~small~~ than the mean free path of slow neutrons, which is only a few millimetres in paraffin.

§ 2. Single Scattering by Protons.

We consider a ~~[illegible]~~ plate of thickness h, containing hydrogen,

1) Wick, Phys. Rev. **49**, 192, 1936; Condon and Breit, ibid. **49**, 229, 1936; Goudsmit, ibid. **49**, 406, 1936; Lamla, Naturwiss. **24**, 251, 1936.

2) Fermi, Zeeman Jubilee Papers, Nijhoff, The Hague, 1935, p. 128.

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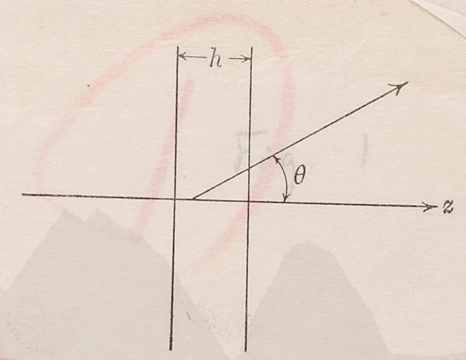
through which a neutron beam of definite velocity v_0 passes perpendicularly from left to right. (Fig. 1.) We neglect at first all the the processes other than the scattering by protons.



If the direction of the incident beam is taken as that of z-axis, the probability for it to be deflected by an angle between θ and $\theta+d\theta$ at a distance between z and z+dz from the left surface is given by

$$\exp\left(-\frac{z}{\lambda_0}\right)\frac{dz}{\lambda_0}\,2\sin\theta\cos\theta\,d\theta, \qquad (1)$$

as the consequence of the assumption of short range force between the neutron and the proton, where λ_0 is the mean free path of the neutron with velocity v_0.

The neutron deflected in this way has the velocity

$$v = v_0\cos\theta \qquad (2)$$

and passes through the plate without further scattering only with the probability $\exp\frac{-(h-z)}{\lambda\cos\theta}$, where λ is the mean free path for velocity v. Hence, the probability, that a neutron passes through the plate in the direction $(\theta, \theta+d\theta)$ after single scattering, becomes

$$2\sin\theta\cos\theta\,d\theta\int_0^h \exp\left\{-\frac{z}{\lambda_0}-\frac{(h-z)}{\lambda\cos\theta}\right\}\frac{dz}{\lambda_0}$$

(3)

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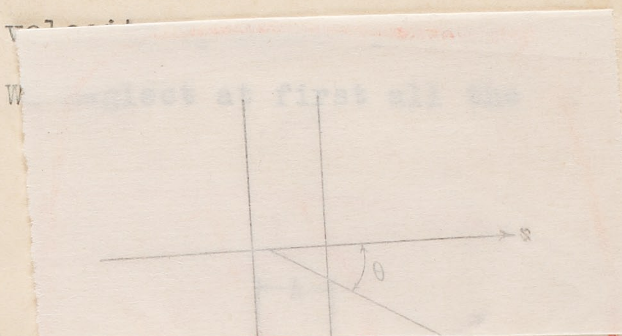

Fig. 1

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$$2\sin\theta\cos\theta\,d\theta\int_0^h \exp\left\{-\frac{z}{\lambda_0}-\frac{(h-z)}{\lambda\cos\theta}\right\}\frac{dz}{\lambda_0}$$

$$= \frac{2\sin\theta\cos\theta\, d\theta}{\lambda_0} \frac{\exp\left(-\frac{h}{\lambda_0}\right) - \exp\left(-\frac{h}{\lambda\cos\theta}\right)}{\frac{1}{\lambda\cos\theta} - \frac{1}{\lambda_0}}, \qquad (3)$$

so that the number of ~~the~~ transmitted neutrons with velocity (v, v+dv) is the fraction

$$p_1(v)dv = \frac{2v\,dv}{\lambda_0 v_0^2} \frac{\exp\left(-\frac{h}{\lambda_0}\right) - \exp\left(-\frac{hv_0}{\lambda v}\right)}{\frac{v_0}{\lambda v} - \frac{1}{\lambda_0}} \qquad (3)'$$

of that of incident neutrons.

