E22 051 P06

17(+12)

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE
NO.

Elementary Problems on Scattering ~~and~~ of Neutrons

Elementary Calculations of single and double scatterings of neutrons by thin ~~layer of hydrogenated substance~~ screen containing hydrogen ~~were~~ made and approximate expressions of the energy distributions of transmitted and reflected neutrons are obtained. Preliminary discussions of the equilibrium ~~distribution for thick plate were~~ also made.

Elementary Examples ~~Theory of Energy Distribution~~

~~Note on~~ ~~Slowing~~ Down of Neutrons Scattered by Thin ~~Layer~~ Plate

Calculations on ~~Scattering~~

Elementary Problems ~~Examples of~~ ~~Calculations on~~ Neutron Scattering Problem the Scattering of Neutrons by Thin Plate.

By Hideki Yukawa

(Read July 4, 1936)

abstract

§1. Introduction

The change of the energy distribution of neutrons due to the presence of ~~effected by the~~ ~~after having~~ passed through a substance containing hydrogen ~~layer of hydrogenated substances~~, in general case, ~~It is not an easy problem to find~~ It is an important but not an easy task. ~~Hitherto this problem was approa~~ attacked ~~to~~ from two sides. ~~One~~ ~~The first method is to~~ in general case.

Of course ~~On the~~ the well known ~~a~~ probability distribution of ~~neutrons~~ the neutron energy after having scattered by protons ~~a fixed time~~ ~~collided several time~~ a given number of ~~fixed~~ number of collisions with protons is ~~can be was found~~ ~~is well known on the one hand,~~ ~~and the homogeneous~~ ~~can~~ will ~~not~~ ~~become~~ ~~have~~ have practical meaning only if the probability distribution of occurrence of for each ~~the~~ number of times of scattering ~~collisions~~ itself is determined theoretically under ~~for~~ ~~each~~ special conditions ~~cases~~. The ~~equilibrium~~ homogeneous and isotropic equilibrium distribution found by Fermi[1] can not easily ~~be extended~~ to inhomogeneous and anisotropic distribution.

On the other hand, We are met with mathematical

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

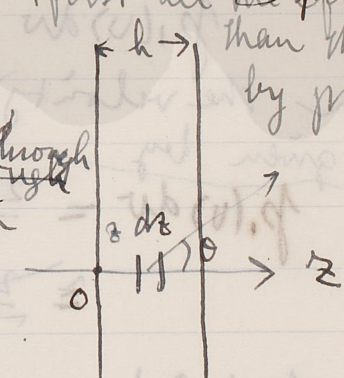
DATE
NO. 2

for the time being

complications, if we want to extend the homogeneous
and isotropic equilibrium distribution found by
Fermi to more general case. to be of some use useless at present
Hence, it ~~seems~~ may ~~not~~ altogether trivial
~~In this note, we~~ & the author want to ~~solve a problem~~ confine our attention to deal with ~~the~~ the
elementary ~~elementary~~ problem ~~by~~ of ~~scatt slowing down~~ scattering passage
of neutrons ~~through~~ by a plate ~~layer~~ so thin that only single
~~and~~ ~~at most~~ double scatterings ~~occur~~ with appreciable
frequency, which can be ~~solved~~ calculations. Other problems which can be solved ~~by~~ in the
similar way, are also ~~treate~~ considering will Probabilities of scattering*
§ 2. ~~Change~~ single and ~~double~~

We ~~want~~ neglect to consider want to neglect at ~~first~~ first all ~~the~~ ~~processes~~, other than the scattering by protons.

~~consi~~ We consider a parallel
plate of ~~thickness h~~ hydrogenated
substance of thickness h, ~~upon~~ ~~through~~ through upon
which a neutron beam of ~~velocity v~~ ~~fixed~~ definite
~~energy~~ ~~E_0~~ velocity v_0
passes perpendicularly from



~~one side~~ left to ~~another~~ right. ~~The~~ Each ~~neutron of the beam has~~
first to be one
~~The~~ probability of ~~each~~ neutron of the beam of being ~~scattered~~ by ~~an~~ angle deflected
to ~~by~~ an angle between (θ and $\theta + d\theta$) at ~~the~~ a depth the distance ~~z~~ between
(z, $z + dz$) from the left surface is given, as well known, by
and

$$e^{-\frac{z}{\lambda_0}} \cdot \frac{dz}{\lambda_0} \cdot 2\sin\theta\cos\theta\, d\theta \qquad (1)$$
,

where ~~λ_0 is~~ the direction of the incident beam is taken as
If ~~we take~~ the direction of the incident be~~am~~ ~~as z-direction~~,
~~and~~ is taken as that of ~~the~~ that z-axis,

~~There~~ No Of course, no neutrons are reflected back by single scattering, as $\theta \leq \frac{\pi}{2}$

(mean free path for ~~critical~~ velocity v_c

$$\frac{-\frac{h}{\lambda_0} + \frac{h v_0}{\lambda v} + \frac{1}{2}\left(\frac{h}{\lambda_0}\right)^2 - \frac{1}{2}\left(\frac{h v_0}{\lambda v}\right)^2}{\frac{\lambda v_0}{\lambda v} - \frac{1}{\lambda_0}} = h - \frac{h^2}{2}\left(\frac{1}{\lambda_0} + \frac{v_0}{\lambda v}\right)$$

On the contrary, it reduces to

$$p_1(v)\,dv = \frac{2v\,dv}{v_0} \ldots$$

for the velocity small compared with the critical velocity v_c, where λ_c is the given by $\frac{\lambda_c v_c}{v_0} = h$,

$$p_1(v)\,dv = \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0}\left\{1 - \frac{h}{2\lambda_0}\left(\frac{1}{\lambda_0} + \frac{\lambda_0 v_0}{\lambda v}\right)\right\} + \ldots \quad (4)$$

$$\cancel{\cong \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0}}$$

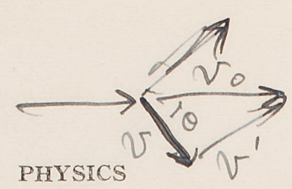
$\lambda v > h v_0$

$$p_1(v)\,dv = \frac{2v\,dv}{v_0^2}\frac{\lambda v_0}{h v_0}\left\{1 - \frac{h}{\lambda_0}\right\} \quad (4)'$$

$$= \frac{2v^2 dv}{v_0} = \frac{dv}{v_0}\cdot\frac{2\lambda}{\lambda_0}\left\{1 - \frac{h}{\lambda_0}\right\} + \ldots \frac{h v_0}{\lambda v} \gg \frac{\lambda_c v_c}{v_0}$$

In this case, ~~the slow~~ ~~the scatt~~ the neutron after ~~slowed down~~ deflected by ~~large~~ an angle nearly equal to $\frac{\pi}{2}$, ~~nearly head on collision deflection~~ has very small energy, and ~~will make a second collision~~ is ~~be~~ almost certainly deflected once more before it comes out of the plate.

