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On the Theory of Collison of Neutrons with Deuterons.

By Hideki YUKAWA and Shoichi SAKATA

(Read March 13, 1937)

ABSTRACT

The problem of the neutron-deuteron collision was reduced to a simple form by taking the structures of ^2H and ^3H into account and neglecting the forces depending explicitly on the spin. (§1.) The form of the effective potential hole was determined by comparing the estimated value of the cross section of scattering of slow neutrons by deuterons with that obtained experimentally. The cross section of capture was found to be small in agreement with the experiment. The energy dependence of the scattering cross section was also discussed. (§2). The theory of Fermi concerning the effect of the chemical binding on the scattering of slow neutrons was extended to more general case and a necessary limitation of the theory was discussed. The cross section for thermal neutrons was found to be nearly twice as large as that for neutrons of several volts. (§3). Theoretical reasons for the small contribution of the deuteron to the slowing down were considered. (§4).

§1. Reduction of the Collision Problem.

The theory of collision of neutrons with protons was developed by many authors and was able to account for the existing experimental results sufficiently well[1][2]. The problem of the neutron-deuteron collision, which is the next to be attacked, is a three body problem already too complicated to be solved rigorously. It will be worth while, however, to reduce it to the form as simple as possible by taking the structures of ^2H and ^3H into account and to find an approximate solution at least valid for neutrons of not too large energies.

The wave equation of the system containing a proton and two neutrons with the coordinates $\vec{r}_1, \vec{r}_2, \vec{r}_3$ and the z components of the spins $\sigma_1, \sigma_2, \sigma_3$ respectively has the general form

$$\left\{\Delta_1+\Delta_2+\Delta_3+\frac{2M}{h^2}(E-V_{12}-V_{13}-V'_{23})\right\}\psi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \sigma_1, \sigma_2, \sigma_3)=0, \quad (1)$$

(1) See, for example, the summary report of Bethe and Bacher, Rev. Mod. Phys. **8**, 82, 1936 and further Fermi, Ric. Scient. VII-**2**, 1, 1936. 太字

(2) Note added in proof: Recent experiments on the angular distribution of recoil protons by collision with D-D neutrons, however, seems to indicate the existence of comparatively long range forces. (Cf. Bull. Amer. Phys. Soc. **12**, 30, 1937)

forces.

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where M is the common mass and V_{12}, V_{13}, V'_{23} are the potentials of forces, each involving the coordinates, spins and exchange operators of two particles in general. If we neglect the terms other than those of Majorana type, they become

$$V_{12}=J(s)P^M_{12},\quad V_{13}=J(s')P^M_{13},\quad V'_{23}=K(s'')P^M_{23}$$

respectively, where

$$\vec{s}=\vec{r}_1-\vec{r}_2,\quad \vec{s}'=\vec{r}_1-\vec{r}_3,\quad \vec{s}''=\vec{r}_3-\vec{r}_2$$

and P^M's are the operators of the coordinate exchange.

Further, if we adopt the coordinates of the centre of mass

$$\vec{R}=\frac{\vec{r}_1+\vec{r}_2+\vec{r}_2}{3},$$

those of the proton relative to the first neutron $\vec{s}$ and those of the second neutron relative to the centre of mass of the former

$$\vec{r}=\vec{r}_3-\frac{\vec{r}_1+\vec{r}_2}{2}$$

as independent variables, (1) reduces to

$$\left\{2\Delta_s+\frac{3}{2}\Delta+\frac{2M}{h^2}(E'-J(s)P^M_{12}-J(s')P^M_{13}-K(s'')P^M_{23})\right\}\psi(\vec{s},\vec{r})=0 \qquad (2)$$

$\hbar^2$

or

$$\left\{2\Delta_s+\frac{3}{2}\Delta+\frac{2M}{h^2}(E'-J(s))\right\}\psi(\vec{s},\vec{r})$$

$+$

$$-\frac{2M}{h^2}J(s')\psi(\vec{s}',\vec{r}')-\frac{2M}{h^2}K(s'')\psi(\vec{s}'',\vec{r}'')=0 \qquad (3)$$

$\hbar^2$

after separating $\vec{s}$ and $\vec{r}$ from $\vec{R}$ and σ's, where

$$\vec{r}'=\vec{r}_2-\frac{\vec{r}_1+\vec{r}_2}{2},\quad \vec{r}''=\vec{r}_1-\frac{\vec{r}_3+\vec{r}_2}{2}$$

and E' is the energy of the system excluding the kinetic energy of the centre of mass.

Now, when the energy E_0 of the incident neutron is not large enough to disintegrate the deuteron, i.e. E_0 is smaller than its binding energy $E_D=2.2\times10^6 eV$, provided that it has no true excited levels, the wave function can be assumed to have the approximate form[1]

(1) In order to avoid the complication of the formulae, we want to omit hereafter the double sign $\pm$, which should be suffixed to each of functions $\varphi(\vec{r})$, $F(\vec{r})$ and $G(\vec{r})$.

