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~~The last case is compared~~ which seems to be in accord with the results of the experiment. (§ 7)

Elementary Calculations on the Scattering of Neutrons by Thin Plate

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(Read July 4, 1936)

Abstract

The energy distribution of neutrons slowed down by ~~the~~ a plate containing hydrogen, whose thickness is small compared with the mean free path of the primary neutrons with a definite energy, is calculated for following cases, taking ~~the~~ single and ~~the~~ double scatterings only into account.

i) Normal Incidence: In this case the ~~number~~ distribution functions for slow neutrons increases with the decreasing ~~as the energy decreases~~ energy tends to zero ~~according to the formula~~ as logarithmically log E. (§2, §3, §4, §5)

ii) A Point Source: In this case, the distribution function is nearly constant for slow neutrons. (§6)

iii) ~~A Point Source~~ ~~Number~~ Energy Distribution of neutrons, which hit a detector ~~after scatt~~ ~~placed on the axis of a circular plate~~ is emitted from a point source and after ~~scattering~~ having slowed down by a ~~circular~~ plate, the source and the detector is placed on ~~the axis of~~ opposite sides of the plate. In this case, the distribution function decreases ~~proportion with decreasing energy~~ ~~is nearly constant~~ ~~throughout~~ for slow neutrons, $E^{\frac{1}{2}}$.

§ 1. Introduction

It is, of course, an important, though not an easy task to find the change of the energy distribution of neutrons due to the presence of a substance containing hydrogen in sufficiently general case. The wellknown probabilty distribution of the neutron energy after having scattered by protons a fixed number of times[1] can be applied to special problems, only if the probability of occurrence for each number of times of scattering is determined theoretically. On the other hand, we are met with mathematical complications, if we want to extend the homogeneous and isotropic equilibrium distribution found by Fermi[2] to more general cases.

Hence, it seems to be justified for the time being to confine our attention to an elementary problem of slowing down of neutrons by a plate whose thickness is small compared with the mean free path of thermal neutrons. In sections from § 2 to § 5, most simple problem of slowing down of parallel beam by a homogeneous plate

§ 2. Single Scattering by Protons

We consider a parallel plate of thickness h, containing hydrogen, through which a neutron beam of definite velocity v_0 passes from left to right perpendicularly. We

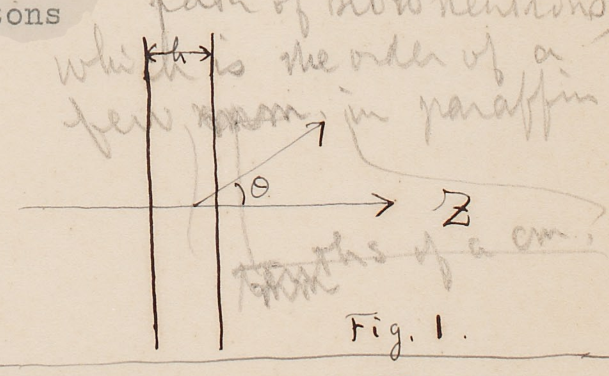


Fig. 1.

1) Wick, Phys. Rev. **49**, 192, 1936; Condon and Breit, ibid. **49**, 229, 1936; Goudsmit, ibid. **49**, 406, 1936; Lamla, Naturwiss. **24**, 251, 1936.

2) Fermi, Zeeman Jubilee Papers, Nijhoff, The Hague, 1935, p.128.

neglect at first all the processes other than the scattering by protons.

According to If the direction of the incident beam is taken as that of z-axis, the probability ~~for~~ for it to be deflected by an angle between θ and $\theta+d\theta$ at a distance between z and z+dz from the left surface is given, ~~according to~~ the result of the assumption of short range force between the neutron and the proton, ~~as generally accepted,~~ by

$$\exp\left(-\frac{z}{\lambda_0}\right)\cdot\frac{dz}{\lambda_0}\cdot 2\sin\theta\cos\theta\, d\theta, \qquad (1)$$

where λ_0 is the mean free path of the neutron with velocity v_0.

The neutron deflected in this way has the velocity

$$v = v_0\cos\theta \qquad (2)$$

and passes through the plate without further scattering only with the probability $\exp -\frac{(h-z)}{\lambda\cos\theta}$, where λ is the mean free path for velocity v. Hence, the probability, that a neutron passes through the plate in the direction $(\theta, \theta+d\theta)$ after single scattering, becomes

$$\int_{z=0}^{h}\exp\left\{-\frac{z}{\lambda_0}-\frac{(h-z)}{\lambda\cos\theta}\right\}\cdot\frac{dz}{\lambda_0}\cdot 2\sin\theta\cos\theta\, d\theta$$

$$= \frac{2\sin\theta\cos\theta\, d\theta}{\lambda_0}\cdot\frac{\exp\left(-\frac{h}{\lambda_0}\right)-\exp\left(-\frac{h}{\lambda\cos\theta}\right)}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}}, \qquad (3)$$

so that the total number of transmitted neutrons with velocity $(v, v+dv)$ is the fraction

$$p_1(v)\,dv = \frac{2v\,dv}{\lambda_0 v_0^2}\,\frac{\exp\left(-\frac{h}{\lambda_0}\right)-\exp\left(-\frac{hv_0}{\lambda v}\right)}{\frac{v_0}{\lambda v}-\frac{1}{\lambda_0}} \qquad (3)'$$

of the number of incident neutrons.

