(1)

Multiple Scattering and Absorption of Neutrons by Heavy Element

Scattering は spherically symmetric とし その mean free path を λ'

$$\lambda' = \frac{1}{\sum_i n_i \sigma_i^{(s)}}$$

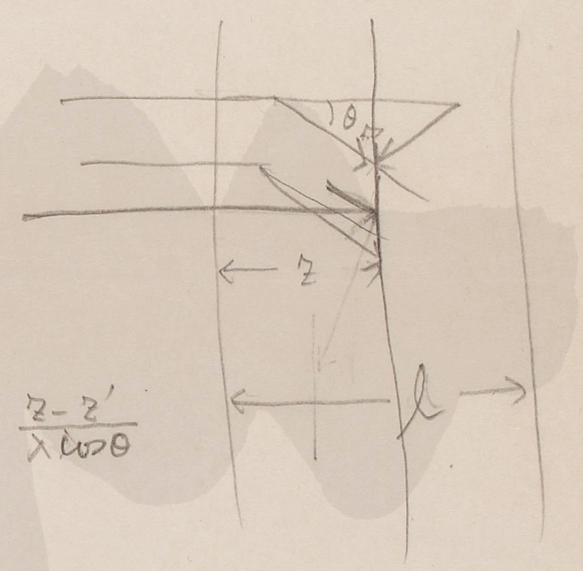
capture の mean free path と inelastic scattering を λ''

$$\lambda'' = \frac{1}{\sum_i n_i \sigma_i^{(c)}}$$

$$\lambda = \frac{1}{\frac{1}{\lambda'} + \frac{1}{\lambda''}}$$

(0) z方向と plate に normal に入って来た neutron が z の深さの plane を通る確率は

$$e^{-\frac{z}{\lambda}}$$



(i) 一回 scatter されて後 z で θ と $\theta+d\theta$) の間の angle で通る確率は

$$\int_{z'=0}^{z} e^{-\frac{z'}{\lambda}} \cdot \frac{dz'}{\lambda'} \cdot \frac{\sin\theta\, d\theta}{2} \cdot e^{-\frac{z-z'}{\lambda\cos\theta}} \quad \text{for } \theta \le \frac{\pi}{2}$$

$$\int_{z'=z}^{l} e^{-\frac{z'}{\lambda}} \frac{dz'}{\lambda'} \cdot \frac{\sin\theta\, d\theta}{2} \cdot e^{\frac{z'-z}{\lambda\cos\theta}} \quad \text{for } \theta \ge \frac{\pi}{2}$$

or

$$\boxed{\begin{aligned} P_1(z,\theta)d\theta &= e^{-\frac{z}{\lambda\cos\theta}} \cdot \frac{\sin\theta\, d\theta}{2\lambda'} \cdot \frac{e^{\frac{z}{\lambda\cos\theta} - \frac{z}{\lambda}} - 1}{\frac{1}{\lambda\cos\theta} - \frac{1}{\lambda}} \quad \text{for } \theta \le \frac{\pi}{2} \\ P_1(z,\theta)d\theta &= e^{-\frac{z}{\lambda\cos\theta}} \cdot \frac{\sin\theta\, d\theta}{2\lambda'} \cdot \frac{e^{-\frac{z}{\lambda} + \frac{z}{\lambda\cos\theta}} - e^{-\frac{l}{\lambda} + \frac{l}{\lambda\cos\theta}}}{\frac{1}{\lambda} - \frac{1}{\lambda\cos\theta}} \quad \text{for } \theta \ge \frac{\pi}{2} \end{aligned}}$$

(ii) ②

$\theta \le \frac{\pi}{2}$ ならば $0 < z' < z$

z'にもう一度scatterされて z なる所を通過するには $0<z'<z$ の範囲でもう一度 ~~$\theta \le \frac{\pi}{2}$~~ なる方向にscatterされたものの総和をとらねばならぬ。

即ち、$\theta \le \frac{\pi}{2}$

$$P_2(z,\theta)d\theta = \int_{z'=0}^{z}\int_{\theta'=0}^{\frac{\pi}{2}} P_1(z',\theta')d\theta' \cdot \frac{dz'}{\lambda'\cos\theta'} \cdot e^{-\frac{z-z'}{\lambda\cos\theta}}$$