Hereafter we assume the thickness h to be small compared with λ_0. Then, (3)' can be reduced to

$$p_1(v)dv \cong \frac{2v\,dv}{v_0^2} \frac{h}{\lambda_0} \left\{1 - \frac{h}{2\lambda_0}\left(1 + \frac{\lambda_0 v_0}{\lambda v}\right)\right\} \qquad (4)$$

for velocity v so large that it satisfies the condition $\lambda v \gg h v_0$ for the scattered neutron to pass through the plate almost certainly without further scattering.

On the contrary, it ~~reduces~~ is reduced to

$$p_1(v)dv \cong \frac{v^2 dv}{v_0^3} \frac{2\lambda}{\lambda_0} \left(1 - \frac{h}{\lambda_0}\right) \qquad (5)$$

for velocity v small in comparison with the critical velocity v_c given by

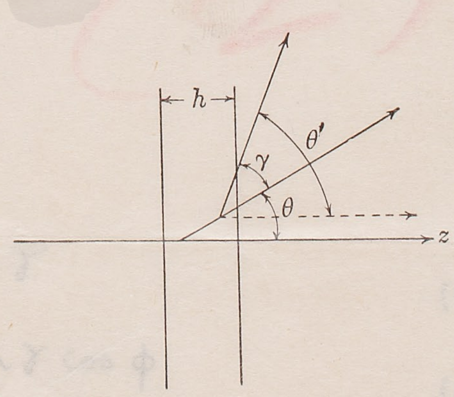
$$\lambda_c v_c = h v_0, \qquad (6)$$

where λ_c is the mean free path for velocity v_c. In this case, the the neutron after having deflected nearly at right angle has very small velocity and will be deflected once more, almost certainly, before it will come out of the plate.

Of course, no neutrons are reflected back by single scttering, as $\theta \leq \frac{\pi}{2}$.

§ 3. Double Scattering by Protons.

We can calculate in the similar manner the probability that a neutron is deflected first by an angle $(\theta, \theta+d\theta)$, scattered once more into a solid angle $\sin\gamma\, d\gamma\, d\phi$ with polar angles (γ, ϕ) in the new coordinate system, in which the direction of motion after first scattering is taken as z-axis and the azymuth of the former z-axis is zero, and finally passes through or is reflected back by the plate, according as the angle θ' between the final direction of motion and the former z-axis is smaller or larger than $\frac{\pi}{2}$. The final velocity v' and the angle θ' are given by



$$v' = v\cos\gamma = v_0 \cos\theta\, c$$

and

$$\cos\theta' = \cos\theta\,\cos\gamma + \sin\theta\, s$$

$$\lambda_c v_c = h v_0, \tag{6}$$

where λ_c is the mean free path for velocity v_c. In this case, the the neutron after having deflected nearly at right angle has very small velocity and will be deflected once more, almost certainly, before it will come out of the plate.

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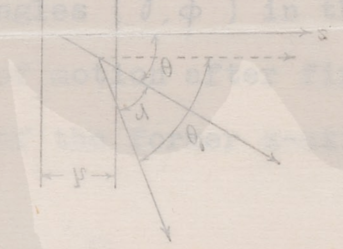

$$v' = v\cos\gamma = v_0 \cos\theta \cos\gamma \tag{7}$$

and

$$\cos\theta' = \cos\theta\cos\gamma + \sin\theta\sin\gamma\cos\phi \tag{8}$$

(2)

respectively.

The result is

$$\frac{2\sin\theta\, d\theta}{\lambda_0}\cdot\frac{2\sin\gamma\cos\gamma\, d\gamma\, d\phi}{2\pi\lambda}\cdot\frac{1}{\frac{1}{\lambda'\cos\theta'}-\frac{1}{\lambda\cos\theta}}$$

$$\times\left\{\frac{\exp(-\frac{h}{\lambda_0})-\exp(-\frac{h}{\lambda\cos\theta})}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}}-\frac{\exp(-\frac{h}{\lambda_0})-\exp(-\frac{h}{\lambda'\cos\theta'})}{\frac{1}{\lambda'\cos\theta'}-\frac{1}{\lambda_0}}\right\} \quad (9)$$

for transmission, i.e. $\theta'<\frac{\pi}{2}$, and

$$\frac{2\sin\theta\, d\theta}{\lambda_0}\cdot\frac{2\sin\gamma\cos\gamma\, d\gamma\, d\phi}{2\pi\lambda}\cdot\frac{\exp(-\frac{h}{\lambda'\cos\theta''})}{\frac{1}{\lambda'\cos\theta''}+\frac{1}{\lambda\cos\theta}}$$

$$\times\left\{\frac{\exp(\frac{h}{\lambda'\cos\theta''})-\exp(-\frac{h}{\lambda_0})}{\frac{1}{\lambda'\cos\theta''}+\frac{1}{\lambda_0}}-\frac{\exp(-\frac{h}{\lambda_0})-\exp(-\frac{h}{\lambda\cos\theta})}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}}\right\} \quad (10)$$

for reflection, i.e. $\theta''=\pi-\theta'<\frac{\pi}{2}$, where λ' is the mean free path for velocity v'.