If we assume hereafter the thickness h to be small compared with the mean free path λ_0, (3)$'$ can be reduced to

$$p_1(v)\,dv = \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0}\left\{1 - \frac{h}{2\lambda_0}\left(1+\frac{\lambda_0 v_0}{\lambda v}\right)\right\} \qquad (4)$$

for velocity v so large that it satisfies the relation $\lambda v \gg h v_0$, which is the condition for the scattered neutron to pass through the plate almost certainly without further scattering.

On the contrary, it reduces to

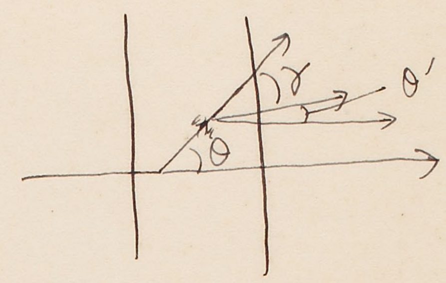
$$p_1(v)\,dv = \frac{v^2 dv}{v_0^3}\frac{2\lambda}{\lambda_0}\left(1 - \frac{h}{\lambda_0}\right) \qquad (5)$$

for the velocity v small compared with the critical velocity v_c given by

$$\lambda_c v_c = h v_0 , \qquad (6)$$

where λ_c is the mean free path for velocity v_c. In this case, the neutron after having deflected by an angle nearly equal to $\frac{\pi}{2}$ has very small velocity and will be deflected once more, almost certainly, before it will come out of the plate.

Of course, no neutrons are reflected back by single scattering, as $\theta \leq \frac{\pi}{2}$.



3. Double Scattering by Protons

In the similar manner, we can calculate the probability that a neutron is deflected first by an angle (θ, $\theta+d\theta$), scattered then into a solid angle $\sin\gamma\, d\gamma\, d\phi$ with polar angles (γ, ϕ) in the new coordinate system, in which the direction of motion after first scattering is taken as z-axis and the azymuth of the former z-axis is zero, and finally passes through or is reflected back by the plate, with the velocity

$$v' = v\cos\gamma = v_0 \cos\theta\cos\gamma, \qquad (7)$$

according as the angle θ' between the final velocity and the former z-axis is smaller or larger than $\frac{\pi}{2}$. The result is

$$\frac{2\sin\theta\, d\theta}{\lambda_0}\cdot\frac{2\sin\gamma\cos\gamma\, d\gamma\, d\phi}{2\pi\lambda}\cdot\frac{1}{\frac{1}{\lambda'\cos\theta'}-\frac{1}{\lambda\cos\theta}} \qquad (8)$$

$$\times\left[\frac{\exp(-\frac{h}{\lambda_0})-\exp(-\frac{h}{\lambda\cos\theta})}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}}-\frac{\exp(-\frac{h}{\lambda_0})-\exp(-\frac{h}{\lambda'\cos\theta'})}{\frac{1}{\lambda'\cos\theta'}-\frac{1}{\lambda_0}}\right]$$

for transmission, where λ' is the mean free path for velocity v' and $\theta'(<\frac{\pi}{2})$ satisfies the relation

$$\cos\theta' = \cos\theta\cos\gamma + \sin\theta\sin\gamma\cos\phi\,(>0) \qquad (9)$$

Similarly, the result is

$$\frac{2\sin\theta\, d\theta}{\lambda_0}\cdot\frac{2\sin\gamma\cos\gamma\, d\gamma\, d\phi}{2\pi\lambda}\cdot\frac{\exp(\frac{-h}{\lambda'\cos\theta''})}{\frac{1}{\lambda'\cos\theta''}+\frac{1}{\lambda\cos\theta}}$$

$$\times\left[-\frac{\exp(-\frac{h}{\lambda_0})-\exp(-\frac{h}{\lambda\cos\theta})}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}}+\frac{\exp(\frac{h}{\lambda'\cos\theta''})-\exp(-\frac{h}{\lambda_0})}{\frac{1}{\lambda'\cos\theta''}+\frac{1}{\lambda_0}}\right] \qquad (10)$$

for reflection, where $\theta'' = \pi - \theta' \left(< \frac{\pi}{2}\right)$.