$\sin\theta d\theta$

$$+\int_{z'=0}^{z}\int_{\theta'=\frac{\pi}{2}}^{\pi} P_1(z',\theta')d\theta' \cdot \frac{dz'}{(-\lambda'\cos\theta')} \cdot e^{-\frac{z-z'}{\lambda\cos\theta}}$$

$$=\int_{\theta'=0}^{\frac{\pi}{2}}\left[e^{-\frac{z}{\lambda\cos\theta}}\int_0^z \frac{\sin\theta' d\theta'}{2\lambda'}\, \frac{e^{\frac{z'}{\lambda\cos\theta}-\frac{z'}{\lambda}} - e^{\frac{z'}{\lambda\cos\theta}-\frac{z'}{\lambda\cos\theta'}}}{\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda}}\, \frac{dz'}{\lambda'\cos\theta'}\right]$$

$$+\int_{\theta'=\frac{\pi}{2}}^{\pi}\left[e^{-\frac{z}{\lambda\cos\theta}}\int_0^z \frac{\sin\theta' d\theta'}{2\lambda'} \cdot \frac{e^{\frac{z'}{\lambda\cos\theta}-\frac{z'}{\lambda}} - e^{-\frac{l}{\lambda}+\frac{z'}{\lambda\cos\theta}+\frac{(l-z')}{\lambda\cos\theta'}}}{\frac{1}{\lambda}-\frac{1}{\lambda\cos\theta'}}\, \frac{dz'}{(-\lambda'\cos\theta')}\right]$$

$$=\int_{\theta'=0}^{\frac{\pi}{2}} e^{-\frac{z}{\lambda\cos\theta}} \frac{\sin\theta' d\theta'}{2\lambda'^2\cos\theta'} \cdot \frac{1}{\frac{1}{\lambda\cos\theta'}-\frac{1}{\lambda}} \left[\frac{e^{\frac{z}{\lambda\cos\theta}-\frac{z}{\lambda}}-1}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda}} - \frac{e^{\frac{z}{\lambda\cos\theta}-\frac{z}{\lambda\cos\theta'}}-1}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda\cos\theta'}}\right]$$

$$+\int_{\theta'=\frac{\pi}{2}}^{\pi} e^{-\frac{z}{\lambda\cos\theta}} \frac{\sin\theta' d\theta'}{(-2\lambda'^2\cos\theta')} \frac{1}{\frac{1}{\lambda}-\frac{1}{\lambda\cos\theta'}} \left[\frac{e^{\frac{z}{\lambda\cos\theta}-\frac{z}{\lambda}}-1}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda}} - \frac{e^{-\frac{l}{\lambda}+\frac{l}{\lambda\cos\theta'}}\cdot\left(e^{\frac{z}{\lambda\cos\theta}-\frac{z}{\lambda\cos\theta'}}-1\right)}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda\cos\theta'}}\right]$$

$\times \sin\theta d\theta$

$\cos\theta' = -x'$

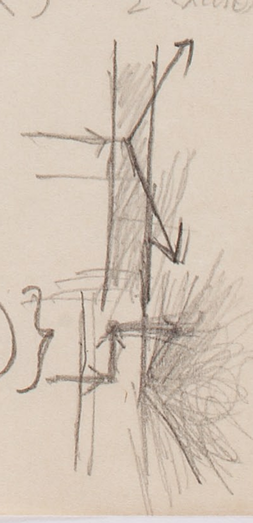
$\sin\theta' d\theta' = dx'$

$\frac{1}{2}\left(\frac{l}{\lambda}\right)^2 - \frac{1}{2}\left(\frac{l}{\lambda\cos\theta}\right)^2$

特に $z=l$ とおくと、

$$P_1(l,\theta)d\theta = \frac{\sin\theta d\theta}{2\lambda'} \frac{e^{-\frac{l}{\lambda}} - e^{-\frac{l}{\lambda\cos\theta}}}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda}}$$

$$P_1(l) = \int_0^{\frac{\pi}{2}} P_1(l,\theta)d\theta = \int_{x=x_0}^{1} \frac{l}{2\lambda'} \cdot dx \left\{1 - \frac{1}{2}\left(\frac{l}{\lambda x}+\frac{l}{x}\right)\right\}$$



(3)

$= \frac{l}{2\lambda'}\left(1 - \frac{1}{2}\frac{l}{\lambda}\log\frac{1}{x_0} + \cdots\right) \qquad x_0 \simeq \frac{\lambda}{l}$!!

