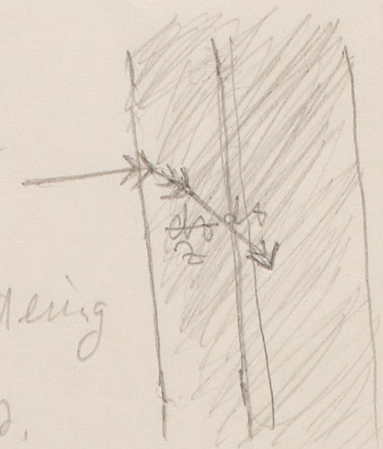
 8(+1)

◎ Capture が ~~大き~~ 少なくて scattering ~~の小さい場合~~ と一緒に起るものは
(例へば slow neutron が Cadmium Sheet を通る場合)

一種類の heavy nucleus を考へ、これによつて elastic scattering と capture をされるとする。elastic scattering は spherically symmetrical であるとする。



t なる時刻に z なる深さの dz の間で unit cross section について ~~(v, v+dv)~~ $(\theta, \theta+d\theta)$ $(\phi, \phi+d\phi)$ の neutron の数を (velocity は v)

$$\rho(z, \theta, t)\, dz\, \sin\theta\, d\theta\, d\phi$$

とする。dt (但し $v\,dt \ll dz$) なる時間の間に之等は scattering によつて

$$(-)\quad \frac{v\,dt}{\lambda_s}\,\rho(z,\theta,t)\,dz\,\sin\theta\,d\theta\,d\phi$$

だけ減り、一方 scattering によつて

$$(+)\quad \iiint \rho(z,\theta',t)\,dz\,\cancel{\sin\theta' d\theta' d\phi'}\ \sin\theta\, d\theta\, d\phi \cdot \frac{v\,dt}{\lambda}\cdot\frac{\sin\gamma\, d\gamma\, d\chi}{4\pi}$$

$\frac{d\phi}{2\pi}$

$\frac{2}{3}\frac{\lambda_c}{\lambda_0}\left(\frac{v_c}{v_0}\right)\ \left(\frac{h}{\lambda_c}\right)\chi$

だけ増える。但し

$$\boxed{\cos\theta = \cos\theta'\cos\gamma + \sin\theta'\sin\gamma\cos\chi}$$

$\cos\theta'\sin\gamma$

$\sin\theta\,\frac{\partial\theta}{\partial\gamma} = \sin\theta'\cos\gamma\cos\chi$

$\int_0^{v_c}$

$\frac{v_c}{v_0}\frac{2\lambda_c}{\lambda_0} = \frac{h}{\lambda_c}\cdot\frac{2\lambda_c}{\lambda_0} = \frac{2h}{\lambda_0}$

$v_c = \frac{h v_0}{\lambda_c}$

~~$d\theta'\,dv'\ \sin\theta\,d\theta\,dv =$~~

$\sin\theta\, d\theta'\, d\theta = \begin{pmatrix} 1 & & \\ & 1 & \\ & & \frac{\partial\theta}{\partial\gamma} \end{pmatrix} d\theta'\, d\gamma\, d\chi = \cancel{\sin\theta'\sin\gamma\sin\chi\, d\theta' d\gamma d\chi}$

$= (\cos\theta'\sin\gamma - \sin\theta'\cos\gamma\cos\chi)\ d\theta\, d\chi\, d\gamma$

$= (\cot\theta' - \cot\gamma\cos\chi)\, d\theta'\, d\chi\, d\gamma$

$$\iiint = \iiint \rho(z',\theta',t)\, dz \,\sin\theta\, d\theta\, d\theta'\, d\chi$$

$$\times \frac{v\,dt}{\lambda} \cancel{\frac{1}{4\pi \sin\chi}} \cdot \frac{1}{2\sin\chi} \cdot \frac{d\phi}{2\pi}$$

$$\times \frac{v\,dt}{\lambda}\ \frac{d\phi}{4\pi}\,,\ \frac{1}{(\cot\theta' - \cot\gamma\cos\chi)}$$

$$\cancel{\cos\theta\left(1+\frac{\dots}{\dots}\right) = \cos\theta' + \sin\theta' \tan\dots}$$

$$\cos^2\theta(1+\cot^2\gamma) = (\cos\theta'\cot\gamma + \sin\theta'\cos\chi)^2$$