Now, if $v' > \frac{v_0}{2}$, θ' is smaller than $\frac{\pi}{2}$ for any values v and ϕ in the intervals (v', v_0) and $(0, 2\pi)$ respectively, so that the total number of neutrons, which pass through with the velocity v', becomes, by integrating (8), the fraction

$$p_2(v')dv' = \int_{v=v'}^{v_0} \int_{\phi=0}^{2\pi} \frac{2\,dv}{\lambda_0 v_0} \cdot \frac{2v'dv'd\phi}{2\pi\lambda} \cdot \frac{1}{\frac{v}{\lambda' v'} - \frac{v_0}{\lambda v}}$$

$$\times \left[\frac{\exp(-\frac{h}{\lambda_0}) - \exp(-\frac{hv_0}{\lambda v})}{\frac{v_0}{\lambda v} - \frac{1}{\lambda_0}} - \frac{\exp(-\frac{h}{\lambda_0}) - \exp(-\frac{hv}{\lambda' v'})}{\frac{v}{\lambda' v'} - \frac{1}{\lambda_0}} \right]$$

$$\cong \left(\frac{h}{\lambda_0}\right)^2 \frac{2(v_0 - v')dv'}{v_0^2} \qquad (11)$$

of the number of the incident neutrons, if we assume the mean free path to be nearly constant for velocity between $\frac{v_0}{2}$ and v_0.

Thus, for fast neutrons, the contribution of the double scattering to velocity distribution is smaller by a factor $\frac{h}{\lambda_0}$ than that of the single scattering. Besides, no neutrons are reflected back with velocity larger than $\frac{v_0}{2}$ to this approximation.

On the contrary, if v' is very small compared with v_0, at least one of θ and γ should be nearly equal to $\frac{\pi}{2}$. The case $\theta \doteqdot \frac{\pi}{2}$ is especially important, since those which are first deflected nearly at right angle will be scattered once more almost certainly, before they will come out of the plate. Roughly speaking, multiple scattering will or will not occur probably according as the angle of the first deflection is larger or smaller than the critical angle $\theta_c \left(\cong \frac{\pi}{2}\right)$ given by

$$\cos\theta_c = \frac{v_c}{v} = \frac{h}{\lambda_c} \ll 1$$

$$\text{or} \quad \theta_c \cong \frac{\pi}{2} - \frac{h}{\lambda_c} \qquad (12)$$

The probability that an incident neutron is deflected by an angle larger than θ_c becomes, by integrating (1),

$$\int_0^h \exp\left(-\frac{z}{\lambda_0}\right)\cdot\frac{dz}{\lambda_0}\cdot\int_{\theta_c}^{\frac{\pi}{2}} 2\sin\theta\cos\theta\, d\theta \cong \left(\frac{h}{\lambda_0}\right)\left(\frac{h}{\lambda_c}\right)^2 \qquad (13)$$

approximately. Nearly a half of such neutrons will pass through the plate after second scattering with the velocity distribution given approximately by

$$p_2(v')dv' \cong \frac{2v'dv'}{v_0^2}\,\frac{h}{\lambda_0}\int_{v'}^{v_c}\frac{dv}{v}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{d\phi}{\pi} = \frac{2v'dv'}{v_0^2}\cdot\frac{h}{\lambda_0}\log\frac{v_c}{v'}, \qquad (14)$$

if the thickness h is even small compared with the mean free path for slow neutrons, which is about one tenth of that for fast neutrons. In this case, other half will be reflected back after second scattering with approximately the same distribution as (14). Only a small fraction will be scattered for the third time before it comes out of the plate.

If h is not small compared with the mean free path for slow neutrons, although it is small compared with that for fast neutrons, those which ~~are first~~ have been deflected nearly at right angle have appreciable probability of being scattered further twice or more. In this case, the distribution

It is very difficult, however, to

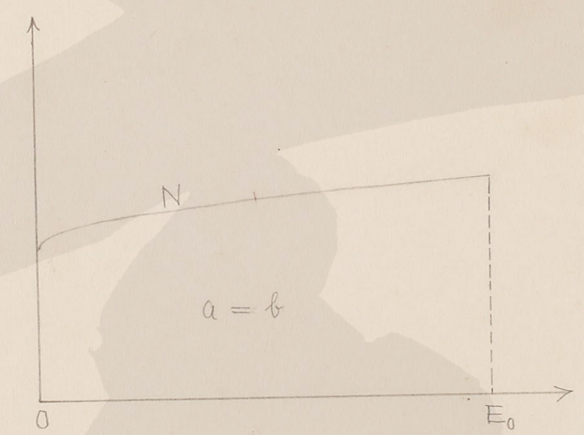


Fig. 5

complicated effects due to the chemical binding of hydrogen with carbon (or oxygen) are expected. Indeed, the mean free path of neutrons with thermal energy was shown by Fermi to be about a quarter of that with energy of the order of 1 eV.) Further, there is a small effect function will increase more sharply than (14) as the velocity due to the capture of slow neutrons by protons. approaches to zero. In the following rough estimations, all these very finer details will be overlooked.

