② Rec. $\cos(\phi-\phi')$ は 0 に近い値をとりうる。その時は

$$\cancel{\tan\theta = \frac{\sin\theta'\cos(\phi-\phi') \pm \sqrt{\sin^2\theta'\cos^2(\phi-\phi') - \frac{4v'}{v_0}\left(\frac{v'}{v_0}-\cos\theta'\right)}}{2\frac{v'}{v_0}}}$$

$$\cancel{= \frac{\sin\theta'\cos(\phi-\phi') - \sin^2\theta'\cos^2(\phi-\phi')}{2\frac{v'}{v_0}}}$$

しかし、$\theta$ を $\frac{\pi}{2}$ に近いとすると、 $\cos\theta \cong 1 - \cancel{\frac{\theta^2}{2}}$ $\quad \frac{\pi}{2} - \theta = \delta$

$$\sin\theta \cong 1 - \frac{1}{2}\left(\frac{\pi}{2}-\theta\right)^2 = 1 - \frac{\delta^2}{2}$$

$$\therefore\ \delta\left(\delta\cdot\cos\theta' + \left(1-\frac{\delta^2}{2}\right)\sin\theta'\cos(\phi-\phi')\right) = \frac{v'}{v_0}$$

∴ 左辺で大きいのは

$$\delta^2\cos\theta' \quad 又は \quad \delta\sin\theta'\cos(\phi-\phi')$$

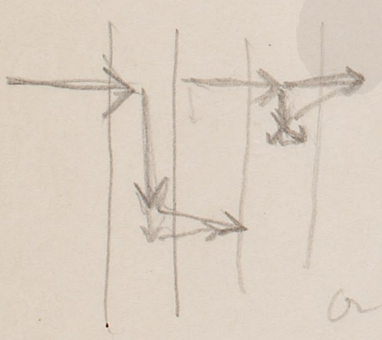
である。後者が大きければ $\delta\sin\theta'\cos(\phi-\phi') = \frac{v'}{v_0} + k\delta^2$

$$\cancel{\delta\cos\theta' - \frac{\delta^2}{2}\sin\theta'\cos(\phi-\phi') + k\delta^2 = 0;}$$

$$\cancel{\cos\theta' - \frac{\delta}{2}\sin\theta'\cos(\phi-\phi')} - k$$

$$\cancel{\cos(\phi-\phi') = \frac{\delta}{2}} \qquad \delta(\delta\cos\theta' \ldots \sin\theta'\cos(\phi-\phi')$$

$$\cancel{\cos\theta' \cong \frac{v'}{2v_0}} \qquad \frac{k}{2\cos\theta'} = -\frac{v'}{2v_0} + k = 0$$



$\cos\theta' \cong 0$ or $(\phi-\phi') \cong \frac{\pi}{2}$,

$$\delta\sin\theta'\cos(\phi-\phi') + \delta^2\cos\theta' - \frac{\delta^3}{2}\sin\theta'\cos(\phi-\phi') - \frac{\delta^4}{3}\cos\theta' = \frac{v'}{v_0}$$

$$\frac{v'}{v_0}\ \delta = \frac{-\sin\theta'\cos(\phi-\phi') \pm \sqrt{\sin^2\theta'\cos^2(\phi-\phi') + 4\left(\frac{v'}{v_0}\cos\theta'\right)^2}}{2\cos\theta'}$$