$$= \cos^2\theta'\cot^2\gamma + 2\sin\theta'\cos\theta'\cos\chi\cdot\cot\gamma + \sin^2\theta'\cos^2\chi$$

$$(\cos^2\theta' - \cos^2\theta)\cot^2\gamma + 2\sin\theta'\cos\theta'\cos\chi\cot\gamma + (\sin^2\theta'\cos^2\chi - \cos^2\theta)$$

$$\cot\gamma = \frac{-\sin\theta'\cos\theta'\cos\chi \pm \sqrt{\sin^2\theta'\cos^2\theta'\cos^2\chi - \dots + \cos^2\theta'\cos^2\theta + \cos^2\theta'\sin^2\theta'\cos^2\chi - \cos^4\theta}}{\cos^2\theta' - \cos^2\theta}$$

$$\pm\cos\theta\sqrt{\cos^2\theta' + \sin^2\theta'\cos^2\chi - \cos^2\theta}$$

boundary の出入は

$$(-)\ \frac{\partial\rho}{\partial z}\, v\cos\theta\, dt\, dz\, \sin\theta\, d\theta\, d\phi$$

capture による

$$(-)\ \frac{v\,dt}{\lambda'_c}\, x$$

$$\therefore \left(\frac{1}{\lambda} + \frac{1}{\lambda'}\right)\rho + \frac{\partial\rho}{\partial z}\cos\theta$$

$$N_0 e^{-\dots z}\ \underset{\delta(\theta)}{} + \frac{1}{\lambda}\iiint \underset{\rho}{\rho(z,\theta',t)}\ \sin\theta'\, d\theta'\, d\phi' \dots$$

$$\frac{\partial \rho}{\partial z}\cos\theta + \frac{1}{\lambda'}\rho = \cancel{\circ N_0 e^{-\left(\frac{1}{\lambda}+\frac{1}{\lambda'}\right)z}}\,\delta(\theta).$$

$$\boxed{\int_0^{\varepsilon}\delta(\theta)\,d\theta = 1.}$$

$$\cancel{x = z\cos\theta \quad y = z\sin\theta.}$$

$$\cancel{\frac{\partial}{\partial z} = \cos\theta\frac{\partial}{\partial x} + \sin\theta\frac{\partial}{\partial y}}$$

$$\frac{\partial}{\partial y} =$$

$$\rho = Z(z)\,\Theta(\theta)$$

$$\frac{1}{Z(z)}\frac{dZ}{dz} + \frac{1}{\lambda'\cos\theta} = N_0 \cancel{e^{-\left(\frac{1}{\lambda}+\frac{1}{\lambda'}\right)z}\delta(\theta)}\Big/\cos\theta\, Z(z)\,\Theta(\theta)$$

$$\rho = e^{-\frac{\mu z}{\cos\theta}}\Theta_\mu(\theta). = e^{-\frac{z}{\lambda'\cos\theta}}\Theta(\theta)$$

$$\cancel{-\mu\cos\theta + \frac{1}{\lambda'}}$$

$$\rho = \cancel{\frac{1}{\frac{1}{\lambda}+\frac{1}{\lambda'}}}N_0\, e^{-\left(\frac{1}{\lambda}+\frac{1}{\lambda'}\right)z}\delta(\theta) + \cancel{C}\,e^{-\frac{z}{\lambda'\sin\theta}}\Theta(\theta).$$

$$\cancel{-N_0\left(\frac{1}{\lambda}+\frac{1}{\lambda'}\right)\delta(\theta) +}$$

$h = 0$: $\sim$

$$\frac{\lambda}{2\lambda_s} e^{-\frac{2h}{x}} \int_0^\infty \frac{e^{-\left(\frac{h}{x}\right) y}}{(2+y)} \frac{dy}{(y+1)^2}$$