4. Effect of Presence of Element other than Hydrogen

Hitherto we have neglected altogether the effect of the presence of element atoms, carbon or oxygen atoms for example, other than hydrogen in the nucleus plate. Among various processes caused by heavier atoms, the elastic scattering is relatively important for fast neutrons, so that the mean free path λ of the neutron with velocity v is given approximately by

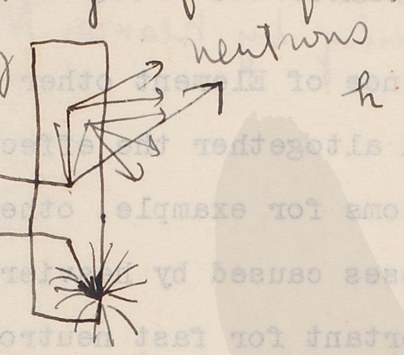
$$\left.\begin{aligned} \frac{1}{\lambda} &= \frac{1}{\lambda'} + \frac{1}{\lambda''}, \\ \frac{1}{\lambda'} &= n'\sigma'(v), \quad \frac{1}{\lambda''} = n''\sigma''(v) \end{aligned}\right\} \qquad (15)$$

where σ' and σ'' are cross sections of the elastic scattering by the hydrogen and the heavier atoms respectively and n' and n'' are their numbers in unit volume of the plate respectively. σ' and σ'' are the quantities of the same order of magnitude for fast neutrons.

For slow neutrons, all the effect due to heavier element such as carbon or oxygen seems to be small compared with that due to the scattering by hydrogen. It should be noticed, however, that if

On these assumption, the velocity distribution of neutrons, which pass through or are reflected by thin plate, can be calculated in the

If the velocity of the neutrons is not too large, the scattering by heavier nuclei will be spherically symmetric approximately. If the velocity becomes very small, the binding energy of hydrogen with carbon (or oxygen) is comparable and it is not negligible, assumption that the angular distribution of scattered by heavier nucleus scattered neutrons to be spherically symmetric, which will be correct for not too large energy of primary neutrons



The number of neutrons with velocity (v, $v+dv$), which are transmitted (or reflected), is the sum of the numbers of those which are transmitted (or reflected) after having scattered

i) once by protons,

ii) twice by protons,

iii) first by protons and then by heavy nuclei,

and

iv) first by heavy nuclei and then by protons,

if we neglect terms higher than h^2.

i) and ii) are calculated in the similar way as in the preceding sections and iii) and iv) are estimated on the

$-\frac{h}{\lambda_0} = -\frac{h}{\lambda_0'} - \frac{h}{\lambda_0''}$.

similar way as in the preceeding sections, the results being as follows. (the mean free paths etc. in (15), corresponding to λ, λ' and λ'' for incident neutrons.)

For velocity v larger than $\frac{v_0}{2}$, the fraction

$$p_1(v)dv + p_2(v)dv \simeq \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}\left\{1 - \frac{h}{2\lambda_0'}\left(\frac{v_0}{v}+1\right) + \frac{h}{\lambda_0'}\left(\frac{v_0}{v}-1\right)\right\} \quad (16)$$

of incident neutrons passes through the plate with the velocity between v and v + dv after having scattered by protons once or twice, ~~respectively,~~ neglecting terms higher than h^2, where λ_0, λ_0' and λ_0'' are

Further, if we neglect the reduction of neutron velocity by the scattering with heavy nucleus and assume the angular distribution of scattered neutron to be spherically symmetric, (which will be justified for not large velocity) the fraction

$$p_2'(v)dv \simeq \frac{v\,dv}{v_0^2}\frac{h}{\lambda_0'}\frac{h}{\lambda_0''}\left[\frac{v_0}{2v} + 1 + \frac{1}{2}\log\left\{\frac{\lambda_0}{h}\left(1-\frac{v^2}{v_0^2}\right)^{\frac{1}{4}}\right\}\right], \quad (17)$$

and the reduction of the neutron velocity is negligibly small.

passes through the plate after having scattered first by proton and then by heavy nucleus or in the reverse order, ~~respectively~~, neglecting terms higher than h^2.[1] Hence, the total number of neutrons with velocity $(v, v+dv)$, v being larger than $\frac{v_0}{2}$, becomes the fraction

$$P(v)\,dv = p_1(v)dv + p_2(v)dv + p_2'(v)dv \quad (18)$$

$$= \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0}\left[1 + \frac{h}{2\lambda_0'} - \frac{v_0}{2v} + \frac{1}{2}\log\frac{\lambda_0}{h}\left(1-\left(\frac{v}{v_0}\right)^2\right)^{\frac{1}{4}}\right]$$

of the incident neutrons to this approximation. (for neutron with velocity in the interval $\left(\frac{v_0}{2}, v_0\sqrt{1-\left(\frac{h}{\lambda_0}\right)^2}\right)$)

Similarly, it becomes approximately

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1) The equation (17) holds only for v smaller than $v_0\sqrt{1-\left(\frac{h}{\lambda_0}\right)^2}$.