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.



has the velocity $v = v_0 \cos\theta$ and (2)

DATE
NO. 3

of

z-axis and ~~θ~~ ~~means~~ the angle ~~between the deflected neutron~~

λ_0 is the mean ~~mean~~ free path of the neutron with velocity v_0. ~~Hence, the proba~~ ~~The neutron~~ The probability of the neutron deflected in this way ~~will~~ passes through the plate ~~on~~ without ~~so~~ further scattering only with the probability

$$\cancel{\ } e^{-\frac{(h-z)}{\lambda\cos\theta}}$$

where λ is the mean free path of the neutron with ~~the~~ ~~velocity~~ velocity v.

~~So that~~ Hence total probability ~~number~~ of transmitted neutrons, with velocity between ~~v and $v+dv$ pass~~ ~~which~~ passed of passing through the plate ~~with the angle~~ in the direction ~~between~~ $(\theta, \theta+d\theta)$ after single scattering of becomes

$$\int_0^h e^{-\frac{z}{\lambda_0}-\frac{(h-z)}{\lambda_0\cos\theta}} \frac{dz}{\lambda_0} \cdot 2\sin\theta\cos\theta\, d\theta$$

$$= \frac{2\sin\theta\cos\theta\, d\theta}{\cancel{\lambda_0 v_0}} \frac{e^{-\frac{h}{\lambda_0}} - e^{-\frac{h}{\lambda\cos\theta}}}{\frac{h}{\lambda\cos\theta} - \frac{h}{\lambda_0}} \quad (3)$$

~~For~~ so that ~~the~~ total number of transmitted neutrons with the velocity between v and $v+dv$ is the fraction,

$$p_1(v)dv = \frac{2v\,dv}{\lambda_0 v_0^2} \frac{e^{-\frac{h}{\lambda_0}} - e^{-\frac{h v_0}{\lambda_0 v}}}{\frac{\cancel{h} v_0}{\lambda_0 v} - \frac{\cancel{h}1}{\lambda_0}} \quad (3)'$$

† If† the thickness h is so small ~~comp~~ that ~~can be neglected~~ compared with the mean free path, λ_0, ~~$\frac{\Delta v}{v_0}$ except for very small value of $\frac{v}{v_0}$, (2) can be~~

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE
NO.

(3)′ can be reduced to

$$p_1(v)\,dv = \frac{2v\,dv}{v_0^2}\,\frac{h}{\lambda_0} \quad (4)$$

for ~~sufficiently large~~ the value of v, for which ~~$\frac{\Delta v}{v_0} \gg h v_0$~~ so large that ~~the~~ the relation $\frac{h v_0}{v} \gg \lambda_0$ is satisfied,‡ is large compared with h††

of the number of ~~the~~ incident neutrons.

Next, the neutron, ~~scatt~~ which was scattered at $(z, z+dz)$ ~~into~~ ~~of~~ ~~by~~ an angle $(\theta, \theta+d\theta)$ with the probability given by (1), will ~~probability~~ of double scattering

§3.

Next, we consider the effect of second scattering, which ~~is~~ becomes all the more important as the deflection angle of the first scattering ~~draws near~~ approaches to $\frac{\pi}{2}$. The neutron, which was ~~scattered~~ scattered at $(z, z+dz)$ by an angle $(\theta, \theta+d\theta)$ with the probability given by (1), ~~will~~ may be ~~scattered~~ deflected ~~for~~ once more at $(z', z'+dz')$ by an angle $(\gamma, \gamma+d\gamma)$ with the probability

$$\cancel{e^{-\frac{z}{\lambda_0}}}\; e^{-\frac{(z'-z)}{\lambda\cos\theta}}\,\frac{dz'}{\lambda\cos\theta}\cdot 2\sin\gamma\cos\gamma\,d\gamma. \quad (5)$$

The neutron after such double scatterings has the velocity.

$$v' = v\cos\gamma = v_0\cos\theta\cos\gamma, \quad (6)$$

~~and Its direction is inclined the angle~~ of ~~it~~ between it and

†† ~~which means~~ This is evidently the condition that ~~almost~~ the scattered neutron passes through the plate without further scattering almost certainly.

N

$$\cos\theta\cos\gamma = \cos^2\theta'\cos\theta' + \frac{\sin\theta}{\cos\theta}\sin\theta'\cos(\phi_1-\phi_2)$$

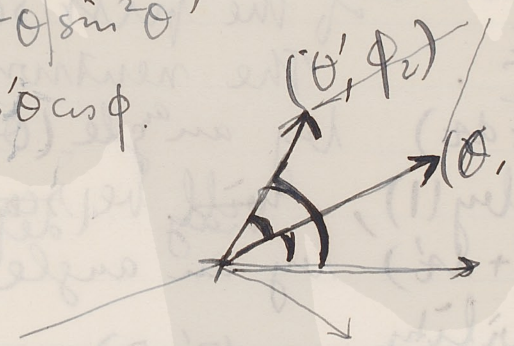
$$\cos\phi\sin\theta\sin\gamma = \sqrt{1-\{\cos^2\theta\cos^2\theta' + \sin^2\theta\sin^2\theta'\cos^2\phi}$$
$$\cos\phi\sin\theta \qquad -2\sin\theta\cos\theta\sin'\theta\cos'\theta\cos\phi$$

$$1 = \cos^2\theta\cos\theta' + \cancel{\cos\theta\sin\theta\sin\theta'\cos\phi}$$
$$\cancel{+\cos\phi\sin\theta} + \cos\phi\sin\theta\,(\cos\theta\sin\theta'$$

$x \quad \sin\theta\cos\phi_1$
$y \quad \sin\theta\sin\phi_1$
$z \quad \cos\theta$

$$+\sqrt{\sin^2\theta + \sin^2\theta\cos\sin^2\theta' + \sin^2\theta\sin^2\theta'}$$
$$-2\sin\theta\cos\theta\sin'\theta\cos'\theta\cos\phi$$

$$\sin\gamma = \sqrt{(\sin\theta\sin\phi_1\quad)^2 + (\quad)^2 + (\quad)^2}$$



$$1-\sin^2\theta\sin^2\phi - 1 + \cos^2\gamma\sin^2\phi = \sin^2\phi\,(\cos^2\gamma-\sin^2\theta)$$

$$(\cos\gamma - \cos\theta\cos\theta')^2 = \sin^2\theta\,(1-\cos^2\theta')\cos^2\phi$$

$$\cos^2\gamma - 2\cos\gamma\cos\theta\cos\theta' + (\cos^2\theta+\sin^2\theta\cos^2\phi)\cos^2\theta' - \sin^2\theta = 0$$