$$= \int_0^\infty \frac{e^{-\frac{h}{x} y}}{(2+y)(1+y)^2}\,dy$$

$$\frac{1}{x} - 1 = y$$

$$\int_0^\infty \frac{e^{-\frac{h}{x} y}\, dy}{y\,(y+1)^2}$$

$$\int_0^\infty \frac{1 - e^{-\frac{h}{x} y}}{y(1+y)^2}\,dy$$

$$= \int \left( \frac{a}{y} + \frac{b + cy}{(1+y)^2} \right) \cdots$$

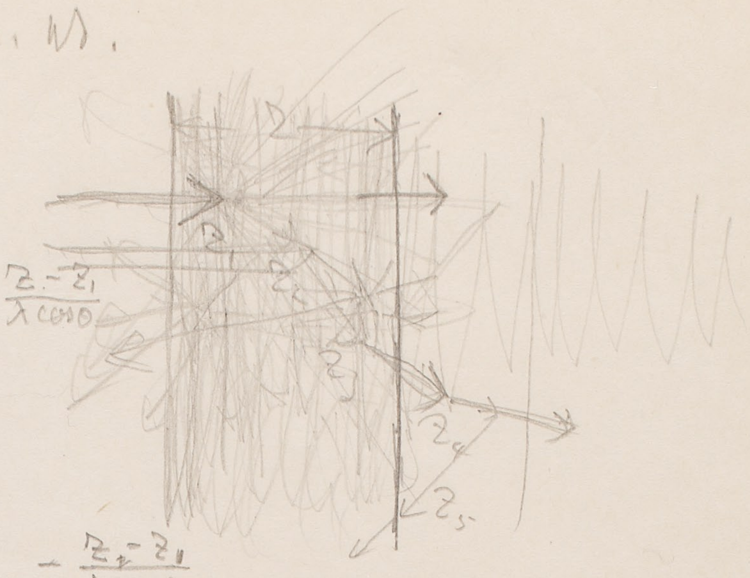
$$= \int_0^\infty \left\{ \frac{1}{y} - \frac{(2+y)}{(1+y)^2} \right\} (\cdots)\, dy$$

~~$a + b + 2a + c$~~

$a + (2a+b)y + (a+c)y^2$

$a = 1$, $b = -2$, $c = -1$.

○ ~~scattering~~ Capture ガ infinitely に多い時.





① $e^{-\frac{z}{\lambda}}$

① $\int_0^z e^{-\frac{z_1}{\lambda}} \cdot \frac{\sin\theta d\theta}{2} \cdot \frac{dz_1}{\lambda_s \cos\theta} \cdot e^{-\frac{z-z_1}{\lambda\cos\theta}}$

for $\theta \leq \frac{\pi}{2}$

$\int_z^h e^{-\frac{z_1}{\lambda}} \frac{\sin d\theta}{2} \frac{dz_1}{(-\lambda_s \cos\theta)} e^{-\frac{z_1-z_1}{\lambda\cos\theta}}$ ~~$\frac{dz_1}{2X}$~~ +

for $\theta > \frac{\pi}{2}$

② $e^{-\frac{z}{\lambda\cos\theta}} \dfrac{e^{(\frac{z}{\lambda\cos\theta}-\frac{1}{\lambda})z}-1}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda}} \frac{\sin\theta d\theta}{2} \frac{1}{\lambda_s\cos\theta}$ $\theta < \frac{\pi}{2}$

$e^{-\frac{z}{\lambda\cos\theta}} \dfrac{e^{-(\frac{1}{(-\lambda\cos\theta)}+\frac{1}{\lambda})z} - e^{-(\frac{1}{(-\lambda\cos\theta)}+\frac{1}{\lambda})h}}{\frac{1}{(-\lambda\cos\theta)}+\frac{1}{\lambda}} \frac{\sin\theta d\theta}{2} \frac{1}{(-\lambda_s\cos\theta)}$

$\theta > \frac{\pi}{2}$

② $z = h:$ $e^{-\frac{h}{\lambda\cos\theta}} \dfrac{e^{(\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda})h}-1}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda}} \to 0$ $h \to \infty$

$z = 0$ $\dfrac{1 - e^{-(\frac{1}{(-\lambda\cos\theta)}+\frac{1}{\lambda})h}}{\frac{1}{(-\lambda\cos\theta)}+\frac{1}{\lambda}} \to \dfrac{1}{\frac{1}{(-\lambda\cos\theta)}+\frac{1}{\lambda}}$ $h \to \infty$

$\frac{\lambda}{2\lambda_s}\int_0^1 \dfrac{e^{-(\frac{h}{\lambda})(\frac{1}{x}+1)}}{\frac{1}{x}+1} dx$

$\frac{1}{\frac{1}{\lambda\cos\theta'}+\frac{1}{\lambda}}$ $\theta' = \pi - \theta$