Now, if $v'>\frac{v_0}{2}$, θ' is smaller than $\frac{\pi}{2}$ for any values of v and ϕ in the intervals (v', v_0) and $(0, 2\pi)$ respectively, so that the total number of neutrons, which pass through with the velocity $(v', v'+dv')$, becomes, by integrating (9), the fraction

$$p_2(v')dv' = \frac{2v'dv'}{\lambda_0 v_0^2}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_{v'}^{v_0}\frac{2dv}{\lambda_0 v^2}\,\frac{1}{\frac{v}{\lambda' v'}-\frac{v_0}{\lambda v}}$$

$$\times\left\{\frac{\exp(-\frac{h}{\lambda_0})-\exp(-\frac{hv_0}{\lambda v})}{\frac{v_0}{\lambda v}-\frac{1}{\lambda_0}}-\frac{\exp(-\frac{h}{\lambda_0})-\exp(-\frac{hv}{\lambda' v'})}{\frac{v}{\lambda' v'}-\frac{1}{\lambda_0}}\right\} \quad \cancel{(11)}$$

$$\cong \cancel{\left(\frac{h}{\lambda_0}\right)^2\frac{2(v_0-v')dv'}{v_0^2}}$$

$$\cong \frac{2v'dv'}{v_0^2}\left(\frac{h}{\lambda_0}\right)^2\left(\frac{v_0}{v'}-1\right) \qquad (11)$$

of the number of the incident neutrons approximately, provided that the mean free path is nearly constant in the interval $(\frac{v_0}{2}, v_0)$.

Thus, for fast neutrons, the contribution of the double scattering to velocity distribution is smaller by a factor of the order of $\frac{h}{\lambda_0}$ than that of the single scattering. Besides, no neutrons are reflected back with velocity larger than $\frac{v_0}{2}$. ~~thus far.~~

Next, if v' is very small compared with v_0, it is required that at least one of θ and γ is nearly equal to $\frac{\pi}{2}$. The case $\theta \cong \frac{\pi}{2}$ is especially important, since those neutrons which are deflected nearly at right angle will be scattered almost certainly in the plate. Roughly speaking, multiple scattering will or will not occur with appreciable probability according as the angle of the first deflection is larger or smaller than the critical angle $\theta_c (\cong \frac{\pi}{2})$ given by

$$\cos\theta_c = \frac{v_c}{v} = \frac{h}{\lambda_c} \ll 1$$

or

$$\theta_c \cong \frac{\pi}{2} - \frac{h}{\lambda_c} \qquad (12)$$

Thus, the probability that an incident neutron undergoes multiple scattering becomes roughly

$$\int_0^h \exp\left(-\frac{z}{\lambda_0}\right)\frac{dz}{\lambda_0}\int_{\theta_c}^{\frac{\pi}{2}} 2\sin\theta\cos\theta\, d\theta \cong \left(\frac{h}{\lambda_0}\right)\left(\frac{h}{\lambda_c}\right)^2 \qquad (13)$$

(8)

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Further, if the thickness h is even small compared with the mean free path for slow neutrons, nearly a half of such slow neutrons that are ~~slowed down~~ produced by first scattering will pass through the plate after second scattering with velocity distribution approximately given by

$$p_2(v')dv' \cong \frac{2v'dv'}{v_0^2}\frac{h}{\lambda_0}\int_{v'}^{v_c}\frac{dv}{v}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{d\phi}{2\pi} = \frac{2v'dv'}{v_0^2}\frac{h}{\lambda_0}\log\frac{v_c}{v'}. \quad (14)$$

In this case, most of the other half will be reflected back after second scattering with approximately the same velocity distribution as (14), while only a small fraction will undergo the third scattering.