$$\cos\theta' = \frac{\cos\gamma\cos\theta \pm \sqrt{\cos^2\gamma\cos^2\theta - (\cos^2\theta+\sin^2\theta\cos^2\phi)(\cos^2\gamma-\sin^2\theta)}}{\cos^2\theta + \sin^2\theta\cos^2\phi}$$

$$= \frac{\cos\gamma\cos\theta \pm \sin\theta\sqrt{\cos^2\theta - \cos^2\gamma\cos^2\phi + \sin^2\theta\cos^2\phi}}{\cos^2\theta + \sin^2\theta\cos^2\phi}$$

$$\cos^2\theta + \sin^2\theta\cos^2\phi = 1 - \sin^2\theta\sin^2\phi$$

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE
NO. 5

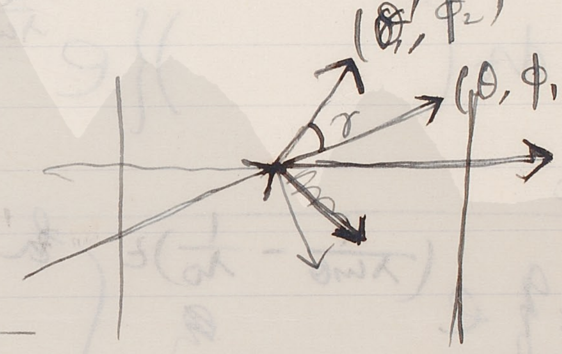
$\cos\delta$

$$= \frac{v'}{v_0} + \sqrt{1-\frac{v'^2}{v_0^2}}\sqrt{1-\left(\frac{v'}{v}\right)^2}\cos\phi \quad (7')$$

but, while θ' ~~the~~ inclination ~~with respect~~ to z-axis, satisfies ~~and the~~ ~~the relation $\cos\theta' =$~~

$$\cos\theta' = \cos\theta\cos\delta + \sin\theta\sin\delta\cos\phi,$$

~~where ϕ is the difference of~~ ~~is determined by~~ θ' depends not only on θ, δ, ~~only~~ ~~but depends only~~ also on ~~the difference ϕ of the azimuths of the first and second scatterings~~ through the relation



$$\cos\theta' = \cos\theta\cos\delta + \sin\theta\sin\delta\cos\phi, \quad (9)'$$

or by (5) and (7)

where ϕ is the azymuth of the direction of ~~the velocity~~ ~~velocity~~ vector after second scattering ~~with respect to the new~~ in new polar coordinate, in which ~~the velocity~~ the direction of velocity after first scattering is taken as z-axis and the former z-axis ~~has~~ ~~lies in the direction~~ ~~has~~ ~~zero~~ the azymuth of the azymuth is zero. ~~taken as~~

~~Thus~~ If θ' is smaller than $\frac{\pi}{2}$, i.e. $\frac{v'}{v_0} + \sqrt{\ }\sqrt{\ }\cos\phi > 0$, the neutron thus formed will passes through the plate ~~with the probability~~ without further scattering

$$e^{-\frac{(h-z')}{\lambda'\cos\theta'}} \quad (8)$$

$$\int_0^h dz\, e^{-\left(\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}\right)z}\, e^{-\frac{h}{\lambda'\cos\theta'}} \int_z^h e^{-\left(\frac{1}{\lambda'\cos\theta'}-\frac{1}{\lambda\cos\theta}\right)z'} dz'$$

$$= \int_0^h dz \left\{ e^{\left(\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}\right)z} \cdot e^{-\frac{h}{\lambda'\cos\theta'}}\, \frac{e^{\frac{h}{\lambda'\cos\theta'}-\frac{h}{\lambda\cos\theta}} - e^{\frac{z}{\lambda'\cos\theta'}-\frac{z}{\lambda\cos\theta}}}{\left(\frac{1}{\lambda'\cos\theta'}-\frac{1}{\lambda\cos\theta}\right)} \right\}$$

$$= \int_0^h dz \left(\quad \right) \left\{ e^{-\frac{h}{\lambda\cos\theta}} \cdot e^{\left(\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}\right)z} - e^{-\frac{h}{\lambda'\cos\theta'}}\, e^{\left(\frac{1}{\lambda'\cos\theta'}-\frac{1}{\lambda_0}\right)z} \right\}$$

$$\int_0^h dz\, e^{\left(\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}\right)z} \int_{\theta'}^{\pi'}$$

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

i.e. $\frac{v'}{v_0} + \sqrt{\ }$ $\cos\phi < 0$ DATE

NO. 6

~~whereas~~ while if θ' is larger than $\frac{\pi}{2}$, ~~the n~~ the neutron will be reflect back without further scattering with the probability

$$e^{-\frac{z'}{\lambda'\cos\theta'}}, \qquad (8')$$

where λ' is the mean free path ~~of the neutron with~~ for velocity v'.

~~For a given value of v', θ, γ If the right hand side of (7') is positive~~

In the former case the ~~total~~ probability ~~that~~ the neutron being transmitted ~~scatt.~~ after having scattered, ~~twice~~ ~~at~~ at arbitrary ~~points~~ depths z, z' ~~between 0 and h~~ (between 0 and h) and z' (z') (between ~~z~~ z and h) ~~in successively~~ first in the direction $(\theta, \theta+d\theta)$ (with azymuth ~~being arbitrary~~ arbitrary azymuth) and second in ~~$(\gamma\phi, \gamma+d\gamma)$~~ $(\gamma, \gamma+d\gamma)$ $(\phi, \phi+d\phi)$.

~~is given by~~

~~$e^{-\frac{z'-z}{\lambda\cos\theta}}$~~

becomes, by using (1), (4) ~~and~~ (8') and the fact that the probability of the second scattering is independent of ϕ,

$$\int_0^h \frac{dz}{\lambda_0}\left\{ e^{-\frac{z}{\lambda_0}} 2\sin\theta\cos\theta\, d\theta \int_z^h e^{-\frac{(z'-z)}{\lambda\cos\theta} - \frac{(h-z')}{\lambda'\cos\theta'}} \frac{dz'}{\lambda\cos\theta} \right\}$$

$$\times\, 2\sin\theta\cos\theta\, d\theta\ \cancel{X}\ 2\sin\gamma\cos\gamma\, d\gamma\ \frac{d\phi}{2\pi}\Big\},$$