$P_2(l,\vartheta)\,d\vartheta = \frac{\sin\vartheta d\vartheta}{d2} e^{-\frac{l}{\lambda x_0}} \int_{x'=0}^{1} \frac{dx'}{2\lambda'^2}\left[\frac{\lambda}{(1-x')}\left\{\frac{\lambda(e^{\frac{l}{\lambda}(\frac{1}{x}-1)}-1)}{\frac{1}{x}-1}\right.\right.$

$\left.- \frac{\lambda(e^{\frac{l}{\lambda}(\frac{1}{x}-\frac{1}{x'})}-1)}{\frac{1}{x}-\frac{1}{x'}}\right\} + \frac{\lambda}{1+x'}\left\{\frac{\lambda(e^{\frac{l}{\lambda}(\frac{1}{x}-1)}-1)}{\frac{1}{x}-1} - \frac{\lambda(e^{-\frac{l}{\lambda}+\frac{l}{\lambda x}}-e^{-\frac{l}{\lambda}-\frac{l}{x'}})}{\frac{1}{x}+\frac{1}{x'}}\right\}$

$= \frac{1}{4}\left(\frac{\lambda}{\lambda'}\right)^2\left(\frac{l}{\lambda}\right)^2 e^{-\frac{l}{\lambda x}}\int^1 dx'\left[\frac{1}{2}\frac{1}{(1-x')}\left(\frac{1}{x'}-1\right) + \frac{1}{2}\frac{1}{(1+x')}\left(\frac{1}{x'}+1\right)\right]$

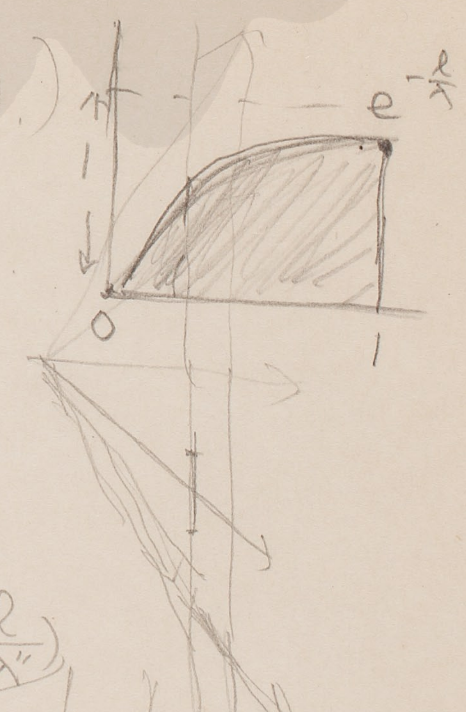
$= \frac{1}{4}\left(\frac{\lambda}{\lambda'}\right)^2\left(\frac{l}{\lambda}\right)^2 e^{-\frac{l}{\lambda x}} \int_{x_0}^{1}\frac{dx'}{x'} = \frac{1}{4}\left(\frac{\lambda}{\lambda'}\right)^2\left(\frac{l}{\lambda}\right)^2 e^{-\frac{l}{\lambda x}}\log\left(\frac{1}{x_0}\right)$

$\frac{1}{2}\left(\frac{l}{\lambda}\right)$

$P_2(l) = \left(\frac{1}{x} - 1\right) - \frac{(\frac{1}{x}-\frac{1}{x'})^2 - (\frac{1}{x'}+1)^2}{\frac{1}{x}+\frac{1}{x'}}$

$\frac{1}{4}\left(\frac{\lambda}{\lambda'}\right)^2\left(\frac{l}{\lambda}\right)^2\int_{x=0}^{1} e^{-\frac{l}{\lambda x}}\log\left(\frac{1}{x_0}\right)dx$

$\frac{1}{x'} - \frac{1}{x}$



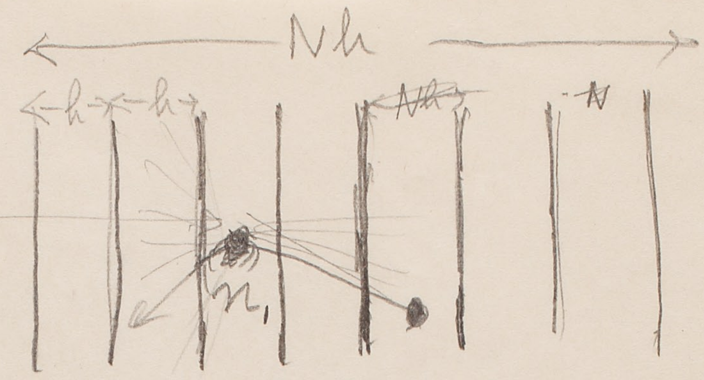
$\simeq \frac{1}{4}\left(\frac{\lambda}{\lambda'}\right)\left(\frac{l}{\lambda}\right)\left(\frac{l}{\lambda'}\right)\cdot\log\frac{1}{x_0}$

