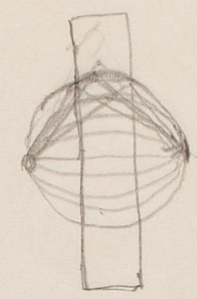
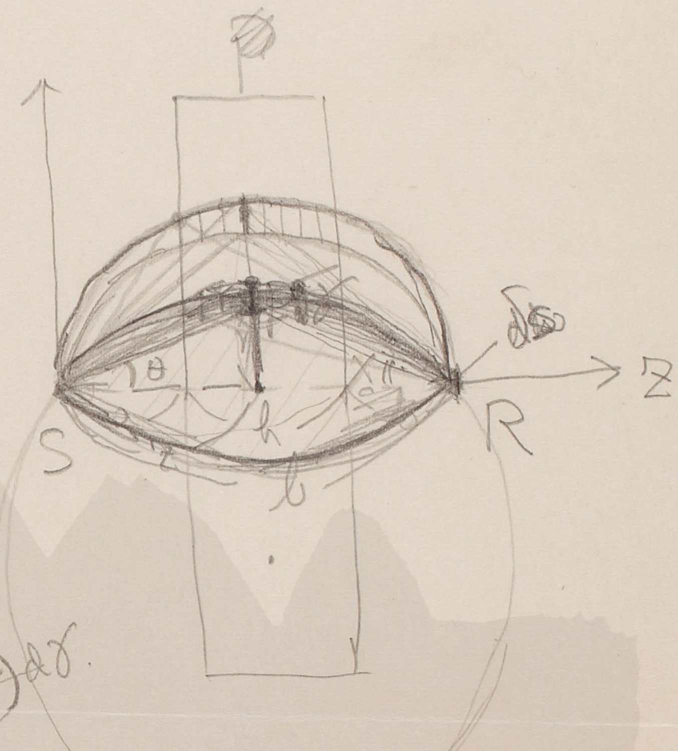
○ Circular Plate による scattering



Sから出て、P点 (~~r~~ r, z) で
scatter されて R へ来って
出た scattering は γ だけ deflection をする。
但し、γ は



~~z^2+h~~

~~$(z^2+r^2)+(b-z)^2+r^2$~~

~~$+2\sqrt{z^2+r^2}\sqrt{(b-z)^2}$~~

$$\gamma = \theta + \chi$$

$$\tan\theta = \frac{r}{z}, \quad \tan\chi = \frac{r}{b-z},$$

$$\cos\theta = \frac{z}{\sqrt{z^2+r^2}} \qquad \cos\chi = \frac{b-z}{\sqrt{(b-z)^2+r^2}}$$

$$\frac{v_0}{v_0} = \cos\gamma$$

$$dk = (1+k^2)\,d\gamma$$

$$k = \tan\gamma = \frac{v_0\sqrt{1-\frac{v^2}{v_0^2}}}{v}$$

$$= \frac{\tan\theta+\tan\chi}{1-\tan\theta\tan\chi} = \frac{\frac{br}{z(b-z)}}{\frac{z(b-z)-r^2}{z(b-z)}} = \frac{br}{z(b-z)-r^2} > 0$$

$$r^2 < z(b-z).$$

故に

$$kr^2+br-kz(b-z)=0$$

$$r = \frac{-b\pm\sqrt{b^2+4k^2z(b-z)}}{2k}$$

+をとる。

$$= \frac{-b+\sqrt{b^2+4k^2z(b-z)}}{2k} \cong$$

$z = \frac{b}{2}$

$$= \frac{b\{\sqrt{(1+k^2)}-1\}}{2k}$$

$$= \frac{b}{2k}\left\{\sqrt{1+k^2\,\frac{z}{b}\left(1-\frac{z}{b}\right)}+1-1\right\} =$$

$z = a + \frac{b}{2}$

$$\frac{-b+\sqrt{b^2+4k^2\cdots}}{2k}$$

$$\frac{b\{k-1\}}{2k}$$

$z(b-z)$

(z, r) の所の dr を通る neutron の数は

$$N_0 e^{-\frac{z-a}{\lambda_0\cos\theta}} \cdot \frac{dr\cdot\cos\theta}{z^2+r^2}$$