~~since the probability of the second scattering is independent of ϕ.~~

2

In the similar, we can ~~easy obtain~~ the probability, that a neutron ~~is~~ after ~~deflected~~ scattered ~~at first~~ by an angles $(\theta, \theta + d\theta)$ ~~is~~ and ~~is deflected successively~~ is scattered ~~between~~ ~~next by~~ and ~~$(\vartheta, \gamma + d\gamma)$~~ further $(\vartheta, \vartheta + d\vartheta)$ respectively into direction $(\gamma, \gamma + d\gamma)$ $(\phi, \phi + d\phi)$ ~~furthered~~ scattered ~~before it transmit~~ during in ~~and~~ passes through the plate without a further scattering, ~~before it and passes through~~ with final velocity

(solid angle $d\omega$, $\sin\gamma\, d\gamma\, d\phi$ with the polar angles around (γ, ϕ), in the)

is given by

$$v' = v \cos\gamma = v_0 \cos\theta \cos\gamma. \qquad (6)$$

making angle θ' ~~given by~~ with ~~z-~~axis, given by

$$\frac{2\sin\theta\, d\theta}{\lambda_0}\cdot\frac{2\sin\vartheta\cos\vartheta\, d\vartheta}{2\pi\lambda\cos\theta}\cdot\Phi$$

$$\times\int\frac{d\phi'}{\frac{1}{\lambda'\cos\theta'}-\frac{1}{\lambda\cos\theta}}\left[\frac{e^{-\frac{h}{\lambda_0}}-e^{-\frac{h}{\lambda\cos\theta}}}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}}-\frac{e^{-\frac{h}{\lambda_0}}-e^{-\frac{h}{\lambda'\cos\theta'}}}{\frac{1}{\lambda'\cos\theta'}-\frac{1}{\lambda_0}}\right],$$

the angle θ' between final velocity vector and z-axis (6) v_0, θ, γ depends not only on the above, but also azymuth ϕ of the velocity vector after second scattering

where θ' is the angle between z-axis and ~~neutron direction of motion~~ velocity vector θ' ~~scattering~~ in ~~and~~ passes through the plate in the direction of solid angle $d\omega = \sin\gamma\, d\gamma\, d\phi$ after second scattering and is given by ~~making angle~~

$$\cos\theta' = \cos\theta\cos\gamma + \sin\theta\sin\gamma\cos\phi,$$

where ϕ ~~being~~ the azymuth of the velocity vector ~~direction of motion~~ after second scattering in the new coordinate system, in which the direction of motion after first scattering is taking as

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE

NO. 7

~~In the latter~~ case, the ~~total probability~~

$$= \cancel{2}\,\tfrac{1}{2}\,\frac{2\sin\theta\, d\theta}{\lambda_0}\cdot\frac{2\sin\gamma\cos\gamma\, d\gamma\, d\phi}{2\pi\cdot\lambda}$$

$$\times\frac{1}{\frac{1}{\lambda'\cos\theta'}-\frac{1}{\lambda\cos\theta}}\left[\frac{e^{-\frac{h}{\lambda_0}}-e^{-\frac{h}{\lambda\cos\theta}}}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}}-\frac{e^{-\frac{h}{\lambda_0}}-e^{-\frac{h}{\lambda'\cos\theta'}}}{\frac{1}{\lambda'\cos\theta'}-\frac{1}{\lambda_0}}\right] \quad (9)$$

,

In the latter case, the ~~total~~ corresponding probab. of the neutron being reflected back ~~after similar~~ becomes, by using (1), (4) and (8') similarly

$$\frac{2\sin\theta\, d\theta}{\lambda_0}\cdot\frac{2\sin\gamma\cos\gamma\, d\gamma\, d\phi}{2\pi\lambda}$$

$$\times\frac{e^{+\frac{h}{\lambda\cos\theta'}}}{\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda\cos\theta}}\left[\frac{e^{-\frac{h}{\lambda_0}}-e^{-\frac{h}{\lambda\cos\theta}}}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}}-\frac{e^{-\frac{h}{\lambda_0}}-e^{-\frac{h}{\lambda\cos\theta'}}}{\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda_0}}\right] \quad (9')$$

The same as (9), except the extra factor $e^{-\frac{h}{\lambda\cos\theta'}}$

$$E^{\frac{1}{2}} = \left(\frac{m}{2}\right)^{\frac{1}{2}} v'$$

$$\frac{dE}{2E^{\frac{1}{2}}} = \left(\frac{m}{2}\right)^{\frac{1}{2}} dv'$$

$$\frac{dE}{E_0} \int \frac{E_0^{\frac{1}{2}} - E^{\frac{1}{2}}}{E^{\frac{1}{2}}} = \frac{dE}{E_0}\left\{\left(\frac{E_0}{E}\right)^{\frac{1}{2}} - 1\right\}$$

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

§4. Number of ~~Doubly scattered~~ Fast Neutrons

$\theta' < \frac{\pi}{2}$ always, i.e., ~~the θ' is always~~

Now, if $v' > \frac{v_0}{2}$, the righthand side of (7') becomes positive for any values v and ϕ bet in the intervals (v', v_0) and $(0, 2\pi)$ respectively, so that ~~θ' is~~ all ~~the~~ total neutrons with the final velocity v' larger than half the initial velocity v_0 after second collision ~~passes through the plate~~, advances forward and the total number which passes through the plate is the fraction

$$p_2(v')dv' = \int_{v=v'}^{v_0}\int_{\phi=0}^{2\pi} \frac{2\,dv}{\lambda_0 v_0} \cdot \frac{2\,v'dv'd\phi}{\lambda \cdot v^2\, 2\pi} \times \frac{1}{\frac{v}{\lambda' v'} - \frac{v_0}{\lambda v}} \left[\frac{e^{-\frac{h}{\lambda_0}} - e^{-\frac{hv_0}{\lambda v}}}{\frac{v_0}{\lambda v} - \frac{1}{\lambda_0}} - \frac{e^{-\frac{h}{\lambda_0}} - e^{-\frac{hv}{\lambda' v'}}}{\frac{v}{\lambda v'} - \frac{1}{\lambda_0}} \right] \quad (10)$$

we assume λ the constant between $\frac{v_0}{2}$ and v_0

of the number of the incident neutrons.

~~In this case~~ If, furthermore, h ~~is~~ be small compared with λ_0, $\frac{\lambda}{2}$, ~~for $v > v' > \frac{v_0}{2}$~~ ~~(10) reduces to almost all~~ the neutrons after second collision passes through the plate almost certainly, so that (10) reduces to

$$p_2(v')dv' = \left(\frac{h}{\lambda_0}\right)^2 \frac{2v_2 dv_2}{v_0^2} \frac{2(v_0 - v_2')dv_2'}{v_0^2} \quad (11)$$

~~for~~ ~~In this case~~ the contribution due to the double scattering ~~number probability~~ $p_2(v')$ is proportional

Thus for fast neutrons with velocity v' larger than $\frac{v_0}{2}$.