$\frac{1}{4}\left(\frac{l}{\lambda'}\right)^2\left\{1 - \frac{\lambda'}{2\lambda}\right\}\log\frac{1}{x_0}$

$\frac{1}{4}\left(\frac{l}{\lambda}\right)\left(\frac{l}{\lambda'}\right)\left\{1 - \frac{\lambda'}{2\lambda}\right\}\log\frac{1}{x_0} = \frac{1}{4}\left(\frac{l}{\lambda}\right)\left(\frac{l}{\lambda'}\right)\log\frac{1}{x_0}$

$P_1(l) + P_2(l) = \frac{l}{2\lambda'}\left\{1 + \frac{1}{2}\left(\frac{l}{\lambda}\right)\log\frac{1}{x_0}\,\cancel{\left(1-\frac{\lambda'}{2\lambda}\right)}\right\} + O(l^2)$

$\left(\frac{l}{\lambda'}\right)$



○ 厚い板を無限な厚さの薄板にわけて考へる。
n_1番目の板を通る primary neutron の数は

$$e^{-\frac{(n_1-1)h}{\lambda}}$$

この中 $e^{-\frac{(n_1-1)h}{\lambda}}\cdot\frac{h}{\lambda}$ は scattering or (inel. scatter or) absorption を起す。

その中 $\frac{1}{2}e^{-\frac{n_1 h}{\lambda}}\frac{h}{\lambda'}$ は el. scattering を起して之へ進み、ほぼ半分が ~~inel~~ reflect back される。之が更に n_2番目の plate に到達すれば

大体 $(n_2>n_1)$

$$p_1\left(\frac{(n_2-n_1)h}{\lambda}\right)=\int_0^{\frac{\pi}{2}} e^{-\frac{(n_2-n_1)h}{\lambda\cos\theta}}\sin\theta\, d\theta = \frac{(n_2-n_1)h}{\lambda}\int_0^{\frac{\lambda}{(n_2-n_1)h}} e^{-\frac{1}{x}}dx$$

$\frac{\lambda\cos\theta}{(n_2-n_1)h}=x$ $\quad \frac{(n_2-n_1)h}{\lambda}dx$ $\quad \frac{1}{x}=y,$ $\quad -y^2dx=dy$

僅へっており

$$=\frac{(n_2-n_1)h}{\lambda}\int_{\frac{(n_2-n_1)h}{\lambda}}^{\infty}\frac{e^{-y}dy}{y^2}$$

$n_2<n_1$ ならば 之までが reflect back して n_2番目 plate を来る。

その中

$$\int_0^{\frac{\pi}{2}} e^{-\frac{(n_2-n_1)h}{\lambda\cos\theta}}\frac{h\sin\theta\, d\theta}{\lambda'\cos\theta}=\frac{h}{\lambda'}\int_{\frac{(n_2-n_1)h}{\lambda}}^{\infty}\frac{e^{-y}dy}{y}=-p_1'\left(\frac{(n_1-n_2)h}{\lambda}\right)$$

$$\frac{1}{\cos\theta}=\frac{\lambda}{(n_2-n_1)h\,x}=\frac{\lambda}{(n_2-n_1)h\,y}$$

まず2回目の scattering を起す。

始めに $(n-\frac{1}{2})h$ なる厚さの所を通る neutron の数は

第一回、 $P_0(n)=e^{-\frac{(n-1)h}{\lambda}}$ $\qquad \frac{h}{2\lambda'}e^{-\frac{n_1h}{\lambda}}$ $\qquad n_1=n-m$

第二回 $P_1(n)=\sum_{n_1=1}^{n-1}p_1\left(\frac{(n-n_1)h}{\lambda}\right)=\sum_{m=1}^{n-1}p_1\left(m\frac{h}{\lambda}\right)\cdot\frac{h}{2\lambda'}e^{-\frac{nh}{\lambda}}\cdot e^{\frac{mh}{\lambda}}$