$$\frac{\delta}{3}\cos\theta' = -\frac{\sin\theta'\cos(\phi-\phi')}{2}$$

$\theta'+d\theta' \cdot \phi'+d\phi' \; v'+dv'$

二間の衝突によって 結局 $(\theta', \phi', v')$ になる prob は

$$\frac{v'}{v_0} = \cos\theta\cos\gamma$$

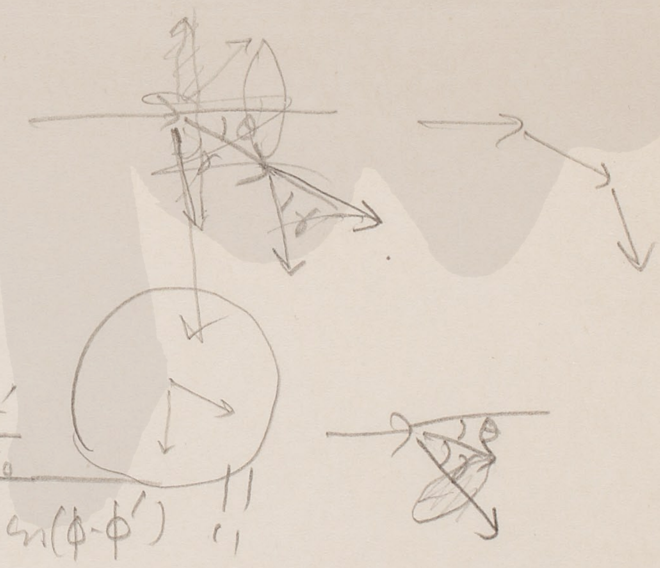
なる関係からして $\cos^2\theta = \dfrac{1}{1+\tan^2\theta}$ $\quad \sin\theta\cos\theta = \dfrac{\tan\theta}{1+\tan^2\theta}$,

$$\frac{v'}{v_0} = \frac{\cos\theta'}{1+\tan^2\theta} + \frac{\sin\theta'\cos(\phi-\phi')\tan\theta}{(1+\tan^2\theta)}$$

$(\phi', v')$ をきめると $\theta'$ の値は二つきまる. ~~…~~ ops.

$$\frac{v'}{v_0}\tan^2\theta - \sin\theta'\cos(\phi-\phi')\tan\theta + \left(\frac{v'}{v_0} - \cos\theta'\right) = 0$$

$$\tan\theta = \frac{\sin\theta'\cos(\phi-\phi') \pm \sqrt{\sin^2\theta'\cos^2(\phi-\phi') - 4\frac{v'}{v_0}\left(\frac{v'}{v_0}-\cos\theta'\right)}}{2\frac{v'}{v_0}}$$



$$\theta \cong \frac{\pi}{2}$$

$$\tan\theta \cong \frac{\sin\theta'\cos(\phi-\phi')}{\frac{v'}{v_0}}$$

$\cos(\phi-\phi') \geq 0$
$|\phi-\phi'| \leq \frac{\pi}{2}$
$$\cos\theta \cong \frac{\frac{v'}{v_0}}{\sin\theta'\cos(\phi-\phi')} \;!!$$

$$\sin\theta \cong 1 - \frac{1}{2}\frac{\frac{v'^2}{v_0^2}}{\sin^2\theta'\cos^2(\phi-\phi')} \;!!$$

~~unit time に横切る~~ $(v'$ から neutron の数は $(v'+dv')(\theta', \theta'+d\theta')(\phi', \phi'+d\phi')$ (二回の衝突によるもの)

$v'\cos\theta' \; d\theta =$ $\qquad d\theta^* = \dfrac{\frac{dv'}{v_0}}{\sin\theta'\cos(\phi-\phi')}$

$$= \frac{v'^2\,dv'\cos\theta'}{v_0^2\sin\theta'\cos(\phi-\phi')}.$$

$\sin\theta\cos\theta\,d\phi\cos\gamma\,d\theta' d\phi' / \pi^2$

なるから $\dfrac{\sin\theta\cos\theta\,d\theta\,d\phi\cos\gamma\sin\theta'\,d\theta' d\phi'}{\pi^2}$ $\quad \dfrac{h}{2\lambda_0\lambda(v)\cos\theta} = \dfrac{v'^2 dv'}{v_0^2\sin\theta\cos(\phi-\phi')}$

$v'\cos\theta$.

~~単位~~ unit time に $(\theta', \theta'+d\theta')$ $(\phi', \phi'+d\phi')$ なる angle で screen の ほうす unit area を横切る $(v', v'+dv')$ の範囲の velocity の neutron の数は（二回の衝突によるもの）

$$\frac{v'^2 dv' \cos\theta'}{\pi^2 v_0^2 \cancel{\sin\theta'} \cos(\phi-\phi')}\left(1-\frac{1}{2}\frac{v'^2}{v_0^2}\frac{1}{\sin^2\theta' \cos^2(\phi-\phi')}\right)\frac{v'}{v_0}\frac{1}{\sin\theta' \cos(\phi-\phi')}$$

$$\times \cos\gamma \cdot d\phi \cdot d\theta' d\phi'$$

ここで $\phi-\phi'$ の範囲 ~~(θ, …) を積分する~~ は $\left(-\arccos\left(\frac{v_0\sin\theta'}{v'}\right), +\arccos(\ )\right)$ の間で積分する。