Only a small fraction ~~part~~ will be scattered for the third time before it comes out of the plate after second scattering. Nearly ~~about~~ a half of such neutrons will pass through the plate ~~without~~ with the velocity v_c ~~distribution given approximately by~~ ~~without further scattering~~.

~~If~~ the thickness h is even small compared with the mean free path for slow neutrons, which is about one tenth ~~slow neutron~~ ~~time~~ as large as of that for fast neutrons, so that the total number of neutrons transmitted or reflected back with velocity smaller than v_c is very small as far as λ_c is ~~sm large~~ small large compared with h. ~~About~~ a half ~~most~~ of such neutrons will ~~be transmitted~~ ~~pass through or~~ be transmitted reflected back without further scattering, ~~in~~ ~~and only a small part~~ ~~being which~~ will equal ~~being their velocity distributions~~ ~~tein~~ of ~~number which each~~ ~~become~~ half of ~~the~~ above the former.

Roughly speaking, ~~the neutrons~~ further ~~double~~ multiple scattering will occur probably or will not according as the angle of first deflection ~~angle~~ is larger or smaller than the critical angle θ_c given by

$$\cos\theta_c = \frac{v_c}{v_0} = \frac{h}{\lambda_c} \ll 1, \quad \left(\cong \frac{\pi}{2}\right)$$

~~Now the single scattering~~ the probability that ~~occurs~~ ~~the~~ incident neutron is deflected in the plate by an angle larger than critical angle θ_c ~~will~~ is given ~~by~~ approximately by

$$\left(\frac{h}{\lambda_0}\right) \cdot \int_{\theta_c}^{\frac{\pi}{2}} 2\sin\theta\cos\theta\, d\theta \cong \left(\frac{h}{\lambda_c}\right)\left(\frac{h}{\lambda_c}\right)^2$$

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

If h is ~~not~~ ~~short~~ small compared with the mean free path for slow neutrons, although small compared with ~~that~~ ~~of~~ for fast neutrons, ~~such~~ neutrons which ~~are~~ first deflected nearly at right angle ~~will~~ have ~~appropriat~~ the appreciable probability back of being further scattered more than once.

if $h \gtrsim \lambda \cos\theta$.

To the square of thickness of the plate and is small compared with $p_1(v')$ ~~due~~ that due to single scattering and ~~there~~ ~~are~~ ~~no neutrons of reflected neutrons of velocity above~~ $\frac{v_0}{2}$ ~~to this approximation.~~
~~no neutrons of velocity above~~ $\frac{v_0}{2}$ ~~are reflected back~~
~~no neutrons~~ reflected back have ~~the~~ velocity ~~above~~ $\frac{v_0}{2}$ ~~to this o~~ no neutrons are reflected back with velocity above $\frac{v_0}{2}$ to this approximation.

§5. Number of Doubly Scattered Slow Neutrons

~~For slow The neutrons are becomes~~ The velocity of the neutron ~~is reduced becomes very small~~ after double scattering is very small only if ~~the~~ the deflection angles θ and χ ~~is nearly at least~~, one of at least, equal to $\frac{\pi}{2}$. The case ~~when~~ $\theta \cong \frac{\pi}{2}$ is more important than the case $\chi \cong \frac{\pi}{2}$ ~~above all~~, since the neutrons ~~scattered~~ deflected first nearly at right angle ~~perpendicularly~~ will be scattered once more almost certainly before ~~it penetrate the plate~~ they can come out of the plate. ~~The case $\chi \cong \frac{\pi}{2}$ is less important~~ In ~~this~~ such a case, only about ~~nearly~~ half after second collision ~~the neutrons will be reflected~~ of them will advance forward, ~~other~~ ~~remaining~~ the other half ~~being reflected~~ advancing backward.

In this case, the ~~veloci~~ distribution function will increase more rapidly than ( ) as the velocity approaches to zero.

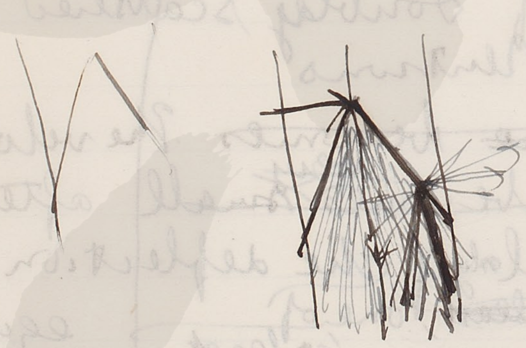
$$\int_{v''}^{v_c} \frac{2dv'}{v'} \log\left(\frac{v_0}{v'}\right)$$

$$= \left(\log \frac{v_c}{v''}\right)^2$$

$$\frac{24 v'^5 dv'}{v_0^6} \left(\log \frac{v}{v'}\right)$$

$$\int_{v''}^{v_c} \boxed{\frac{2v'dv'}{v_0^2} \cdot \frac{h}{\lambda_0} \cdot \log \frac{v_0}{v'}}$$

$$\frac{2v''dv''}{v'^2}$$



λ_c

If we assume
So rough estimation
~~Rough estimation~~
The case $\theta \cong \frac{\pi}{2}$ is more important than the case $\gamma \gtrsim \frac{\pi}{2}$, since main part of ~~contribution~~ ~~to the double~~ slow neutrons, ~~transmitted passed through~~ after double scattering ~~can be those which $\theta \gtrsim \frac{\pi}{2}$, since~~ will correspond to first scattered —

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

As ~~The exact calculation is difficult, we assume simply that these neutron~~ We assume ~~further~~ moreover that these neutrons ~~are~~ suffer no further scattering, ~~so that~~ i.e. h is ~~large~~ small compared with $\lambda' \cos\theta'$, which can be satisfied ~~exp~~ except for a small region of θ' near $\frac{\pi}{2}$, (9) to reduce then to

$$\frac{2\sin\gamma\cos\gamma\, d\gamma\, d\phi}{2\pi} \cdot \frac{h}{\lambda_0} \qquad (12)$$

for $\cos\theta \lesssim \frac{h}{\lambda}$, or

$$\frac{2v\,dv}{v_0^2} \cdot \frac{2v'dv'\,d\phi}{2\pi v^2} \cdot \frac{h}{\lambda_0} \qquad (12)'$$

for v ~~<~~ $v_c = \frac{hv_0}{\lambda}$ smaller than the critical velocity $v_c = \frac{hv_0}{\lambda} \ll v_0$.