2)

$$P(v)\,dv \cong \frac{2v\,dv}{v_0^2}\,\frac{h}{\lambda_0'}\left\{\frac{\lambda v}{h v_0} + \log\left(\frac{v_c}{v}\right) + \frac{1}{2}\frac{\lambda}{\lambda''}\right.$$

$$\left. + \frac{h}{4\lambda_0''}\left(\frac{1}{2} + \frac{1}{4}\log\frac{\lambda_0}{h}\right)\right\} + O(h^2) \qquad (19)$$

for velocity v smaller than the critical ~~valocity~~ velocity v_c, given by $\left(\frac{v_c}{v_0}\right)^2 = \frac{h}{\lambda_c}$, provided that the thickness is small compared with the mean free path for slow neutrons.

Next, the ~~number of neutrons~~ fraction which is reflected back with velocity $(v, v+dv)$ becomes approximately

~~$$P(v)\,dv \cong \frac{v\,dv}{v_0^2}\,\frac{h}{\lambda_0}\ \frac{\lambda_0}{\lambda_0''}\ \frac{h}{\lambda_0'}\left\{\frac{v_0}{2v} + 1 + \frac{1}{2}\log\frac{\lambda_0}{h}\right.$$~~

$$R(v)\,dv \cong \frac{2v\,dv}{v_0^2}\,\frac{h}{\lambda_0'}\,\frac{h}{4\lambda_0''}\left[\frac{v_0}{2v} + 1 + \frac{1}{2}\log\left\{\frac{\lambda_0}{h}\left(1-\frac{v^2}{v_0^2}\right)^{\frac{1}{4}}\right\}\right] \qquad (20)$$

for ~~large~~ velocity ~~and~~ between $\frac{v_0}{2}$ and $v_0\sqrt{1-\left(\frac{h}{\lambda_0}\right)^2}$ and $+O(h^3)$

$$R(v)\,dv \cong \frac{2v\,dv}{v_0^2}\,\frac{h}{\lambda_0'}\left\{\log\left(\frac{v_c}{v}\right) + \frac{1}{2}\frac{\lambda}{\lambda''}\right.$$

$$\left. + \frac{h}{4\lambda_0''}\left(\frac{1}{2} + \frac{1}{4}\log\frac{\lambda_0}{h}\right)\right\} + O(h^2) \qquad (21)$$

for ~~small~~ velocity smaller than v_c.

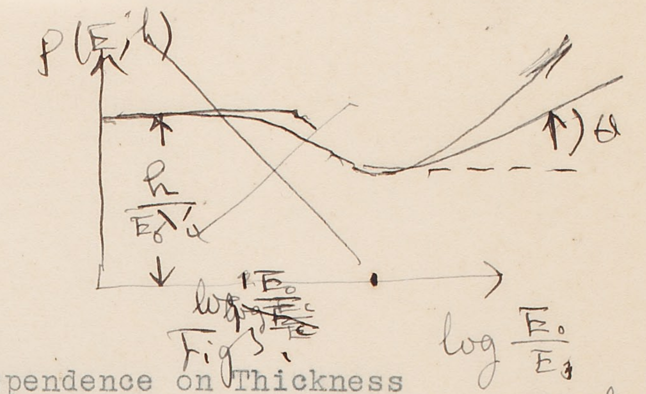
Finally, the fraction, which ~~is reflected~~ passes through with velocity nearly equal to ~~v_0 after having scattered by heavy nuclei once or twice~~ the initial velocity v_0, is given approximately by

$$R(v_0) \cong \frac{h}{2\lambda_0''}\left(1 + \frac{h}{\lambda_0''}\log\frac{\lambda_0}{h}\right) + O(h^2), \qquad (22)$$

$$\left(1 - \frac{h}{\lambda_0} + \right)$$

if we take the single and the double scatterings by heavy nuclei into account. Similarly, the fraction, which is reflected with velocity nearly equal to v_0, is given appr. by

$$R(v_0) \cong \frac{h}{2\lambda_0''}\left(1 + \frac{h}{\lambda_0''}\log\frac{\lambda_0}{h}\right) + O(h^2) \qquad (23)$$



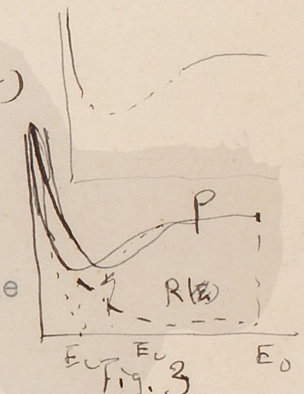
5. Energy Distribution and its Dependence on Thickness