$\frac{\lambda}{\lambda_s}\int_0^{\frac{\pi}{2}} \frac{\sin\theta' d\theta'}{2(1+\cos\theta')} = -\frac{1}{2}\log(1+\cos\theta')\Big|_0^{\frac{\pi}{2}}$

$\frac{1}{x} = y.$

$= \frac{1}{2}\log 2 \cdot \frac{\lambda}{\lambda_s} = \frac{\log 2}{2} \frac{\sigma_s}{\sigma_s+\sigma_c}$

$$\begin{array}{r} 0.34 \\ 8686\overline{)30100} \\ 26058 \\ \hline 40420 \end{array}$$

$= \frac{1}{2} \cdot \frac{0.3010}{0.4343} = 0.35 \frac{\lambda}{\lambda_s}$

② $\frac{\lambda}{\lambda_s}\int_0^{\frac{\pi}{2}} \frac{\sin\theta' d\theta'}{2 \cdot (\frac{1}{\cos\theta}+1)} = \frac{\lambda}{\lambda_s} \frac{1}{2}(1-\log 2)$

$= 0.15 \frac{\lambda}{\lambda_s}$

$$\int_{z_2=0}^{\infty}\int_{z_1=0}^{z_2} e^{-\frac{z_1}{\lambda}} \frac{\sin\theta_1 d\theta_1}{2} \cdot \frac{dz_1}{\lambda_s\cos\theta_1} e^{-\frac{z_2-z_1}{\lambda\cos\theta_1}}$$

$$\times \frac{dz_2}{\lambda_s\cos\theta_2} e^{-\frac{z_2}{\lambda\cos\theta_2}} \qquad \theta_1<\frac{\pi}{2}$$

$$\int_{z_2=0}^{\infty}\int_{z_1=z_2}^{\infty} e^{-\frac{z_1}{\lambda}} \frac{\sin\theta_1' d\theta_1'}{2} \frac{dz_1}{\lambda_s\cos\theta_1'} e^{+\frac{z_2-z_1}{\lambda\cos\theta_1'}} \qquad \theta_1>\frac{\pi}{2}$$

$$\theta_1'<\frac{\pi}{2}$$

$$\times \frac{dz_2}{\lambda_s\cos\theta_2} e^{-\frac{z_2}{\lambda\cos\theta_2}}$$

$$\int_{z_2=0}^{\infty}\int_{z_1=0}^{z_2} e^{-\frac{z_2}{\lambda}} \frac{\sin\theta_1' d\theta_1'}{2} \frac{dz_2}{\lambda_2\cos\theta_1'} e^{\frac{z_1-z_2}{\lambda\cos\theta_1}}$$

$$\times \frac{dz_1}{\lambda_2\cos\theta_2} e^{-\frac{z_1}{\lambda\cos\theta_2}} \qquad e^{-\left(\frac{1}{\lambda\cos\theta_1}+\frac{1}{\lambda\cos\theta_2}\right)z_2}$$

$$\int_{\theta_1=0}^{\frac{\pi}{2}}\int_{z_2=0}^{\infty}\int_{z_1=0}^{z_2} \left\{\frac{\sin\theta_1 d\theta_1}{2} \frac{1}{\lambda_1\cos\theta_1\lambda_2\cos\theta_2}\right\} e^{\left(\frac{1}{\lambda\cos\theta_1}-\frac{1}{\lambda}\right)z_1}$$

$$+ e^{-\left(\frac{1}{\lambda\cos\theta_1}+\frac{1}{\lambda\cos\theta_1}\right)z_2} e^{\left(\frac{1}{\lambda\cos\theta_1}-\frac{1}{\lambda\cos\theta_1}\right)z_1} \Big\} dz_1 dz_2$$

$$= \frac{\sin\theta_1 d\theta_1}{2} \frac{1}{\lambda_s\cos\theta_1\lambda_s\cos\theta_2} \left\{ \frac{e^{-\left(\frac{1}{\lambda\cos\theta_1}+\frac{1}{\lambda\cos\theta_2}\right)z_2}\left(e^{\left(\frac{1}{\lambda\cos\theta_1}-\frac{1}{\lambda}\right)z_2}-1\right)}{\frac{1}{\lambda\cos\theta_1}-\frac{1}{\lambda}} \right.$$