Hence the ~~fraction~~ of number of neutrons ~~wi~~ ~~pa~~ which passes through the plate with the velocity $(v, v+dv)$ after double scattering is approximately equal to the fraction

$$p_2(v)dv \cong \frac{2v'dv'}{v_0^2} \int \frac{h}{\lambda_0} \int_{v'}^{v_c} \frac{dv}{v} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{d\phi}{2\pi}$$

$$= \frac{2v'dv'}{v_0^2} \frac{h}{\lambda_0} \log \frac{v_c}{v'} \qquad (3)$$

$$\frac{h}{\cos\theta_c} = \lambda_c$$

$$\cos\theta_c \cong \frac{v_c}{v} = \frac{h}{\lambda_c}$$

$$\cong \frac{\pi}{2} - \theta_c \quad \text{or} \quad \theta_c \cong \frac{\pi}{2} - \frac{h}{\lambda_c}.$$

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

Thus the number of ~~next~~ slow neutrons increases as $v' \log \frac{1}{v'}$ as their velocity tends to zero.

reflect back

of the number of the incident neutron. The ~~number~~ ~~frac~~ number of the neutrons with velocity ~~v~~ smaller than v_c ~~will be~~ is ~~equ~~ nearly ~~equal to~~ the same with (13), ~~as~~ this was noticed above.

~~Although~~ These estimations are very rough, ~~but~~ we can expect that ~~more exact~~ the general trend of ~~the~~ the ~~neutron~~ distribution of slow neutrons ~~after~~ produced by ~~triple~~ double ~~state~~ scattering can be expressed by (13) except that the value of ~~the~~ v_c may ~~change somewhat~~ differ from $\frac{h\nu_0}{\lambda}$ by some factor.

If we take ~~the tri~~ triple ~~and higher~~ scattering × into account, the number of slow neutrons will increase more rapidly than (13) above, as the velocity ~~become~~ tends to zero, for it will be proportional to

$$v' \log^2 \frac{1}{v'}\ .\ \cancel{etc}.$$

§6. ~~The~~ Energy Distribution of Scattered Neutrons.

From the ~~calcu~~ results in the preceding sections, the distribution of energy of ~~scattered~~ transmitted ~~and~~ or reflected neutrons ~~can be~~ ~~at once~~ ~~become~~ can be ~~easi~~ readily obtained. Namely, ~~the fraction~~ the number of ~~the~~ the neutrons ~~of~~ by (3),

$$\frac{e^{-\frac{h}{\lambda_0}} - e^{-\frac{h}{\lambda}\left(\frac{E_0}{E}\right)^{\frac{1}{2}}}}{\frac{h}{\lambda}\left(\frac{E_0}{E}\right)^{\frac{1}{2}} - \frac{h}{\lambda_0}}$$

$$= e^{-\frac{h}{\lambda_0}} \frac{1-e^{-x}}{x} \qquad x = \frac{h}{\lambda}\left(\frac{E_0}{E}\right)^{\frac{1}{2}} - \frac{h}{\lambda_0}$$

$$\frac{1-e^{-x}}{x} = 1 - e^{-1} = 0,63$$

$$x \simeq 1 : \boxed{\frac{h}{\lambda}\left(\frac{E_0}{E}\right)^{\frac{1}{2}} \simeq 1,} \qquad \lambda^2 E = h^2 E_0$$

$$
\begin{array}{r}
0,37 \\
2,7\overline{)\cancel{1,8=8}\ 1,00} \\
81 \\
\hline
190 \\
189
\end{array}
$$

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE
NO. 12

~~transmitted~~ with energy between E and E + dE ~~transmitted~~ after single collision is, ~~given by~~ ~~from (3)'~~ by using (3)', the fraction

$$P_1(E)dE = \frac{dE}{\lambda_0 E_0}\cdot\frac{e^{-\frac{h}{\lambda_0}} - e^{-\frac{h}{\lambda}\left(\frac{E_0}{E}\right)^{\frac{1}{2}}}}{\frac{1}{\lambda}\left(\frac{E_0}{E}\right)^{\frac{1}{2}} - \frac{1}{\lambda_0}} \qquad (14)$$

of the number of incident neutrons with ~~velocity~~ energy E_0. For the ~~energy so~~ large; such that $\lambda\left(\frac{E}{E_0}\right)^{\frac{1}{2}} \gg h$, it reduces ~~to~~, by (4), to

$$P_1(E)dE = \frac{dE}{E_0}\cdot\frac{h}{\lambda_0}\left(1 + \frac{h}{2\lambda}\left(\frac{E_0}{E}\right)^{\frac{1}{2}} + \frac{h}{\lambda_0}\right) \quad (15)$$

~~The distribution (14)~~

Thus ~~the number~~ $P_1(E)$ is nearly constant for large E ~~value~~ ~~of energy~~, ~~and~~ ~~decreases~~ ~~zero for small value~~ about becomes $\frac{0.632\,h}{3\lambda_0 E_0}$ for ~~about~~ E_c given by $\frac{h}{\lambda}\left(\frac{E_0}{E}\right)^{\frac{1}{2}} \simeq 1$ and decreases $(1-e^{-1})$ ~~and~~ rapidly to zero as the energy tends to zero.

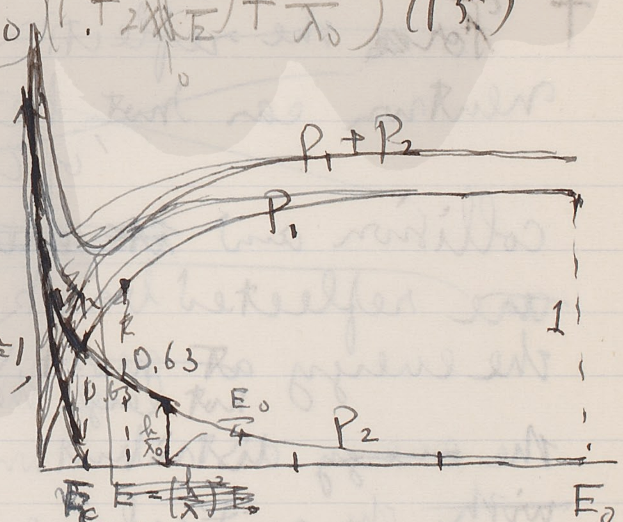


Fig. 1

Next the ~~total~~ approximate number of transmitted ~~neutrons~~ with energy ~~in~~ between E and E + dE after double collisions is ~~given~~ ~~approximately~~, by (11), the fraction

$$P_2(E)dE \simeq \left(\frac{h}{\lambda_0}\right)^2 \frac{dE}{E_0}\left\{\left(\frac{E_0}{E}\right)^{\frac{1}{2}} - 1\right\} \qquad (16)$$

$$\{P_1(E) + P_2(E)\}dE \simeq \left(\frac{h}{\lambda_0}\right)\frac{dE}{E_0}\left\{1 + \frac{3h}{2\lambda_0}\left(\frac{E_0}{E}\right)^{\frac{1}{2}} - \frac{h}{2\lambda_0}\right\}$$

† ~~For~~ ~~the reflection~~ Next, the ~~the reflection energy~~ neutron can ~~not~~ be reflected back by single collision ~~, of course,~~ and ~~its energy ha~~ ~~it is refl.~~ Next ~~The~~ neutrons ~~those which are~~ reflected back by double collisions ~~has~~ the energy ~~at most~~ ~~half of~~ not larger than a quarter the initial energy and the ~~energy~~ distribution for the energy smaller ~~compared~~ than ~~with~~ the critical energy E_c are given approximately by (17) being the same as ~~those~~ that ~~of~~ for transmitted neutrons.