From above results, the energy distribution of neutrons can be obtained at once. Namely, the fraction, which passes through with energy between E and E+dE, is given by

$$P(E,h)\,dE \cong \frac{dE}{E_0}\frac{h}{\lambda_0'} \qquad (24)$$

for $\frac{E_0}{4} < E < E_0$, where E_0 is the energy of the incident neutrons, and

$$P(E,h)\,dE \cong \frac{dE}{E_0}\frac{h}{\lambda_0'}\frac{1}{2}\log\left(\frac{E_c}{E}\right) \qquad (25)$$



for $E \ll E_c$, where $E_c (\ll E_0)$ is the critical energy satisfying the relation

$$\frac{E_c}{E_0} = \left(\frac{h}{\lambda_{0c}}\right)^2$$

Thus the energy distribution of the transmitted neutrons is nearly constant for large energy and increases logarithmically as the energy tends to zero, (Fig. 3, P(E)) as shown in logarithmic scale.

Similarly, the fraction, which is reflected, becomes

$$R(E,h)\,dE \cong \frac{dE}{E_0}\frac{h}{\lambda_0'}\frac{h}{\lambda_0'}\left[\frac{1}{2}\left(\frac{E_0}{E}\right)^2 + 1 + \frac{1}{2}\log\left\{\frac{\lambda_0}{h}\left(1-\frac{E}{E_0}\right)^{\frac{1}{4}}\right\}\right] \qquad (26)$$

In the intermediate region, It is difficult, however, to decide whether P(E,h) has a minimum or not in the intermediate region, so that the distribution curve takes the general form in Fig. 3.

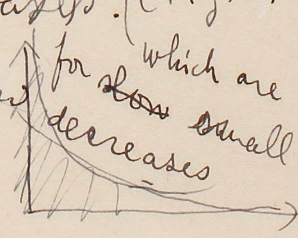
for $\frac{E_0}{4} < E < E_0$ and

$$R(E,h)\,dE \cong \frac{dE}{E_0}\frac{h}{\lambda_0'}\frac{1}{2}\log\left(\frac{E_c}{E}\right) \qquad (27)$$

from (27) P_1 or P_2 respectively.

for $E \ll E_c$. Hence, The energy dist number of reflected neutrons increases is approx. the same as those

Thus, both for transmission and reflection, rapidly steadily with as the increasing energy decreases. (Fig. 3, R(E)) which are 1 rem. for slow small energy and decreases



In order to compare these results with the experiment on the activation of a certain element by slow neutrons of a certain group, ~~produced~~ ~~which are~~

and are ~~measured~~ for two groups ~~of~~ one element or of two different elements, ~~with their~~ the above result shows that the ratio of their energies has the order of magnitude

These results can not ~~be com~~ be immediately compared with the experiment on the activation of a certain element by ~~slow neutrons of~~ a certain group of slow neutrons, which are produced by thin layers of paraffin, for

In practice, however, there remains always an appreciable amount of low velocity neutrons, even if the layer containing hydrogen is removed. Moreover, the actual energy distribution of neutrons which hit the detector will be affected considerably by the sizes and ~~the geo~~ arrangements ~~by geometrical~~ configurations of the source, the scatterer and the detector. ~~Thus, the complicated~~ In the next section, the ~~effec~~problem will be dealt with under conditions more closely related to the experiment ~~effects of inhomogeneity of primary beam, its oblique incidence etc.~~ ~~should be estimated before the consequences above obtained are compared with experiments.~~ ~~For such an estimation, the intermadiate region between small and large angle scatterings, which was not dealt with in any detail, becomes more important. At any rate, no conspicu results can be expected from the theory as long as both the energy and the direction of incident neutron beam extend over wide ranges.~~ for a limiting case of oblique incidence will be considered and applied to the case when the primary neutron is emitted from a point source.

§7. ~~Reflection and Transmission by Highly Absorbing Layer of a Heavy Element~~

~~As wellknown, the fact that the layer either of certain elements, Cd for example, absorbs a large part of a certain group of slow neutrons~~

and are measured for two groups of two each of two elements of or

§6. Effect of Oblique Incidence of Primary neutrons ~~Scattering of Neutrons from a Point Source by a Thin Plate.~~

~~We consider first the effect~~
The probability that a ~~neutron~~ primary neutron ~~inclined~~ making an angle θ with the ~~angle~~ normal of the plate ~~is scattered once by a proton~~ passes through the plate with energy between E and E+dE after single collision with a proton is given approx. by

$$P(E,\theta)\,dE \cong \frac{dE}{E_0}\cdot\frac{h}{\lambda_0\cos\theta} \qquad (28)$$

for $0 \leq \theta \lesssim \arccos\sqrt{1-\frac{E}{E_0}}$,

$$P(E,\theta)\,dE \cong \frac{dE}{E_0}\cdot\frac{h}{\lambda_0\cos\theta}\cdot\frac{\arccos\left(\cot\theta\cdot\sqrt{\frac{E_0}{E}-1}\right)}{\pi} \qquad (29)$$

for $\arccos\sqrt{1-\frac{E}{E_0}} \lesssim \theta \lesssim \frac{\pi}{2}-\frac{h}{\lambda_0}$

and

$$P(E,\theta)\,dE \cong \frac{dE}{2E_0}. \qquad (30)$$

for $\frac{\pi}{2}-\frac{h}{\lambda_0} \lesssim \theta < \frac{\pi}{2}$.