On the contrary, if h is not small compared with the mean free path for slow neutrons, although it is small compared with that for fast neutrons, considerable part of those which have been deflected nearly at right angle will be scattered further twice or more [redacted], so that the distribution function will increase more sharply than (14) as the velocity approaches to zero. For still larger thickness, the number of neutrons of not very small velocity will be saturated and will begin to decrease as further increase of the thickness. It is very difficult, however, to infer the behavior of the distribution function in such a region from complicated formulae of multiple scattering.

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§ 4. Effect of Scattering by Heavier Nuclei.

Hitherto we have neglected altogether the effect of the presence of element such as carbon or oxygen in the plate. Among various processes caused by the collision of neutrons with nuclei of such a heavy element, the elastic scattering is relatively important for fast neutrons, so that the mean free path λ of the neutron with velocity v is given approximately by

$$\left.\begin{aligned} \frac{1}{\lambda} &= \frac{1}{\lambda'} + \frac{1}{\lambda''} \\ \frac{1}{\lambda'} &= n'\sigma'(v), \quad \frac{1}{\lambda''} = n''\sigma''(v) \end{aligned}\right\} \qquad (15)$$

where σ' and σ'' are cross sections of the elastic scattering by the hydrogen and the heavier atoms respectively and n' and n'' are their numbers in the unit volume of the plate respectively. σ' and σ'' are the quanties of the same order of magnitude for fast neutrons. Corresponding quantities for primary neutrons of velocity v_o will be characterized by the suffix o.

For slow neutrons, all the effect due to heavier element such as carbon or oxygen seems to be small in comparison with that due to the scattering by hydrogen, i.e. $\sigma' \gg \sigma''$ or $\lambda' \ll \lambda''$.

It should be noticed, however, that if the velocity of the neutron becomes very small, ~~the binding energy of hydrogen with carbon (or oxygen)~~ complicated effects due to the chemical binding of hydrogen with carbon (or oxygen) are expected. Indeed, the mean free path of neutrons with thermal energy was shown by Amaldi and Fermi to be about a quarter of that with energy of the order of 1~~6~~ eV.[1)] Further, there is a small effect due to the capture of slow neutrons by protons In the following rough estimations, all these finer details will be ignored.

~~Now, the number of neutrons, with velocity smaller than the initial~~

Now, the number of transmitted or reflected neutrons is the sum of numbers of those which are transmitted or reflected after having scattered

i) once by protons,

ii) twice by protons,

iii) first by protns and then by heavy nuclei

and

iv) first by heavy nuclei and then by protons,

if we neglect the terms of the order higher than h^2.

i) and ii) are calculated in the similar way as in the pre~~d~~ceeding sections and iii) and iv) are estimated on the assumption that the angular distribution of neutrons scattered by heavy nuclei is spherically symmetric, which will be correct for not too large energy of primary

1) Amaldi and Fermi, Ric. Scient. 7, 223, 1936.

neutrons, and the reduction of the neutron velocity is negligibly small. Only the results of these elementary calculations will be mentioned.

For the velocity v larger than $\frac{v_0}{2}$, the number of neutrons, which pass through the plate with velocity between v and v+dv, is the fraction

$$P(v)dv \cong \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}\Big[1-\frac{h}{2\lambda_0'}\left(3-\frac{v_0}{v}\right) + \frac{h}{4\lambda_0''}\Big\{\log\Big(\frac{\lambda_0}{h}\sqrt[4]{1-\frac{v^2}{v_0^2}}\Big)-\frac{v_0}{v}\Big\}\Big] \quad (16)$$

of that of incident neutrons.[1)] Similarly, the number of those which are reflected is the fraction

$$R(v)dv \cong \frac{2v\,dv}{v_0^2}\cdot\frac{h}{\lambda_0'}\cdot\frac{h}{4\lambda_0'}\Big[2+\frac{v_0}{v}+\log\Big\{\frac{\lambda_0}{h}\sqrt[4]{1-\frac{v^2}{v_0^2}}\Big\}\Big] \quad (17)$$

For the velocity v small ~~than~~ (compared with) the critical velocity v_c, the corresponding expressions are

$$P(v)dv \cong \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}\Big\{\frac{\lambda v}{h v_0}+\log\Big(\frac{v_c}{v}\Big)+\frac{1}{2}\frac{\lambda}{\lambda''} + \frac{h}{4\lambda_0''}\log\frac{\lambda_0}{h}\Big\}+O(h^2) \quad (18)$$

and

$$R(v)dv \cong \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}\Big\{\log\Big(\frac{v_c}{v}\Big)+\frac{1}{2}\frac{\lambda}{\lambda''} + \frac{h}{4\lambda_0''}\log\frac{\lambda_0}{h}\Big\}+O(h^2) \quad (19)$$

respectively, if the thickness of the plate is small compared with the

1) The expressions (16) and (17) ~~hold~~ (are valid) only for v smaller than $v_0\sqrt{1-\left(\frac{h}{\lambda_0}\right)^2}$

mean free path of slow neutrons.