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE
NO. 13

of the number of the incident neutrons, for ~~provided that~~ E is larger than $\frac{E_0}{4}$, and by (13)

$$P_2(E)\,dE \simeq \frac{h}{\lambda_0}\frac{dE}{2E_0}\,\tfrac{1}{2}\log\frac{E_c}{E} \qquad (17)$$

for E ~~smaller than than~~ compared with E_c. although approximate expression for $P_2(E)$ can not be easily ~~to~~ be found for intermediate region, ~~we expect the~~ the distribution will be expressed by the P_2 the ~~curve of P_2 does the form~~ will perhaps has the ~~form given~~ curve P_2 plotted in Fig. 1 approximately.

The ~~resultant total~~ number of transmitted neutrons due to single and double scattering ~~resultant~~ resultant distribution will be expressed by the curve P_1+P_2 ~~and~~ in Fig. 1, having approximately a minimum ~~at~~ for energy E_m of the order of the critical energy E_c.

The forms of the curves P_1, P_2 ~~and~~ and the P_1+P_2 depend, of course, on the thickness h, and the energy E_m ~~the minimum of P_1+P_2~~ increases as h increases; the ~~the relative importance of P_2 with~~ P_2 increasing more rapidly than P_1 as h increases. ~~For h still more large h~~ at the same time, the triple ~~third~~ and higher scatterings ~~becomes~~ the effect of begin to be more and more ~~important~~ prominent, ~~as~~ till the distribution becomes entirely different from that given above. for h of the order of λ_0.[†]

$$p_1(v)\,dv \cong \frac{2v^2\,dv}{v_0^3}\,\frac{\lambda_*(v)}{\lambda_1(v_0)} \quad \text{for } v < v_c$$

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE
NO. 14

Reflection ~~by protons.~~

~~the~~ atomic cross section being ~~about the~~ the order as that of scattering ~~for~~ slow neutrons all the by ~~protons~~ effects seems to be ~~sa~~ smaller ~~negligible~~ compared with the scattering by protons.

§ 7. Problem of Equilibrium Distribution of ~~for thick~~ Plate

C or O for example, ~~Carbon and oxygen~~

~~§ 7. of Transmitted Neutrons~~ Transmission

§ 7. Effect of Presence of Element ~~Atoms~~ Nuclei other than Protons Hydrogen various due to

1) Velocity Distribution

Hitherto we have altogether neglected the effects of the ~~the presence of atoms C or O other than hydrogen atoms.~~ collisions of ~~the~~ neutrons with the ~~nucleus~~ nuclei, such as other than protons. For fast neutrons, the elastic Since ~~Among which~~ the effect due to the elastic scattering is relatively important ~~effects as due to~~ ~~for~~ by ~~the~~ heavier ~~atoms~~ ~~fast neutrons~~. ~~Then taking this effect into account at least~~ If we take only, the mean free path of the neutron ~~can~~ with velocity v is given by

$$\frac{1}{\lambda(v)} = \frac{1}{\lambda_1(v)} + \frac{1}{\lambda_2(v)} = n\sigma + NS \quad (18)$$

$$\frac{1}{\lambda_1} = n_1\sigma_1(v) \quad \frac{1}{\lambda_2} = n_2\sigma_2(v)$$

where σ_1 and σ_2 are the cross sections of the elastic scattering by the proton and the ~~other~~ heavier nucleus (C or O) respectively and n_1 and n_2 are ~~their~~ numbers respectively in unit volume of the plate. In this case ~~After~~ the fraction of the neutrons ~~scatt~~ transmitted with velocity between v and $v+dv$ after single scattering with the proton becomes

$$p_1(v)\,dv = \frac{2v\,dv}{\lambda_1(v_0)\,v_0^2}\,\frac{e^{-\frac{h}{\lambda(v_0)}} - e^{-\frac{h v_0}{\lambda(v) v}}}{\frac{v_0}{\lambda(v)v} - \frac{1}{\lambda(v_0)}} \quad (19)$$

Iより $\cos\theta = 0$ の近く

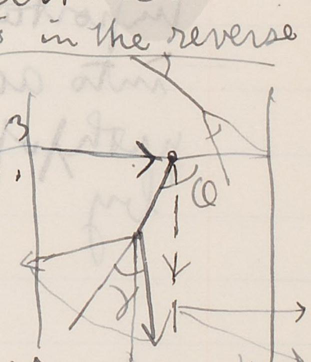
$$\int_{\theta=0}^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{2\lambda_2(v_0)} \cdot \frac{2\cos\gamma \sin\gamma\, d\gamma\, d\phi}{\lambda_1(v_0)\cos\theta \cdot 2\pi} \left(\frac{1}{\lambda(v)\cos\theta'} - \frac{1}{\lambda(v_0)\cos\theta}\right)^{-1}$$

$$\times \left[\frac{e^{-\frac{h}{\lambda(v_0)}} - e^{-\frac{h}{\lambda(v_0)\cos\theta}}}{\frac{1}{\lambda(v_0)\cos\theta} - \frac{1}{\lambda(v_0)}} - \frac{e^{-\frac{h}{\lambda(v_0)}} - e^{-\frac{h}{\lambda(v)\cos\theta'}}}{\frac{1}{\lambda(v_0)\cos\theta'} - \frac{1}{\lambda(v_0)}} \right]$$

$$+ \int_{\theta=\frac{\pi}{2}}^{\frac{\pi}{2}+\gamma} \quad '' \quad \left[\cancel{\quad} '' \times e^{-\frac{h}{\lambda(v)\cos\theta'} + \frac{h}{\lambda(v_0)\cos\theta}} \right]$$

$$(4^{-} \quad '')$$

Further, ~~if~~ we neglect the reduction of velocity (neutron) by ~~elasti~~ scattering with heavy nucleus and ~~we~~ assume ~~the uniform~~ the angular distribution to be spherically symmetric, the fractions pass through the plate after having scattered first by proton, then by heavy nucleus and in the reverse order respectively, neglecting terms heavy nucleus で scatter されてくる terms higher than h,



$$\int_0^{h}\int_0^{\frac{\pi}{2}} e^{-\frac{z}{\lambda_2(v_0)}} dz \frac{\sin\theta\, d\theta}{2\lambda_2(v_0)} \cdot e^{-\frac{(h-z)}{\lambda(v_0)\cos\theta}} =$$

$$\cos\gamma = \frac{v}{v_0} \qquad v = \frac{v_0}{2} \qquad \gamma = \frac{\pi}{3} \qquad \theta \le \frac{2\pi}{3}$$