Hence, if ~~the number of neutron a point source~~ N_0 neutrons of energy E_0 are emitted per unit time from a point source, the ~~number~~ ~~the fraction of neutron~~ number per unit time of those which hit the plate with an ~~angle~~ inclination between θ and $\theta+d\theta$ becomes

$$N_0\frac{\sin\theta\,d\theta}{2},$$

so that, the number per unit time of those which passes through the plate with energy between E and $E+dE$ is given ~~by~~ approximately by

$$N(E)\,dE \cong \frac{N_0}{2}\frac{dE}{E_0}\frac{h}{\lambda_0}\log\frac{\lambda_0}{h} \quad (31)$$

for

$$E_0 > E > E_0\left(1-\left(\frac{h}{\lambda_0}\right)^2\right)$$

and

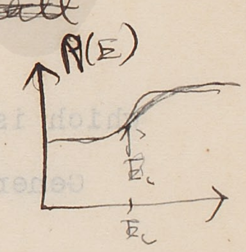
$$N(E)\,dE \cong \frac{N_0}{2}\frac{dE}{E_0}\frac{h}{2\lambda_0}\left\{\log\frac{\lambda_0}{h}\right\} \quad (32)$$

for

$$E_0\left(1-\left(\frac{h}{\lambda_0}\right)^2\right) > E,$$

$E_0 \gg E$, it can be easily seen that

~~Thus~~ In this case, the effect of double scattering is ~~not large~~ small even for slow neutrons and thus the ~~energy~~ distribution is nearly constant for ~~slow neutrons,~~ ~~energy small compared with small~~ the range of the interval of energy small compared with E_0, in contrast to the previous case.



Similarly the number per unit time of neutrons which are reflected ~~is smaller than~~ with energy between E and $E+dE$ will be smaller than those which are transmitted ~~and~~ for ~~large energy~~ fast neutrons and will be about the same as (32) for slow neutrons.

and reflects or transmits only a small part, is attributed to the smallness of probability of scattering in comparison with that of capture with γ-ray emission. In such a case, only single scattering occurs with appreciable frequency, so that the coefficient of reflection becomes approximately

for the absorber of thickness h, where n is the number of atoms per unit volume of the plate and σ_s and σ_c are the cross sections for scattering and absorption respectively, the latter being large compared with the former. For thick plate, ~~for which h is large compared with $\frac{1}{n\sigma_c}$~~, the coefficient of reflection reduces to

$$0.15\,\frac{\sigma_s}{\sigma_c} \ll 1.$$

Similarly, the coefficient of transmission becomes

(9)

which is smaller than

General The curves for R and P is plotted in Fig. 5.

Fig. 5

$$f\left(\frac{E}{E_0}\right) = \qquad f(x) = \frac{a^{\frac{1}{2}} b \left(y - \frac{a+b}{2} x \right)}{\left(a + \frac{b-a}{2} x^2 - x y\right)\left(b - \frac{b-a}{2} x^2 - x^2 y\right)^{\frac{3}{2}}}$$

$$y = \left\{ab + \left(\frac{a-b}{2}\right)^2 x^2\right\}^{\frac{1}{2}}$$

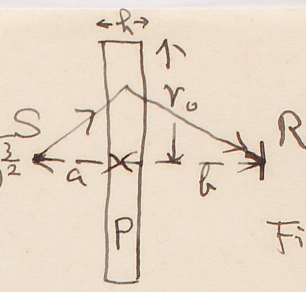


Fig. 5

§ 9. ~~Scattering~~ Slowing Down of Neutrons from a Point Source by a ~~circular~~ Thin Plate

~~In~~ ~~We want~~ In this section, we want to ~~deal with~~ apply ~~another simple example,~~ the following ~~problem~~, similar problem, a more concrete problem, which is ~~more practical than the previous one.~~ similar to that in the previous section, but ~~consider a circular disc~~