$R_1(n)=\sum_{n_1=n+1}^{N}p_1\left(\frac{(n_1-n)h}{\lambda}\right)=\sum_{m=1}^{N-n}p_1\left(m\frac{h}{\lambda}\right)e^{-\frac{mh}{\lambda}}\cdot\frac{h}{2\lambda'}e^{-\frac{nh}{\lambda}}$ $\qquad n_1=m+n$ $\qquad \frac{h}{2\lambda'}e^{-\frac{nh}{\lambda}}$

z位深さの所を通るneutronの数は

$$P_0(z)=e^{-\frac{z}{\lambda}}$$

途中 $z'<z$ なる所でscatterされて zを通る数は

$$P_1(z)=\int_0^z e^{-\frac{z'}{\lambda}}\frac{dz'}{2\lambda'}\int_0^{\frac{\pi}{2}} e^{-\frac{(z-z')}{\lambda\cos\theta}}\cdot\sin\theta\, d\theta$$

$$=\int_0^z e^{-\frac{z'}{\lambda}}\frac{dz'}{2\lambda'}\int_0^{\frac{\pi}{2}} e^{-\frac{(z-z')}{\lambda\cos\theta}}\sin\theta\, d\theta$$

$$=\int_0^z \frac{e^{-\frac{z'}{\lambda}}}{2\lambda'} f(z-z')dz'=\int_0^z \frac{e^{-\frac{(z-z')}{\lambda}}}{2\lambda'} f(z')dz'=\frac{e^{-\frac{z}{\lambda}}}{2\lambda'}\int_0^z e^{\frac{z'}{\lambda}}f(z')dz'$$

この中で z〜dz の間でscatterされる~~数~~数は

$$p_2(z)dz=\iint \frac{dz}{\lambda'\cos\theta}\frac{dz'}{2\lambda'}e^{-\frac{z'}{\lambda}}e^{-\frac{(z-z')}{\lambda\cos\theta}}\sin\theta\, d\theta$$

$$=-\int_{z'=0}^{z}\frac{e^{-\frac{z'}{\lambda}}\lambda}{2\lambda'^2}\cdot dz\,dz' f'(z-z')=\frac{-\lambda e^{-\frac{z}{\lambda}}}{2\lambda'^2}\int_0^z e^{\frac{z'}{\lambda}}f'(z')dz'$$

又 $h>z'>z$ の間でscatterされて zにreflect backされる数は

$$R_1(z)=\int_{z'=z}^{h} e^{-\frac{z'}{\lambda}}\frac{dz'}{2\lambda'}\int_0^{\frac{\pi}{2}} e^{-\frac{(z'-z)}{\lambda\cos\theta}}\sin\theta\, d\theta=\int_{z'=z}^{h} e^{-\frac{z'}{\lambda}}\frac{dz'}{2\lambda'}f(z'-z)$$

$$=\frac{e^{-\frac{z}{\lambda}}}{2\lambda'}\int_0^{h-z} e^{-\frac{z'}{\lambda}}f(z')dz'$$

$$\frac{\lambda}{\lambda'}f(z'-z)$$

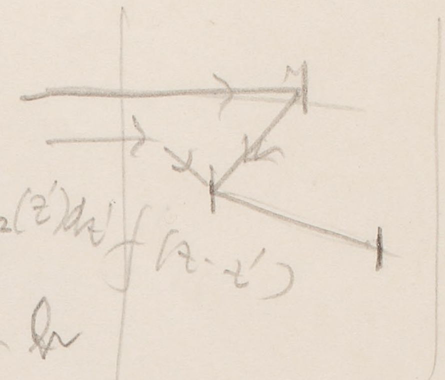
この中で z〜dz の間でscatterされる数は

$$r_2(z)dz=\int_{z'=z}^{h} e^{-\frac{z'}{\lambda}}\frac{dz'}{2\lambda'}\int_{\theta=0}^{\frac{\pi}{2}}\frac{dz}{\lambda'\cos\theta}e^{-\frac{(z'-z)}{\lambda\cos\theta}}\sin\theta\, d\theta$$

$$=\frac{-\lambda\, dz}{2\lambda'^2}\int_{z'=z}^{h} e^{-\frac{z'}{\lambda}}dz' f'(z'-z)=\frac{-\lambda e^{-\frac{z}{\lambda}}}{2\lambda'^2}\int_0^{h-z} e^{-\frac{z'}{\lambda}}f'(z')dz'$$