$$\int \frac{d\phi}{\cos^2(\phi-\phi')} = \tan\{\arccos(\ )\} - \tan\{-\arccos(\ )\}$$

$$= 2\tan\left\{\arccos\left(\frac{v_0\sin\theta'}{v'}\right)\right\} = \frac{2\sqrt{1-\left(\frac{v_0\sin\theta'}{v'}\right)^2}}{\frac{v_0}{v'}\sin\theta'}$$

$$\int_{-x}^{x} \frac{d(\phi-\phi')}{\cos(\phi-\phi')} = \log\tan\left(\frac{x}{2}+\frac{\pi}{4}\right) - \log\left\{\frac{\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)}\right\}$$

$$= 2\log\left\{\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right\}$$

$$\tan(\phi-\phi') = 2\,\frac{1+\sqrt{1-x^2}}{}$$

$$\frac{d}{dx}\{\tan x\} = \frac{d}{dx}\frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin x \cdot \sin x}{\cos^2 x} =$$

$$= \frac{2\left\{1+\sqrt{1-\left(\frac{v_0}{v'}\sin\theta\right)^2}\right\}}{\frac{v_0}{v'}\sin\theta}$$

$$\tan \arccos x = y = \frac{\sqrt{1-x^2}}{x}$$

$$\arccos x = \arctan y$$

$$x = \cos(\arctan y)$$

$$\tan\left\{\frac{\arccos x}{2}+\frac{\pi}{4}\right\} = \frac{\tan\frac{\arccos x}{2}+1}{1-\tan\frac{\arccos x}{2}}$$

$$\tan\frac{\alpha}{2} = \frac{1-\cos\alpha}{\sin\alpha} = \frac{1-x}{\sqrt{1-x^2}} = \sqrt{\frac{1-x}{1+x}}$$

$$= \frac{\sqrt{\frac{1-x}{1+x}}+1}{1-\sqrt{\frac{1-x}{1+x}}} = \frac{\frac{1-x}{1+x}+1+2\sqrt{\frac{1-x}{1+x}}}{1-\frac{1-x}{1+x}}$$

$$= \frac{2+2\sqrt{1-x^2}}{2x}$$

$\frac{\pi}{4}+\frac{x}{2}$, $\frac{\pi}{2}-\frac{\pi}{4}$, $\frac{\pi}{4}-\frac{x}{2}$, $\frac{\pi}{2}$

$$\frac{v'^2\cos\theta'}{\pi^2 v_0} \cdot \frac{v'}{v_0} \cdot \frac{d\theta' d\phi'}{\sin\theta'}$$

$$\times\left\{ \cancel{\cos\theta\cos\theta' \frac{2\sqrt{1-\left(\frac{v_0}{v'}\sin\theta'\right)^2}}{\frac{v_0^2}{v'}\sin\theta'}} + \cancel{\sin\theta\sin\theta'} \right.$$

$$\times \sin\theta' \frac{2\left\{1+\sqrt{1-\left(\frac{v_0}{v'}\sin\theta'\right)^2}\right\}}{\frac{v_0}{v'}\sin\theta'}$$

$\boxed{v_0\sin\theta' < v'}$

$$= \frac{v'^4 \cos\theta' d\theta' d\phi'}{\pi^2 v_0^3} \cancel{2\{1+\sqrt{1-\frac{v_0}{v'}}} v' - v_0\sin\theta$$

$$\times 2 \cdot \frac{v'}{v_0 \sin\theta'}\left\{1+\sqrt{1-\left(\frac{v_0\sin\theta'}{v'}\right)^2}\right.$$

更に $\phi'$について $(0, 2\pi)$ の間で積分し、又
$\theta'$について、$0 \le \sin\theta' \le \frac{v'}{v_0}$ or $0 \le \theta' \le \arcsin\frac{v'}{v_0}$
の間で積分すると、

$$\cancel{\phantom{x}} \cdot \frac{4}{\pi}\frac{v'^5}{v_0^4} \cdot \int_0^{\frac{v'}{v_0}} \frac{dx}{x}\left(1-\sqrt{1-\left(\frac{v_0}{v'}x\right)^2}\right)$$

$$\int_0^1 \frac{dx}{x}\left(1-\sqrt{1-x^2}\right)$$