Fast Neutron

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① Proton による Single Scattering

$$\int_{z=0}^{h} e^{-\frac{z}{\lambda_0}} \frac{dz}{\lambda_0'} \, 2\sin\theta\cos\theta\, d\theta \; e^{-\frac{(h-z)}{\lambda\cos\theta}}$$

$$= \frac{2v\,dv}{v_0^2\lambda_0'} \frac{e^{-\frac{h}{\lambda_0}} - e^{-\frac{h v_0}{\lambda v v_0}}}{\frac{v_0}{\lambda v v_0} - \frac{1}{\lambda_0}}$$

$$\cong \frac{2v\,dv\,h}{v_0^2\lambda_0'}\left\{1 + \frac{h}{2}\left(\frac{v_0}{\lambda v} + \frac{1}{\lambda_0}\right)\right\}$$

$$\cong \boxed{\frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}\left\{1 - \frac{h}{2\lambda_0}\left(\frac{v_0}{v}+1\right)\right\}}$$

② Proton による Double Scattering

$$\int_{z=0}^{h}\int_{z'=z}^{h} e^{-\frac{z}{\lambda_0}} \frac{dz}{\lambda_0'} \, 2\sin\theta\cos\theta\, d\theta \; e^{-\frac{(z'-z)}{\lambda\cos\theta}}$$

$$\times \frac{dz'}{\lambda'\cos\theta} \frac{2\sin\gamma\cos\gamma\, d\gamma\, d\phi}{2\pi} e^{-\frac{h-z'}{\lambda\cos\theta'}}$$

$$= \frac{2\sin\theta\,\cancel{\cos\theta}\,d\theta}{\lambda_0'} \frac{2\sin\gamma\cos\gamma\,d\gamma\cdot d\phi}{2\pi\lambda''(v^3)} \frac{1}{\frac{1}{\lambda\cos\theta'} - \frac{1}{\lambda\cos\theta}}$$

$$\left\{\frac{e^{-\frac{h}{\lambda_0}} - e^{-\frac{h}{\lambda\cos\theta}}}{\frac{1}{\lambda\cos\theta} - \frac{1}{\lambda_0}} - \frac{e^{-\frac{h}{\lambda_0}} - e^{-\frac{h}{\lambda\cos\theta'}}}{\frac{1}{\lambda\cos\theta'} - \frac{1}{\lambda_0}}\right\}$$

$v' > \frac{v_0}{2}$

$$\frac{2v'dv'}{v_0^2}\int_0^{2\pi}\frac{d\phi}{2\pi}\int_{v'}2\frac{dv}{v^2}\frac{h}{2\lambda_0'\lambda'}$$

$$\fallingdotseq \frac{2v'dv'}{v_0^2}\left(\frac{v_0}{v'}-1\right)\frac{h^2}{\cancel{2}\lambda_0'\lambda'}$$

$$\fallingdotseq \frac{2v'dv'}{v_0^2}\frac{h}{\lambda_0'}\left\{\frac{h}{\cancel{2}\lambda_0'}\left(\frac{v_0}{v'}-1\right)\right)$$

$$\boxed{\frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}\frac{h}{\cancel{2}\lambda_0'}\left(\frac{v_0}{v'}-1\right)}$$

$$\{p_1(v)+p_2(v)\}dv = \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}$$

$$\times\left\{1 - \frac{h}{2\lambda_0}\left(\frac{v_0}{v}+1\right)\cancel{+\frac{h}{2\lambda_0'}\left(\frac{v_0}{v}-1\right)}\right\} = \{1 -$$

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③ 又 proton が次に heavy nucleus に scatterされる;

$$e^{-\frac{z}{\lambda_0}}\frac{dz}{\lambda_0'}\,2\sin\theta\cos\theta\,d\theta\cdot e^{-\frac{(z'-z)}{\lambda\cos\theta}}$$

$$\times\frac{dz'}{\lambda''\cos\theta}\,\frac{\cancel{2}\sin\gamma\,d\gamma\,d\phi}{4\pi}\,e^{-\frac{h-z'}{\lambda\cos\theta'}}$$

$$=\frac{2\sin\theta\,d\theta}{\lambda_0'}\,\frac{\sin\gamma\,d\gamma\,d\phi}{4\pi\lambda''}\,\frac{1}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda\cos\theta}}$$

$$\left\{\frac{e^{-\frac{h}{\lambda_0}}-e^{-\frac{h}{\lambda\cos\theta}}}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}}-\frac{e^{-\frac{h}{\lambda_0}}-e^{-\frac{h}{\lambda\cos\theta'}}}{\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda_0}}\right\}$$