⑥

次に 二回のscatteringの後に
zなる所を通る数は　$p_2+r_2=q_2$ とおくと



$$P_2(z)=\int_0^z \{p_2(z')+r_2(z')\}dz'\cdot f(z-z')=\int_0^z q_2(z')dz'\,f(z-z')$$

$$R_2(z)=\int_z^h \{p_2(z')+r_2(z')\}dz'\,f(z'-z)=\int_z^h q_2(z')dz'\,f(z'-z)$$

この中で(dzの間で)更に三回目のscatteringを起す数は

$$p_3(z)dz=-\int_0^z q_2(z')dz'\,\frac{dz}{2\lambda'}\,\frac{\lambda}{\lambda'}\,f'(z-z')$$

$$r_3(z)dz=-\int_z^h q_2(z')dz'\,\frac{dz}{2\lambda'}\,\frac{\lambda}{\lambda'}\,f'(z'-z)$$

$$q_3(z)dz=p_3(z)+r_3(z).$$

一般に n回のscatteringの後に zなる所を通るneutronの数は

$$P_n(z)=\int_0^z q_n(z')f(z-z')dz'=\int_0^z q_n(z-z')f(z')dz'$$

$$R_n(z)=\int_z^h q_n(z')f(z'-z)dz'=\int_0^{h-z} q_n(z'-z)f(z')dz'$$

従って z なる深さの所を(左から來る)通る neutronの総数(確率)は (これは勿論 nに依る が何回も通ってる故 1より大かも知れぬ)

$$f(z)=\int_0^1 e^{-\frac{z}{\lambda x}}dx$$

$$=\frac{z}{\lambda}\int_{\lambda/z}^{\infty}\frac{e^{-x}dx}{x^2}$$

$$P(z)=\sum_{n=0}^{\infty}P_n(z)$$

$$R(z)=\sum_{n=1}^{\infty}R_n(z)$$

である。

$f(z-z')=$

⑰

然るに $z=h$ に於ては

$$P_n(h) = \int_0^h q_n(z')f(h-z')dz' = \int_0^h q_n(h-z')f(z')dz'$$

$z=0$ に於ては

$$R_n(0) = \int_0^h q_n(z')f(z')dz'$$

[illegible]して

又 $$q_n(z)dz = -\int_0^z q_{n-1}(z')dz' \frac{dz}{2\lambda'}\frac{\lambda}{\lambda'} f'(z-z')$$

$$-\int_z^h q_{n-1}(z')dz' \frac{dz}{2\lambda'}\frac{\lambda}{\lambda'} f'(z'-z)$$

$$= \frac{-\lambda dz}{2\lambda'^2}\left[\int_0^z q_{n-1}(z')dz' f'(z-z') + \int_z^h q_{n-1}(z')dz' f'(z'-z)\right]$$

$$f'(z) = \frac{-1}{\lambda}\int_0^1 e^{-\frac{z}{\lambda x}}\frac{dx}{x}$$

$$= \frac{-1}{\lambda}\int_{\frac{\lambda}{z}}^{\infty}\frac{e^{-x}dx}{x}$$

$y=\frac{1}{x}$

$$-\lambda f'(z) = \int_{\frac{\lambda}{z}}^{\infty}\frac{e^{-x}dx}{x} = -E_i\left(-\frac{\lambda}{z}\right).$$

$$q_n(z)dz = \frac{-dz}{2\lambda'^2}\left\{\int_0^z q_{n-1}(z')E_i\left(-\frac{\lambda}{z-z'}\right)dz' + \int_z^h q_{n-1}(z')E_i\left(-\frac{\lambda}{z'-z}\right)dz'\right\}$$

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$$P_0(z) = e^{-\frac{z}{\lambda}}$$

$$P_1(z,\theta)\,d\theta = \frac{\sin\theta\, d\theta}{2\lambda'} \cdot e^{-\frac{z}{\lambda\cos\theta}} \cdot \frac{e^{\frac{z}{\lambda\cos\theta}-\frac{z}{\lambda}}-1}{\frac{1}{\lambda\cos\theta}-\frac{1}{\lambda}} \qquad \theta \le \frac{\pi}{2}$$