その中、r が deflection をやつて R の所の δs が area (cross section) を通る数は、

$$\times \frac{dz}{\lambda_0\cos\theta}\,\frac{2\cos\gamma}{(b-z)^2+r^2}\,\delta s\cdot\cos\chi\cdot e^{-\frac{h+a-z}{\lambda_k\cos\chi}}$$

velocity に平の neutron の通る数といへば、上式を、

$$\frac{b}{2} > r > \frac{b}{2}\left\{\frac{\sqrt{4k^2\frac{z}{b}\left(1-\frac{z}{b}\right)+1}-1}{k}\right\}$$

$$a > z > a+h$$

の間で積分すればよい

$$e^{-\frac{z-a}{\lambda_0\cos\theta}} \cong 1. \qquad e^{-\frac{h+a-z}{\lambda_k\cos\chi}} \cong 1-\frac{h+a-z}{\lambda_k\cos\chi}$$

とおく。

$$\frac{N_0\delta s}{\lambda_0}\iint\frac{dr\cdot dz}{z^2+r^2}\,\frac{2\cos\gamma\cos\chi}{\{(b-z)^2+r^2\}(z^2+r^2)} + \cdots$$

$$= \frac{N_0}{\lambda_0}\delta s\iint dz\int dr\,\frac{2(b-z)\,dz\,\cos\gamma}{\{(b-z)^2+r^2\}^{\frac{3}{2}}(z^2+r^2)}$$

$$dk\{r^2 - z(b-z)\} + 2kr\,dr + b\,dr = 0.$$

$$\frac{dr}{dk} = \frac{r^2 - z(b-z)}{2kr+b}$$

$$= \frac{1}{k}\cdot\frac{-br}{\sqrt{\quad}}$$

$$2kr\frac{dr}{dk} + b\frac{dr}{dk} - z(b-z) = 0.$$

$$\frac{dr}{dk} = \frac{z(b-z) - r}{2kr+b} = +\frac{b}{k}\,\frac{\sqrt{\cdots} - b}{\sqrt{b^2+4k^2 z(b-z)}}$$

$$\delta s\frac{N_0\,dk}{\lambda_0}\int_{z=a}^{a+h} e^{-\frac{z-a}{\lambda_0\cos\theta}}\,\frac{2\cos\gamma\cos\chi\,dz}{(z^2+r^2)((b-z)^2+r^2)}\,e^{-\frac{h+a-z}{\lambda_k\cos\chi}}$$

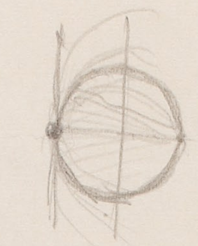
$$\times\frac{b}{k}\,\frac{\sqrt{\cdots}-b}{\sqrt{\cdots}}\,dz$$

$$= \delta s\frac{N_0}{\lambda_0}\frac{2b\,dk}{k(1+k^2)}\cdot\int_{z=a}^{a+h}\frac{(b-z)\,dz}{(z^2+r^2)((b-z)^2+r^2)}$$

r = const $= r_k$ とすると

$$\delta s\frac{N_0}{\lambda_0}dk\int$$

$$\frac{2N_0\delta s}{\lambda_0}\frac{dk}{(2kr_k+b)(1+k^2)}\cdot\int^{a+h}_{a}\frac{(z(b-z)-r_k^2)\,dz}{(z^2+r_k^2)\{(b-z)^2+r_k^2\}}\quad\frac{(b-z)}{\sqrt{(b-z)^2+r_k^2}}$$



$$(b-a-\tfrac{h}{2})$$

$$\cong\frac{2N_0\delta s\,h}{\lambda_0}\cdot\frac{dk}{(2kr_k+b)(1+k^2)}\frac{\{(a+\frac{h}{2})(b-a-\frac{h}{2})-r_k^2\}}{\{(a+\frac{h}{2})^2+r_k^2\}\{(b-a-\frac{h}{2})^2+r_k^2\}}$$

$$\times\frac{b\,r_k}{k}\qquad r_k\cong$$

$$\frac{E}{E_0}=\frac{1}{1+k^2}\qquad\frac{1}{k}=\frac{\frac{E}{E_0}}{\sqrt{1-\frac{E}{E_0}}}$$

$$dk=d\left(\frac{E}{E_0}\right)\left(1-\frac{E}{E_0}\right)^{\frac{1}{2}}\}$$

$$=\left\{-\frac{1}{2}\left(\frac{E}{E_0}\right)^{-\frac{3}{2}}\left(1-\left(\frac{E}{E_0}\right)\right)^{\frac{1}{2}}\right.$$