Besides, the number of neutrons, which passes through with velocity nearly equal to the initial velocity v_0, is approximately the fraction

$$P(v_0) \cong 1 - \frac{h}{\lambda_0} + \frac{h}{2\lambda_0''}\left(1 + \frac{h}{2\lambda_0''}\log\frac{\lambda_0}{h}\right) + O(h^2) \qquad (20)$$

of that of the incident neutrons, if we take the single and the double scattering by heavy nuclei into account. Similarly, the number of those which is reflected is the fraction

$$R(v_0) \cong \frac{h}{2\lambda_0''}\left(1 + \frac{h}{2\lambda_0''}\log\frac{\lambda_0}{h}\right) + O(h^2). \qquad (21)$$

§5. Energy Distribution for Normal Incidence.

From above results, the energy distribution of scattered neutrons for normal incidence can be obtained at once. Namely, the fraction, which passes through with energy between E and E+dE, is given roughly by

$$P(E)\,dE \cong \frac{dE}{E_0}\frac{h}{\lambda_0'} \qquad (22)$$

for $\frac{E_0}{4} < E < E_0$, where E_0 is the energy of the incident neutrons, and

$$P(E)\,dE \cong \frac{dE}{E_0}\frac{h}{2\lambda_0'}\log\left(\frac{E_c}{E}\right) \qquad (23)$$

for $E \ll E_c$, where $E_c (\ll E_0)$ is the critical energy satisfying the relation

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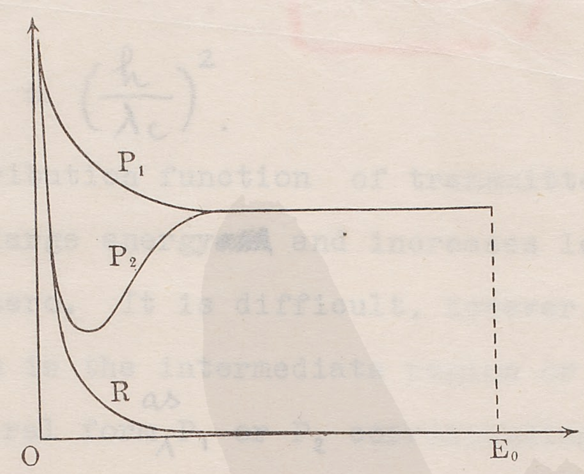
この辺に Fig. 3 を入れること

$$\frac{E_c}{E_0} = \left(\frac{h}{\lambda_c}\right)^2 . \qquad (24)$$



Thus the energy distribution function of transmitted neutrons is nearly constant for large energy and increases logarithmically as the energy tends to zero. It is difficult, however, to decide whether P(E) has a minimum in the intermediate region or not, the distribution curve taking the general form as P1 or P2 accordingly. (Fig. 3).

Similarly, the fraction of those which are reflected becomes

$$R(E)\,dE \cong \frac{dE}{E_0}\frac{h}{\lambda_0'}\frac{h}{\lambda_0''}\left[2+\left(\frac{E_0}{E}\right)^{\frac{1}{2}}+\log\left\{\frac{\lambda_0}{h}\left(1-\frac{E}{E_0}\right)^{\frac{1}{4}}\right\}\right] \qquad (25)$$

for $\frac{E_0}{4} < E < E_0$ and

$$R(E)\,dE \cong \frac{dE}{E_0}\frac{h}{2\lambda_0'}\log\left(\frac{E_c}{E}\right) \qquad (26)$$

for $E \ll E_c$. Hence, the number of reflected neutrons is approximately the same as those which are transmitted for small energy and decreases steadily with the increasing energy, the distribution curve having the general form as R in Fig. 3.