$\cos\theta = x$

$\frac{z}{\lambda} = \zeta$

$$= \frac{\lambda\, dx}{2\lambda'} \cdot e^{-\frac{\zeta}{x}} \cdot \frac{e^{\zeta(\frac{1}{x}-1)}-1}{\frac{1}{x}-1} \qquad 1 \ge x \ge 0 \quad \cancel{\theta \ge \frac{\pi}{2}}$$

$$\frac{h}{\lambda} = \zeta_0 \quad R_1(z,\theta)\,d\theta = \frac{\sin\theta\, d\theta}{2\lambda'}\, e^{-\frac{z}{\lambda\cos\theta}} \cdot \frac{e^{-\frac{z}{\lambda}+\frac{z}{\lambda\cos\theta}} - e^{-\frac{h}{\lambda}+\frac{h}{\lambda\cos\theta}}}{\frac{1}{\lambda}-\frac{1}{\lambda\cos\theta}} \qquad \theta \ge \frac{\pi}{2}$$

$$= \frac{\lambda\, dx}{2\lambda'}\, e^{-\frac{\zeta}{x}}\, \frac{e^{-\zeta(1-\frac{1}{x})} - e^{-\zeta_0(1-\frac{1}{x})}}{1-\frac{1}{x}} \qquad 0 \ge x \ge -1$$

$$R_1(z,\theta)\ \left(= \frac{\lambda\, dx\, d\theta}{2\lambda'}\, e^{\frac{\zeta}{x}}\, \frac{e^{-\zeta(1+\frac{1}{x})} - e^{-\zeta_0(1+\frac{1}{x})}}{1+\frac{1}{x}} \qquad 0 \le x \le 1 \right)$$

$$P_1(z) = \int_0^z e^{-\frac{z'}{\lambda}} \frac{dz'}{2\lambda'} \int_0^{\frac{\pi}{2}} e^{-\frac{(z-z')}{\lambda\cos\theta}} \sin\theta\, d\theta$$

$$= \left(\frac{\lambda}{\lambda'}\right) \int_0^{\zeta} e^{-\zeta'} \frac{d\zeta'}{2} \int_0^1 e^{-\frac{(\zeta-\zeta')}{x}}\, dx = \left(\frac{\lambda}{2\lambda'}\right) \int_0^1 \frac{x(e^{-\zeta}-e^{-\frac{\zeta}{x}})}{(x-1)}\, dx.$$

$$p_2(z)\,dz = \int_0^z e^{-\frac{z'}{\lambda}} \frac{dz'}{2\lambda'} \cdot \int_0^{\frac{\pi}{2}} \frac{dz}{\lambda'\cos\theta} \cdot e^{-\frac{(z-z')}{\lambda\cos\theta}} \sin\theta\, d\theta$$

$$= \left(\frac{\lambda}{\lambda'}\right)^2 d\zeta \int_0^{\zeta} e^{-\zeta'} d\zeta' \int_0^1 e^{-\frac{(\zeta-\zeta')}{x}} \frac{dx}{x}$$

$$R_{x,1}(z) = \int_z^h e^{-\frac{z'}{\lambda}} \frac{dz'}{2\lambda'} \int_0^{\frac{\pi}{2}} e^{-\frac{(z'-z)}{\lambda\cos\theta}} \sin\theta\, d\theta$$

$$= \left(\frac{\lambda}{\lambda'}\right) \int_{\zeta}^{\zeta_0} e^{-\zeta'} \frac{d\zeta'}{2} \int_0^1 e^{-\frac{(\zeta'-\zeta)}{x}}\, dx = \left(\frac{\lambda}{2\lambda'}\right) \int_0^1 \frac{x(e^{-\zeta} - e^{\frac{\zeta}{x}-\zeta_0(1+\frac{1}{x})})}{x+1}\, dx$$

$$r_2(z)\,dz = \int_{z}^{h} e^{-\frac{z'}{\lambda}} \frac{dz'}{2\lambda'} \int_0^{\frac{\pi}{2}} \frac{dz}{\lambda'\cos\theta}\, e^{-\frac{(z-z')}{\lambda\cos\theta}} \sin\theta\, d\theta$$

$$= \left(\frac{\lambda}{\lambda'}\right) d\zeta \int_{\zeta}^{\zeta_0} e^{-\zeta'} \frac{d\zeta'}{2} \int_0^1 e^{-\frac{(\zeta'-\zeta)}{x}} \frac{dx}{x}$$

(9)

$$q_2(z)dz = p_2(z)dz + \gamma_2(z)dz$$

$$= \frac{d\zeta}{2}\left(\frac{\lambda}{\lambda'}\right)^2\left\{\int_0^{\zeta} e^{-\zeta'}d\zeta' \int_0^1 e^{-\frac{(\zeta-\zeta')}{x}}\frac{dx}{x} + \int_{\zeta}^{\zeta_0} e^{-\zeta'}d\zeta' \int_0^1 e^{-\frac{(\zeta'-\zeta)}{x}}\frac{dx}{x}\right.$$

$$= \frac{d\zeta}{2}\left(\frac{\lambda}{\lambda'}\right)^2\left\{\int_0^1 \frac{dx}{x}\left[\frac{e^{-\frac{\zeta}{x}}\left(e^{\zeta(\frac{1}{x}-1)}-1\right)}{\frac{1}{x}-1} + \frac{e^{\frac{\zeta}{x}}\left(e^{-\zeta(\frac{1}{x}+1)} - e^{-\zeta_0(\frac{1}{x}+1)}\right)}{\frac{1}{x}+1}\right]\right.$$