$$\left. + \frac{e^{-\left(\frac{1}{\lambda\cos\theta_1}+\frac{1}{\lambda}\right)z_2}\left(e^{\left(\frac{1}{\lambda_1\cos\theta_1}-\frac{1}{\lambda\cos\theta_2}\right)z_2}-1\right)}{\frac{1}{\lambda\cos\theta_1}-\frac{1}{\lambda\cos\theta_2}} \right\}$$

$$= \frac{1}{\frac{1}{\lambda\cos\theta_1} - \frac{1}{\lambda}} \left( \frac{1}{\frac{1}{\lambda\cos\theta_2} + \frac{1}{\lambda}} - \frac{1}{\frac{1}{\lambda\cos\theta_1} + \frac{1}{\lambda\cos\theta_2}} \right)$$

$$+ \frac{1}{\frac{1}{\lambda\cos\theta_1} - \frac{1}{\lambda\cos\theta_2}} \left( \frac{1}{\frac{1}{\lambda\cos\theta_2} + \frac{1}{\lambda}} - \frac{1}{\frac{1}{\lambda\cos\theta_1} + \frac{1}{\lambda}} \right)$$

$$= \frac{1}{\cancel{\frac{1}{\lambda\cos\theta_1\lambda\cos\theta_2} + \frac{1}{\lambda}}} \frac{\lambda\cos\theta_1\cos\theta_2}{\frac{1}{\lambda\cos\theta_1} - \frac{1}{\lambda}} \left( \frac{1}{\cos\theta_1(1+\cos\theta_2)} - \frac{1}{\cos\theta_2+\cos\theta_1} \right)$$

$$+ \frac{\lambda\cos\theta_1\cos\theta_2}{\frac{1}{\lambda\cos\theta_1} - \frac{1}{\lambda\cos\theta_2}} \left( \frac{1}{\cos\theta_1(1+\cos\theta_2)} + \frac{1}{\cos\theta_2(1+\cos\theta_1)} \right)$$

$$= \frac{\lambda^2\cos^2\theta_1\cos\theta_2}{1-\cos\theta_1} \left( \frac{\cos\theta_2\cancel{(1-\cos\theta_1)}}{\cancel{\cos\theta_1}(1+\cos\theta_2)(\cos\theta_1+\cos\theta_2)} \right.$$

$$+ \frac{\lambda^2\cos^2\theta_1\cos^2\theta_2}{\cos\theta_2-\cos\theta_1} \frac{\cancel{\cos\theta_2 - \cos\theta_1}}{\cancel{\cos\theta_1\cos\theta_2}(1+\cos\theta_2)(1+\cos\theta_1)}$$

$$= \lambda^2\cos\theta_1\cos\theta_2 \frac{}{\cancel{(1+\cos\theta_2)}(1+\cos\theta_1)(\cos\theta_2-\cos\theta_1)(\cos\theta_1+\cos\theta_2)}$$

$$\frac{\cos\theta_2(1+\cos\theta_1) + \cos\theta_1+\cos\theta_2}{(1+\cos\theta_2)(1+\cos\theta_1)(\cos\theta_1+\cos\theta_2)}$$

$\cos\theta_2+\cos\theta_1$
$\cos\theta_1(1+\cos\theta_2)$
$+2\cos\theta_2$

$\sin\theta_1 d\theta_1 \cancel{\sin\theta_2}$

$$\frac{1}{4}\left(\frac{\lambda}{\lambda_s}\right)^2 \int_0^1\int_0^1 dx_1\, dx_2 \left( \frac{x_1 + x_2}{(1+x_2)(x_1+x_2)} + \frac{1}{(1+x_1)(1+x_2)} \right)$$

$= \frac{1}{\cancel{x_1+x_2}} - \frac{1}{(1+x_2)(x_1+x_2)}$

$\cancel{1x_1+x_2}$

$\cancel{\log \frac{1}{x_1+1} - \log}$

$$\int_0^1 \left( \log(x_1+1) - \log x_1 \right) dx_1 - \iint_0^1 \frac{1}{1+x_1} \left( \log(x_1+1) - \log x_1 \right) dx_1$$