These results can not immediately be compared with the experiment on the activation of certain elements by certain groups of slow neutrons, which ~~was~~ were produced by thin layers of paraffin, since the actual energy distribution of neutrons which hit the detector will be affected considerably by sizes and arrangements of the source,

(13)

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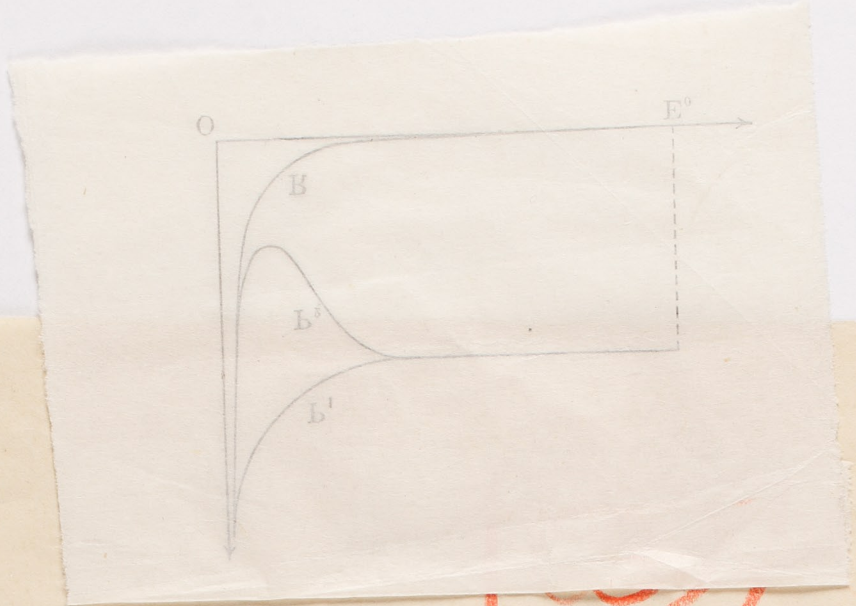

$$\frac{E_c}{E_0} = \left(\frac{h}{\lambda_c}\right)^2. \qquad (24)$$

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for $\frac{E_0}{4} < E < E_0$ and

$$R(E)\,dE \cong \frac{dE}{E_0}\,\frac{h}{2\lambda_0'}\,\log\left(\frac{E_c}{E}\right) \qquad (26)$$

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(14)

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the scatterer and the detector. In the following sections, the effect of oblique incidence will be considered and will be applied to the case , when the neutrons are emitted from a point source.

§6. Effect of Oblique Incidence of Primary Neutrons.

The probability, that a ~~primary~~ neutron, initially making an angle θ with the normal of the plate, passes through with energy between E and E+dE after single ~~scattering~~ collision with a proton, is given approximately by

$$P(E,\theta)\,dE \cong \frac{dE}{E_0}\,\frac{h}{\lambda_0'\cos\theta} \qquad (27)$$

for $0 \leq \theta < \arccos\sqrt{1-\frac{E}{E_0}}$ ~~and~~,

$$P(E,\theta)\,dE \cong \frac{dE}{E_0}\,\frac{h}{\lambda_0'\cos\theta}\,\frac{\arccos\left(\sqrt{\frac{E_0}{E}-1}\cdot\cot\theta\right)}{\pi} \qquad (28)$$

for $\arccos\sqrt{1-\frac{E}{E_0}} < \theta < \frac{\pi}{2}-\frac{h}{\lambda_0}$ and

$$P(E,\theta)\,dE \cong \frac{dE}{2E_0} \qquad (29)$$

for $\frac{\pi}{2}-\frac{h}{\lambda_0} < \theta < \frac{\pi}{2}$.

Thus, if N_0 neutrons of energy E_0 are emitted per unit time from a point source and consequently the fraction

$$N_0\,\frac{\sin\theta\,d\theta}{2}$$

hits the plate with the inclination between θ and $\theta+d\theta$, the number per unit time of those which pass through the plate with energy (E, E+dE) is given approximately by

$$N(E)dE \cong \frac{N_0}{2}\frac{dE}{E_0}\frac{h}{\lambda_0'}\log\frac{\lambda_0}{h} \qquad (30)$$

for $E_0 > E > E_0\{1-(\frac{h}{\lambda_0})^2\}$ and

$$N(E)dE \cong \frac{N_0}{2}\frac{dE}{E_0}\frac{h}{2\lambda_0'}\log\frac{\lambda_0}{h} \qquad (31)$$

for $E \ll E_0$.