$v > \frac{v_0}{2}$

$$\cong \frac{2\cancel{v}\,dv}{v_0}\cdot\frac{h^2}{2\lambda_0'\lambda_0''}\frac{1}{2}$$

$$= \boxed{\frac{2v\,dv}{v_0^2}\cdot\frac{h^2}{2\lambda_0'\lambda_0''}\frac{v_0}{2v}}$$

$$(p_1+p_2+p_2')\,dv = \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}\left\{1-\frac{h}{2\lambda_0''}\right.$$

$$\left(\frac{v_0}{v^2}-1-\frac{v_0}{2v}\right) = \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}\left\{1+\frac{h}{2\lambda_0''}\right.$$

$$\left.\times\left(1-\frac{v_0}{2v}\right)\right\}$$

4 先 heavy ~~pr~~nucleus に 次に proton に scatter される。

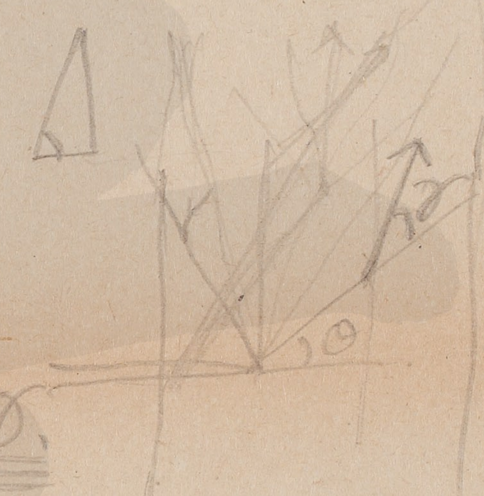
$$e^{-\frac{z}{\lambda_0}}\frac{dz}{\lambda_0''}\cdot\frac{\sin\theta\,d\theta}{2}\cdot e^{-\frac{z'-z}{\lambda\cos\theta}}$$

$$\times\frac{dz'}{\lambda_0'\cos\theta}\cdot\frac{2\sin\gamma\cos\gamma\,d\gamma\,d\phi}{2\pi}\cdot e^{-\frac{h-z'}{\lambda\cos\theta'}}$$

$\theta < \frac{\pi}{2}$. $\underline{\cos\theta' = \cos\theta\cos\gamma + \sin\theta\sin\gamma\cos\phi}$

$$\frac{\sin\theta\, d\theta}{2\,\cancel{\cos}\,\theta} \cdot \frac{2\sin\gamma\cos\gamma\, d\gamma\, d\phi}{2\pi \quad \lambda_0'' \lambda_0'} \quad \frac{1}{\frac{1}{\lambda\cos\theta'} - \frac{1}{\lambda\cos\theta}}$$

$$\left\{ \frac{e^{-\frac{h}{\lambda_0}} - e^{-\frac{h}{\lambda\cos\theta}}}{\frac{1}{\lambda\cos\theta} - \frac{1}{\lambda_0}} - \frac{e^{-\frac{h}{\lambda_0}} - e^{-\frac{h}{\lambda\cos\theta'}}}{\frac{1}{\lambda\cos\theta'} - \frac{1}{\lambda_0}} \right\}$$



$\nu > \frac{\nu_0}{2}$ $\quad \cos\gamma\ (<) > \frac{1}{2}$

$\gamma \lessgtr \frac{\pi}{3}$

$\cos\phi > -\cot\theta\, \underline{\underline{\cot\gamma}}$.

$\left(> -\cot\theta \cdot \frac{1}{\sqrt{3}} \right)$

$\frac{1}{\sqrt{1 - \frac{\nu_0}{4\nu}}} = \frac{1}{\sqrt{3}}$ $\quad \sqrt{\frac{3}{4}}$