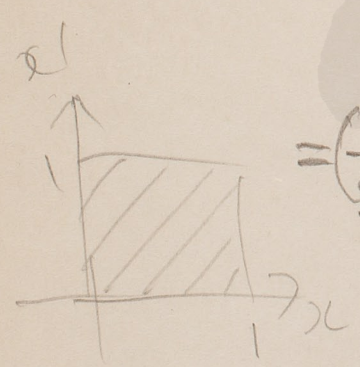
$$= \frac{d\zeta}{2}\left(\frac{\lambda}{\lambda'}\right)^2\left\{\int_0^1 dx\left[\frac{e^{-\zeta}-e^{-\frac{\zeta}{x}}}{1-x} + \frac{e^{-\zeta} - e^{-\frac{\zeta_0-\zeta}{x}-\zeta_0}}{1+x}\right]\right\}$$

$$P_2(z) = \int_0^z dz' q_2(z')\int_0^{\frac{\pi}{2}}\frac{\sin\theta\, d\theta}{2}\cdot e^{-\frac{z-z'}{\lambda\cos\theta}}$$

$$= \int_0^{\zeta}\frac{d\zeta'}{2}\left(\frac{\lambda}{\lambda'}\right)^2\left\{\int_0^1 dx'\left[\frac{e^{-\zeta'}-e^{-\frac{\zeta'}{x'}}}{1-x'} + \frac{e^{-\frac{\zeta'}{x'}} - e^{\frac{\zeta'}{x'}-\zeta_0(\frac{1}{x'}+1)}}{1+x'}\right]\right\}$$

$$\times\int_0^1 \frac{dx}{2}e^{-\frac{\zeta-\zeta'}{x}}$$

$$= \left(\frac{\lambda}{2\lambda'}\right)^2\int_0^1 e^{-\frac{\zeta}{x}}dx\int_0^1 dx'\left[\frac{\frac{e^{\zeta(\frac{1}{x}-1)}-1}{\frac{1}{x}-1} - \frac{e^{\zeta(\frac{1}{x}-\frac{1}{x'})}-1}{\frac{1}{x}-\frac{1}{x'}}}{(1-x')} + \frac{\frac{e^{\zeta(\frac{1}{x}-\frac{1}{x'})}-1}{\frac{1}{x}-\frac{1}{x'}} - e^{-\zeta_0(\frac{1}{x'}+1)}\frac{e^{\zeta(\frac{1}{x}+\frac{1}{x'})}-1}{\frac{1}{x}+\frac{1}{x'}}}{(1+x')}\right]$$



$$= \left(\frac{\lambda}{2\lambda'}\right)^2\int_0^1 dx\int_0^1 dx'\left[\frac{x(e^{-\zeta}-e^{-\frac{\zeta}{x}})}{(1-x')(1-x)} - \frac{xx'(e^{-\frac{\zeta}{x'}}-e^{-\frac{\zeta}{x}})}{(1-x')(x'-x)}\right.$$

$$\left. + \frac{xx'(e^{-\frac{\zeta}{x'}}-e^{-\frac{\zeta}{x}})}{(1+x')(x'-x)} - \frac{xx'e^{-\zeta_0(\frac{1}{x'}+1)}\left(e^{+\frac{\zeta}{x'}}-e^{-\frac{\zeta}{x}}\right)}{(1+x')(x'+x)}\right]$$

$$= \left(\frac{\lambda}{2\lambda'}\right)^2\int_0^1\int_0^1 dx\,dx'\left[\frac{xe^{-\zeta}}{(1-x')(1-x)} - \frac{x(x'^2-x^2) - xx'(1+x')(x'+x) - xx'(1-x)(x'+x)}{(1-x')(1+x')(x'-x)(x'+x)}\right.$$

$$R_2(z) = \int_{z}^{h} dz' q_2(z') \int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{2} e^{-\frac{z'-z}{\lambda\cos\theta}}$$

$$= \left(\frac{\lambda}{2X}\right)^2.$$