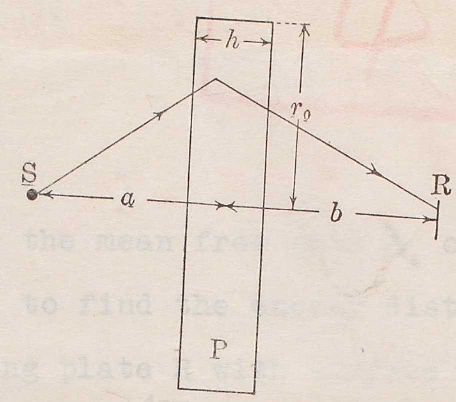
In this case, it can be easily seen that the effect of double scattering is small even for slow neutrons, so that the distribution is nearly constant for energy small compared with E_0, in contrast to the previous case of normal incidence.

Similarly, the number of reflected neutrons ~~can be calcuted~~ is about the same as that of transmitted neutrons for slow neutrons and ~~and~~ is smaller for fast neutrons.

§7. The Case of a Point Source and a Small Detector.

Now we want to consider a more concrete problem. As shown in Fig. 4, a point source S emitting N_0 neutrons of energy E_0 per unit time is placed at a distance a on the axis from the centre of a circular plate P, containing hydrogen, with radius r_0 and thickness h, the latter the mean free path λ_0'.

In this and the following sections λ_0' means the ~~partial~~ ~~mean~~ free path ~~for proton~~ for collision with proton reciprocal cross section of



being small compared with the mean free path λ of the primary neutrons. It is required to find the energy distribution of neutrons, which hit a small detecting plate R with sensitive area s placed parallel to P at a distance b from the centre of P on the side opposite to S. We consider the case, where the relation $ab < r_0^2$ is satisfied, so that the neutrons slowed down to small velocity by single scattering can ~~hit~~ reach R. In this case, the number of those which hit R per unit time with energy between E and E+dE after having scattered once in P, is given approximately by

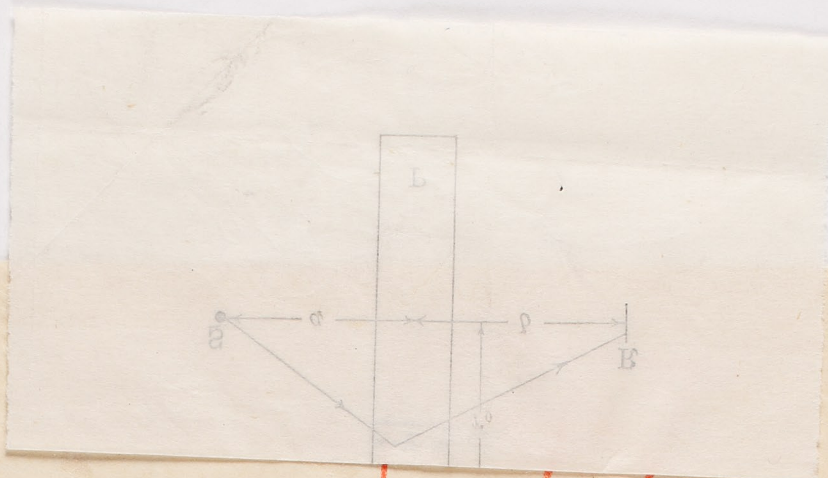
$$N(E)dE \cong N_0 \frac{2\pi s h}{\lambda_0'} \frac{dE}{E_0} \frac{b(a+b)^2 r^3}{\left(1-\frac{E}{E_0}\right)^{\frac{3}{2}}\left\{4ab+(a-b)^2\frac{E}{E_0}\right\}^{\frac{1}{2}}(a^2+r^2)^{\frac{3}{2}}(b^2+r^2)^2}, \qquad (32)$$

provided that the mean free path for energy E is large in comparison with h, where $r = \dfrac{\sqrt{4ab+(a-b)^2\frac{E}{E_0}} - (a+b)\sqrt{\frac{E}{E_0}}}{2\sqrt{1-\frac{E}{E_0}}}$.