$|\cos\phi| < \cot\theta\, \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6} \quad \cot\theta = \sqrt{3}$

$$\frac{2\nu\, d\nu}{\nu_0^2} \left\{ \left(\frac{\pi}{2}\right) \int_0^{\frac{\pi}{2} - \frac{h}{\lambda_0}} \frac{\sin\theta\, d\theta}{2\cos\theta} \cdot \frac{h^2}{2\lambda_0'\lambda_0''} \right.$$

$$\left. + \frac{1}{2} \frac{h\lambda}{\lambda''\lambda'} \cdot \frac{1}{2} \cdot \frac{h}{\lambda_0} \right\} = \frac{\nu\, d\nu}{\nu_0^2} \frac{h^2}{2\lambda_0''\lambda_0'}$$

$$\left\{ \frac{1}{2}\log\frac{\lambda_0}{h} + 1 \right\}$$

$\theta > \frac{\pi}{2}$

$$e^{-\frac{h}{\lambda\cos\theta'}}\int_0^h e^{\left(\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}\right)z}dz\int_0^z e^{\left(\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda\cos\theta}\right)z'}dz'$$

$$=\frac{e^{-\frac{h}{\lambda\cos\theta'}}}{\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda\cos\theta}}\int_0^h\left\{e^{\left(\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda_0}\right)z}-e^{\left(\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}\right)z}\right\}dz$$

$$=\frac{e^{-\frac{h}{\lambda\cos\theta'}}}{\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda\cos\theta}}\left\{\frac{e^{\left(\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda_0}\right)h}-1}{\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda_0}}-\frac{e^{\left(\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}\right)h}-1}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda_0}}\right\}$$

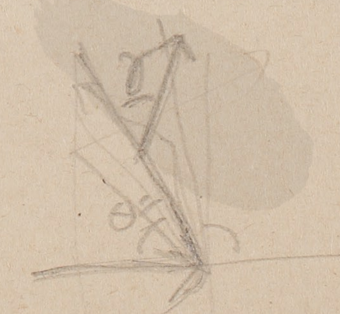
$$=\frac{e^{\mp\frac{h}{\lambda\cos\theta'}}}{\frac{1}{\lambda\cos\theta'}+\frac{1}{\lambda\cos\theta''}}\left\{\frac{e^{\left(\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda_0}\right)h}-1}{\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda_0}}\mp\frac{1-e^{-\left(\frac{1}{\lambda\cos\theta''}+\frac{1}{\lambda_0}\right)h}}{\frac{1}{\lambda\cos\theta''}+\frac{1}{\lambda_0}}\right\}$$

$\frac{\pi}{2}$ $\int$ $\frac{\pi}{2}-\frac{h}{\lambda_0}$

$+\frac{1}{2}\int$

$\frac{\pi}{2}-$

$\frac{\pi}{2}-\gamma$



$$=\frac{v\,dv}{v_0^2}\frac{h^2}{2\lambda_0\lambda_0'}\left\{1+\frac{1}{2}\log\frac{\lambda_0}{h}-\frac{1}{2}\log\frac{1}{\sqrt{1-\frac{h^2}{\lambda_0^2}}}\right\}$$

$\theta''+\gamma=\frac{\pi}{2}$

$\frac{\pi}{2}$

$$p_2''(v)\,dv = \boxed{2}\frac{v\,dv}{v_0^2}\frac{h}{2\lambda_0'\lambda_0''}\left\{\cancel{2}\,1+\frac{1}{2}\log\frac{\lambda_0}{h}\right.$$

$$\left. - \frac{1}{4}\log\frac{1}{\sqrt{1-\frac{v^2}{v_0^2}}}\right\}$$

$$(p_1+p_2+p_1'+p_2')\,dv = \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}\left[1\right.$$

$$\left. + \frac{h}{2\lambda_0''}\left\{2 - \frac{v_0}{2v} + \frac{1}{2}\log\frac{\lambda_0}{h}\left(1-\left(\frac{v}{v_0}\right)^2\right)^{\frac{1}{4}}\right\}\right]$$

$$\frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}\left\{1 - \frac{h}{2\lambda_0}\left(\frac{v_0}{v}+1\right) + \frac{h}{\lambda_0'}\left(\frac{v_0}{v}-1\right)\right.$$

$$\left. + \frac{h}{4\lambda_0''}\frac{v_0}{v} + \frac{h}{4\lambda_0''}\left(\cancel{1+}\,2 + \log\frac{\lambda_0}{h}\sqrt{1-\frac{v^2}{v_0^2}}\right)\right\}$$

$$= \frac{2v\,dv}{v_0^2}\frac{h}{\lambda_0'}\left[1 \cancel{+ \frac{h}{2\lambda_0'}\left(\frac{v_0}{v}-3\right)} - \frac{h}{2\lambda_0'}\left(3-\frac{v_0}{v}\right)\right.$$

$$\left. + \frac{h}{4\lambda_0''}\left\{\log\left(\frac{\lambda_0}{h}\sqrt{1-\frac{v^2}{v_0^2}}\right) - \frac{v_0}{v}\right\}\right]$$