As shown in Fig. 4, a point source S emitting N neutrons of energy E per unit time is placed on the axis at a distance a from the centre of a circular plate P containing hydrogen of radius r_0 and thickness h, ~~which is~~ the latter being small compared with the mean free path λ_0 of primary neutrons. We wish to find the energy distribution of neutrons, which hit a small ~~and thin~~ ditecting plate R with surface area s, placed parallel to P at a distance b from the centre of P on th side opposite to S. We consider the case, when the relation $ab < r_0^2$ is satisfied, so that the neutrons slowed down to small velocity by single scattering can ~~hit~~ reach R. In this case, the number of neutrons, which hit R per unit time with energy between E and E+dE after having ~~deflected~~ scattered once ~~by~~ in P, is given approximately by

$$N(E)dE \doteqdot \frac{4\pi s h N_0}{\lambda_0 a^{\frac{1}{2}}(a+b)^{\frac{3}{2}}} \left(\frac{E}{E_0}\right)^{\frac{1}{2}} \left(1-\frac{E}{E_0}\right)^{\frac{1}{2}} \frac{dE}{E_0} f\left(\frac{E}{E_0}\right), \qquad \text{(33)} ~~(29)~~$$

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provided that, the mean free path for energy E, ~~is large in comparison with h.~~ h is small compared with, where ~~f(x)~~ ~~In this expression~~ Even $f\left(\frac{E}{E_0}\right) = f(x) =$ . .

If $E \ll E_0$, this is reduced to

$$N(E)dE \doteqdot \frac{4\pi s h N_0}{\lambda_0 a^{\frac{1}{2}}(a+b)^{\frac{3}{2}}} \left(\frac{E}{E_0}\right)^{\frac{1}{2}} \frac{dE}{E_0} \qquad \text{(34)} ~~(30)~~$$

for sufficiently small h.

so that the number of slow neutrons per unit energy range, which hit the detector, decreases as the energy tends to zero. Thus, the average energy of a certain group of slow neutrons, which activate the detector strongly

$$\lim_{x \to 0} f(x) = 1$$

$$\lim_{x \to 1} f(x) = \frac{(a+b)^{\frac{3}{2}}}{a^{\frac{1}{2}} b} \cdot \frac{1}{\left(1-\frac{E}{E_0}\right)^{\frac{3}{2}}} \cdot \frac{a^2 b^2}{(a+b)^2} \left(1 - \frac{E}{E_0}\right) \qquad \frac{ab}{(a+b)^2}$$

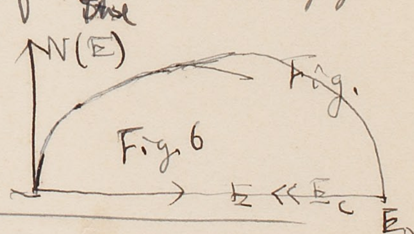
can be estimated from the rate of increase of activity with thickness h at the limit $h=0$, if the detector is thick enough to absorb almost all the impinging neutrons of this group.

These conclusions seems to be in agreement with the experiment of Nishikawa, Nakagawa and Sumoto[1)] on the slowing down of neutrons from a Rn-Be source by thin layers of paraffin, which shows that the rate of increase of activity of Ag (or I) detector with the thickness is nearly quadratic, for neutrons of C group with thermal energy, whereas it is undoubtedly linear for those of A group with larger energy. For, if the expression (30) is correct, the ratio of the numbers of A and C groups will be have the order of magnitude

$$\frac{E^{\frac{1}{2}}\Delta E}{kT^{\frac{3}{2}}}, \qquad (35)$$

where E and ΔE are the mean energy and the energy breadth of A group, and (31) will be large compared with 1, unless ΔE is small compared with kT.

Further, we can estimate the breadth ΔE of a group from the data of such an experiment, if the mean energy E is determined from other experiments.



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In conclusion, the author should acknowledge

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1) Proc. Imp. Acad. Tokyo 12, 128.

From these formulae, the dependence of P(E,h) and R(E,h) on h in the neighborhood of ϕ 0 is, ~~also apparent and the~~ determined (shown in Fig. 2 and Fig. 3) curves for a certain fixed value of E. ~~have~~ general forms as ~~written~~ plotted in Fig. 2 ~~and Fig. 3.~~

The inclinations of these curves at $h = 0$ can be determined at once from (24), (25), (26) and (27) respectively. For example, the angle θ is common to P(E,h) and R(E,h) for $E \ll E_c$ (Fig. 3) and is given by

$$\tan\theta \cong \frac{1}{2\lambda_0'} \log\left(\frac{E_c}{E}\right). \qquad (28)$$

~~for E ≪ E.~~

~~Thus~~ Now the numerouscity average energy of a certain group, ~~the resonance band~~ of slow neutrons, which activate a certain detector, can be estimated, ~~irrespective of the breadth of the band~~, from the rate of increase of activity with the thickness of the thin layer containing hydrogen, provided that the primary fast neutrons ~~have~~ are nearly homogeneous and their direction is normal to the plate. ~~In this case, if the inclinations~~

Now, the number of slow neutrons ~~whi~~ of a certain group, with energy E and ~~which~~ activate a certain det can be ~~esti~~ given by integrating