$$\left.-\frac{1}{2}\left(\frac{E}{E_0}\right)^{-\frac{1}{2}}\left(1-\frac{E}{E_0}\right)^{-\frac{1}{2}}\right\}d\left(\frac{E}{E_0}\right)$$

$$=\frac{1}{2\left(\frac{E}{E_0}\right)^{\frac{3}{2}}\sqrt{1-\frac{E}{E_0}}}\,d\left(\frac{E}{E_0}\right)\qquad\left(1-\frac{E}{E_0}\right)+\left(\frac{E}{E_0}\right)$$

$$\cong\frac{N_0\delta s\,h}{\lambda_0}\cdot\frac{\frac{E}{E_0}\,d\left(\frac{E}{E_0}\right)}{\sqrt{1-\frac{E}{E_0}}}\cdot\frac{b\,(b-a-\frac{h}{2})\,r_k}{\{(a+\frac{h}{2})^2+r_k^2\}\{(b-a-\frac{h}{2})^2+r_k^2\}\{2kr_k+b\}}\qquad\left(\frac{b}{2}\right)^2=$$

slow neutron $k\to\infty$, $kr_k\simeq\frac{b}{2}$, $a+\frac{h}{2}=\frac{b}{2}$, $b-a-\frac{h}{2}=\frac{b}{2}$

とおく。 $$\cong\frac{N_0\delta s h}{\lambda_0}\frac{\frac{E}{E_0}d\left(\frac{E}{E_0}\right)}{(1-\frac{E}{E_0})^{\frac{3}{2}}}\frac{b\frac{b}{2}\cdot\frac{b}{2}}{b\frac{b^2}{4}\cdot\frac{b^2}{2}}\cong\frac{N_0\delta s h}{\lambda_0 b^2}\cdot\frac{E}{E_0}d\left(\frac{E}{E_0}\right)$$

$$-\frac{h+a-z}{\lambda_k \cos\chi}, \quad -\frac{\sqrt{(b-z)^2+r_k^2}}{\lambda_k (b-z)} (h+a-z) \Big|_a^{a+h}$$

$$\int_a^{a+h} (h+a-z)\, dz = (h+a)h - \frac{(a+h)^2}{2} + \frac{a^2}{2}$$

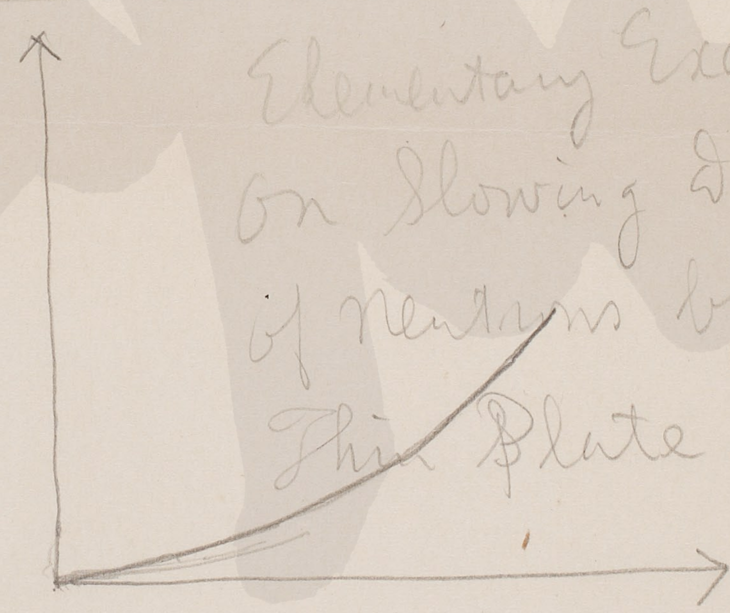
$$= \frac{h^2}{2}$$

$$\sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

$$b - \frac{l}{2}$$

slow neutron.

$$\frac{N_0 \delta S h}{\lambda_0 b^2} \frac{E}{E_0} d\left(\frac{E}{E_0}\right) \left\{1 - \frac{h}{\sqrt{2}\lambda_k}\right\}$$



Elementary Examples on Slowing down of Neutrons by Thin Plate.

§ Scattering of Neutrons from a Point Source by ~~Circular~~ Thin Plate.