もう一方 proton で scatter されてくるのは

$\cos\theta' = \theta - \gamma \le \frac{\pi}{2}$

$$\iint_0^{\frac{\pi}{2}+\gamma} e^{-\frac{z}{\lambda(v_0)}} dz \frac{\sin\theta\, d\theta}{2\lambda_2(v_0)} \cdot e^{-\frac{(z'-z)}{\lambda(v_0)\cos\theta}} \cdot \frac{\cancel{dz}}{\lambda_1(v_0)\cos\theta} \frac{2\cos\gamma\sin\gamma\, d\gamma\, d\phi}{2\pi}$$

$$\times e^{-\frac{(h-z')}{\lambda(v)\cos\theta'}} dz'$$

$$\cos\theta' = \cos\theta\cos\gamma + \sin\theta\sin\gamma\cos\phi$$

(z'についての積分は θ が $(0, \frac{\pi}{2})$ の間なら (z, h), $(\frac{\pi}{2}, \pi)$ の間なら $(0, z)$)

$$\cos\phi > \cot\theta\cot\gamma \qquad \dagger$$

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE
NO.

$+\frac{1}{2}\frac{h^2}{\lambda_1(v_0)}\left(\frac{v_0}{\lambda(v)v}+\frac{1}{\lambda(v_0)}\right)\}$

, which reduces ~~to~~ for ~~v~~ large v to

$$p_1(v)dv = \frac{2vdv}{v_0^2}\left\{\frac{h}{\lambda_1(v_0)},\right. \quad (20)$$

instead of (3′) and (4), ~~while~~ that after having scattered by two ~~double scattering with~~ protons successively p_2 becomes approximately ~~for~~

$$p_2(v)dv \cong \left(\frac{h}{\lambda_1(v_0)}\right)^2 \frac{2(v_0-v)dv}{v_0^2} \quad (21)$$

instead of (11) for ~~the velocity~~ v larger than $\frac{v_0}{2}$, ~~while and~~ It becomes

$$p_2(v)dv \cong \frac{2vdv}{v_0^2}\frac{h}{\lambda_1(v_0)}\log\left(\frac{v_c}{v}\right) \quad (22)$$

instead of (13) for v smaller than the critical velocity ~~to~~ given by

$$v_c \cong \frac{h\,v_0}{\lambda(v_c)},$$

provided that h is small compared with the mean free path for slow neutrons.

Next, the fraction of ~~the neutrons~~ the neutrons ~~which~~ transmitted with velocity ~~v and~~ between v and $v+dv$ after having scattered by a heavy nucleus and a proton successively is given approximately by

$$p_2'(v)dv \cong \frac{vdv}{v_0^2}\frac{h^2}{\lambda_1(v_0)\lambda_2(v_0)}\left\{1+\frac{1}{2}\log\left\{\frac{\lambda(v_0)}{h}\left(1-\frac{v^2}{v_0^2}\right)^{\frac{1}{4}}\right\}\right\} \quad (24)$$

for velocity v smaller than $v_0\sqrt{1-\left(\frac{h}{\lambda(v_0)}\right)^2}$.

Finally, the fraction of the neutrons transmitted with velocity between v and $v+dv$ after having scattered by a proton and a heavy nucleus successively

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE
NO.

is given approximately by

$$p_2''(v)dv = \frac{dv}{v_0} \frac{h}{\lambda_1(v_0)\lambda_2(v_0)} \qquad (25)$$

for velocity v ~~small~~ ~~not small compared with~~ large compared with ~~the critical velocity~~ v_c ~~$v_c = \frac{hv_0}{\lambda(v_0)}$~~,
and by

$$p_2''(v)dv = \frac{v\,dv}{v_0^2} \frac{h\,\lambda(v)}{\lambda_1(v_0)\lambda_2(v)} \qquad (26)$$

for v smaller than ~~the critical velocity~~ v_c.
Summing up all these ~~contributions we~~, the fraction of transmitted neutrons with velocity between v and $v+dv$ becomes, by using (20), (21), (24) and (25),

$$P(v)dv = \left(p_1(v) + p_2(v) + p_2'(v) + p_2''(v)\right)dv$$

$$\cong \frac{2v\,dv}{v_0^2} \Big\{ \frac{h}{\lambda_1(v_0)} + \cancel{\left(\frac{h}{\lambda_1(v_0)}\right)^2\left(\frac{v_0}{v}-1\right)}$$

$$+ \cancel{\frac{2h}{}} \Big\{ 1 + \frac{h}{\lambda_1(v_0)}\left(\frac{v_0}{v}-1\right)$$

$$+\frac{h}{2}\left(\frac{v_0}{\lambda(v)v}+\frac{1}{\lambda(v_0)}\right)$$

$$+ \frac{h}{\lambda_2(v_0)}\left(\frac{1}{2}+\frac{1}{4}\log\left\{\frac{\lambda(v_0)}{h}\left(1-\frac{v^2}{v_0^2}\right)^{\frac{1}{4}}\right\}\right)$$

$$+ \cancel{\frac{v_0}{v}} \frac{h}{\lambda_2(v_0)} \frac{v_0}{2v} \Big\} \qquad (27)$$

~~, for~~ for velocity larger than $\frac{v_0}{2}$,

DEPARTMENT OF PHYSICS
OSAKA IMPERIAL UNIVERSITY.

DATE

NO. 17

~~It becomes, by~~ whereas, by using (19), (22), (24) and (25), it becomes

$$P(v)dv \cong \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_1(v_0)}\left\{\frac{\lambda(v)v}{\cancel{h v_0}\;\lambda(v_c)v_c} + \log\left(\frac{v_c}{v}\right)\right.$$
$$\left. + \frac{h}{\lambda_2(v_0)}\left(\frac{1}{2} + \frac{1}{4}\log\frac{\lambda(v_0)}{h}\right) + \frac{1}{2}\frac{\lambda(v)}{\lambda_2(v)}\right\} \quad (28)$$

for velocity smaller than v_c.

ii) Reflection

Next ~~the~~ the number of neutrons which is reflected back with the velocity ~~nearly~~ equal to v_0 after ~~sca~~ having scattered by ~~m~~ heavy nuclei once or ~~twice~~, is given ~~by~~ app. by the fraction

$$R(v_0) = \frac{h}{2\lambda_0''}\left(1 + \frac{h}{2\lambda_0''}\log\frac{\lambda_0}{h}\right)$$

of the number of incident neutrons.

§ Reflection by highly Absorbing Layer.

§ Summary and Numerical Results