$x(\log x - 1)$ $\quad (x_1+1)\left(\log(x_1+1) - 1\right)\Big|_0^2$

$$+ \int_0^1 \frac{dx_1}{1+x_1} \log 2 \cancel{-\log} = 2(\log 2 - 1) + 1$$

$$+ 1 + (\log 2)^2 - \int_0^1 \frac{\log(1+x_1)}{1+x_1} dx_1$$

$\cancel{1} \frac{\pi^2}{12}$ $\quad = \quad \cancel{\int \frac{\log(1+x)}{1+x}} - \frac{1}{2}(\log 2)^2$

$$\int_0^1 \frac{\log(1+x)\,dx}{x} = \frac{\pi^2}{12}$$

$$\frac{1}{x(1+x)} = \frac{1}{1+x} - \frac{1}{1+x}$$

$\cancel{\int_0^1 \frac{\log(1+x)\,dx}{x(1+x)}} =$

$$\int_0^1 \frac{\log(1+x)\,dx}{x(1+x)} = \frac{1}{12}\pi^2 - \frac{1}{2}(\log 2)^2$$

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$$\int_0^1 \frac{\log(1+x)}{1+x} dx = \frac{1}{2}(\log 2)^2$$

$$= 2\log 2 \cancel{-4} + \frac{3}{2}(\log 2)^2 - \frac{\pi^2}{12}$$

$0.7$

$$= 1.4 \cancel{-4} \cancel{- 0.75} - 0.83 =$$

$+0.25$

$$= 0.8, \qquad \times \left(\frac{\lambda_s}{\lambda_s}\right)^2 \times \frac{1}{4} = 0.2 \times \left(\frac{\lambda}{\lambda_s}\right)^2$$

$0.25$

$1.5$
$0.59$
$0.75$

$0.83$
$12) 10$
$9.6$
$40$

$1.65$
$.83$
$0.82$

Transmission の方は $R = \int_{\frac{\pi}{2}}^{0}$

$$\int_{\theta=0}^{\frac{\pi}{2}} \int_0^h e^{-\frac{z}{\lambda}} \frac{\sin\theta d\theta}{2} \frac{dz}{\lambda_s} e^{-\frac{(h-z)}{\lambda\cos\theta}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin\theta d\theta}{2\lambda_s} e^{-\frac{h}{\lambda\cos\theta}} \frac{\left(e^{\frac{h}{\lambda\cos\theta}-\frac{h}{\lambda}}-1\right)}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda}}$$

$$= \frac{\lambda}{2\lambda_s} \int_0^1 \frac{dx}{\left(\frac{1}{x}-1\right)} e^{-\frac{h}{\lambda}} \left(1-e^{-\left(\frac{1}{x}-1\right)h}\right)$$

$$= \frac{\lambda}{2\lambda_s} e^{-\frac{h}{\lambda}} \int_0^1 \frac{1-e^{-\left(\frac{1}{x}-1\right)h}}{\frac{1}{x}-1} dx$$

$Ei\left(-\frac{h}{\lambda}\right) < \frac{\lambda}{h} \cdot e^{-\frac{h}{\lambda}}$

for

$\frac{1}{x}-1=y,\quad \frac{dx}{x^2}=dy$

$\frac{1}{x}+1=y$

$x=0,\ y=\infty$

$x=1,\ y=2$

$\frac{1}{x^2}=(1+y)^2$

$-Ei\left(\frac{1}{2}\right)=0.56$

$-Ei(-1)=0.22$

$-Ei(-2)=0.049$

$$\frac{\lambda}{2\lambda_s} e^{-\frac{h}{\lambda}} \int_0^\infty \frac{1-e^{-hy}}{y(1+y)^2} dy$$

$$\int_0^\infty \frac{dy}{(1+y)^2} = \frac{1}{1+y}\Big|_\infty^0 = 1$$

$$\int e^{-hx} \frac{dx}{(1+x)^2} = 1 + h e^{h} Ei(-h)$$

$$= \frac{\lambda}{2\lambda_s} e^{-\frac{h}{\lambda}} \int_0^\infty \frac{1-e^{-z}}{} dz$$

$-\frac{h}{2\lambda_s} Ei\left(-\frac{h}{\lambda}\right)$

$Ei\left(-\frac{h}{\lambda}\right) = \int_{\frac{h}{\lambda}}^\infty \frac{e^{-x}dx}{x}$

$$= \frac{\lambda}{2\lambda_s} e^{-\frac{h}{\lambda}} \left(1+\frac{h}{\lambda} e^{\frac{h}{\lambda}} Ei\left(-\frac{h}{\lambda}\right)\right)$$