If $E \ll E_0$, this is reduced to

$$N(E)dE \cong N_0 \frac{\pi s h}{\lambda_0'} \frac{1}{a^{\frac{1}{2}}(a+b)^{\frac{3}{2}}} \frac{dE}{E_0}, \qquad (33)$$

so that the number per unit energy range of slow neutrons, which hit the detector, is nearly independent of the enrgy. Thus, the rate of increase with the thickness h of the activity of the detector due to a certain group of slow neutrons ~~with the thickness h~~ will be given approximately by the product of the breadth ΔE of ~~the~~ this group and the energy of ~~the fraction of neutron~~



この処に Fig. 4. を入れる

being small compared with the mean free path λ_0 of the primary neutrons. It is required to find the energy distribution of neutrons, which hit a small detecting plate R with surface area s placed parallel to P at a distance b from the centre of P on the side opposite to S. We consider the case, when the relation $ab < r_0^2$ is satisfied, so that the neutrons slowed down to small velocity by single scattering can ~~hit~~ reach R. In this case, the number of those which hit R per unit time with energy between E and E+dE after having scattered once in P, is given approximately by

$$N(E)dE \cong N_0 \frac{2\pi s h}{\lambda_0} \frac{dE}{E_0} \frac{b(a+b)^2 r^3}{(1-\frac{E}{E_0})^{\frac{3}{2}} \{4ab+(a-b)^2\frac{E}{E_0}\}^{\frac{1}{2}} (a^2+r^2)^{\frac{3}{2}} (b^2+r^2)^2}, \qquad (32)$$

provided that the mean free path for energy E is large in comparison with h, where $r = \dfrac{\sqrt{4ab+(a-b)^2\frac{E}{E_0}} - (a+b)\sqrt{\frac{E}{E_0}}}{2\sqrt{1-\frac{E}{E_0}}}$.

If $E \ll E_0$, this is reduced to

$$N(E)dE \cong N_0 \frac{\pi s h}{\lambda_0} \frac{1}{a^{\frac{1}{2}}(a+b)^{\frac{3}{2}}} \frac{dE}{E_0}, \qquad (33)$$

so that the number per unit energy range of slow neutrons, which hit the detector, is nearly independent of the enrgy. Thus, the rate of increase with the thickness h of the activity of the detector due to a certain group of slow neutrons ~~with the thickness h~~ will be given approximately by the product of the breadth ΔE of ~~the~~ this group and the energy of ~~the fraction of neutron~~

(17)

17

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~~assuming~~ to (assuming that of C group to be the order of kT) Thus, we can estimate the breadth of energy of the latter group, if we know ~~the~~ the percentages ~~of abs~~ of neutrons of two groups ~~abs~~ absorbed ~~in~~ by detector. are known

the probability for neutrons of this group to be absorbed by the detector, ~~provided that~~ if the thickness h is extremely small.

It will be interesting to compare these conclusions with the results of the experiment of Nishikawa, Nakagawa and Sumoto[1] on the slowing down of neutrons from a Rn-Be source by thin layers of paraffin. Their results seems to show that the rate of increase of activity of Ag detector with h is nearly quadratic for neutrons of C group with thermal energy, whereas it is undoubtedly linear for those of A group with larger energy. Thus, the breadth of energy of A group should be large compared with kT, if the detector is so thick ~~we assume~~ that the probabilities for the neutrons of these two groups to be absorbed by the detector do not differ greatly from each other.

In conclusion, the author desires to acknowledge the assistance rendered by Mr. S. Sakata to the completion of this paper.

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(Received July 20, 1936)

1) Proc. Imp. Acad. Tokyo, **12**, 128, 1936. I am much obliged to Mr. S. Nakagawa for his kindness of communicating the details of the experiment to me.

~~The vali approximate~~

~~17 If (33) is correct the~~ According to (33),

It can also be inferred from (33) that the activity changes as $a^{-\frac{1}{2}}$, if ~~we move~~ the thin plate ~~&~~ P is moved ~~with fixed~~ between $\underbrace{S}_{\text{fixed}}$ and R. ~~Thus~~

~~the approximate validity of (33) can be check~~ by

~~Further, The approximate~~ validity of the equation (33) can ~~all be~~ check by

This can be verified by experiment. ~~directly~~

verified by experiment

大阪帝國大學理學部　**試驗答案用紙**　(　　)

星石 健三

Ultrarote Absorptionsspektra photochemisch sensibilisierter Alkalihalogenidkristalle
von K. Korth

Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen　Band 1.　221　1935

山村 律.

Equivalent Two Body Method

(E. Feenberg, Phys. Rev. 47, June 1,
48, Dec 1.)

岡小天

分光解離に就ての解離電圧結果に就て.

湯川秀樹

重粒子と輕粒子間に作用する一般的力場に就て.