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$$\psi_{\pm}(\vec{s}\vec{r})=\varphi(\vec{r})\chi(s)\pm\varphi(\vec{r}')\chi(s'), \tag{4}$$

where $\chi(s)$ is the normalized wave function of the deuteron, satisfying the equation

$$\left\{\Delta_s-\frac{M}{h^2}(E_D+J(s))\right\}\chi(s)=0 \tag{5}$$

$\hbar^2$

and the approximate orthogonality

$$\iiint\chi^*(s)\chi(s')d\vec{s}\cong 0 \tag{6}$$

$\varphi(\vec{r}')$

is assumed.

The wave function ψ_+ should be multiplied by either of two spin functions

$$\left.\begin{matrix}\alpha(\sigma_1)\\ \beta(\sigma_1)\end{matrix}\right\}\{\alpha(\sigma_2)\beta(\sigma_3)-\beta(\sigma_2)\alpha(\sigma_3)\}$$

antisymmetric with respect to the neutrons, corresponding to ^2S, ^2P, ... states of ^3H, while ψ_- should be multiplied by either of six symmetric spin functions

$$\left.\begin{matrix}\alpha(\sigma_1)\\ \beta(\sigma_1)\end{matrix}\right\}\left\{\begin{matrix}\alpha(\sigma_2)\alpha(\sigma_3)\\ \alpha(\sigma_2)\beta(\sigma_3)+\beta(\alpha_2)\alpha(\sigma_3)\\ \beta(\sigma_2)\beta(\sigma_3)\end{matrix}\right.$$

σ_2 σ_3

corresponding to the mixture of states ^2S, ^2P, ... and ^4S, ^4P, ..., where

$$\alpha(\sigma)=1 \quad \text{or} \quad 0$$
$$\beta(\sigma)=0 \quad \text{or} \quad 1$$

according as $\sigma=1$ or -1.

If we insert (4) in the wave equation (3), multiply both sides by $\chi^*(s)$ and integrate it with respect to $\vec{s}$, we obtain an integro-differential equation

$$\Delta\varphi+\frac{4M}{3h^2}(E'+E_D)\varphi=\frac{4M}{3h^2}\iiint\chi^*(s)\{J(s')\pm J(s)\}\varphi(\vec{r}'')\chi(s'')d\vec{s}$$
$$+\frac{4M}{3h^2}\iiint\chi^*(s)K(s'')\{\varphi(\vec{r}')\chi(s')\pm\varphi(\vec{r})\chi(s)\}d\vec{s}. \tag{7}$$

$\hbar^2$ $\hbar$

This can be transformed into an integral equation with the asymptotic form(1)

$$\varphi(\vec{r})\cong F(r,\theta)+\frac{e^{ikr}}{4\pi r}\iiint G(\vec{q})F(q,\pi-\omega)d\vec{q} \tag{8}$$

(1) Mott, Theory of Atomic Collisions, Oxford, 1933, Chapt. VI.

and Massey,

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for large r, where

$$G(\vec{r})=\frac{4M}{3h^2}\int\int\int\chi^*(s)\{J(s')\pm J(s)\}\varphi(\vec{r}'')\chi(s'')d\vec{s}$$
$$+\frac{4M}{3h^2}\int\int\int\chi^*(s)K(s'')\{\varphi(\vec{r}')\chi(s')\pm\varphi(\vec{r})\chi(s)\}d\vec{s}+U_\pm(r). \quad (9)$$

θ denotes the angle between the vector $\vec{r}$ and z axis, i.e. the direction of motion of the incident neutron and ω the angle between $\vec{r}$ and $\vec{q}$. $F(r,\theta)$ is an axially symmetric solution of the homogeneous differential equation

$$\Delta F+\frac{4M}{3h^2}(E'+E_D-U_\pm(r))F=0 \quad (10)$$

with the asymptotic form

$$F(r,\theta)\cong e^{ikz}+\frac{e^{ikr}}{r}f(\theta) \quad (11)$$

where $k=\frac{2}{h}\sqrt{\frac{M(E'+E_D)}{3}}$ and $U_\pm(r)$ are auxiliary potentials, which will be determined in the following section.

§2. Estimation of the Scattering Cross Section.

In order to obtain an approximate value of the scattering cross section without solving the complicated integral equation, it is needed to choose the auxiliary potentials $U_\pm(r)$ in (9) and (10) such that $F_\pm(r,\theta)$ become already approximate expressions for the required function $\varphi_\pm(r,\theta)$. If we consider the deuteron to be a sphere of diameter $\frac{1}{\alpha}=\frac{h}{\sqrt{ME_D}}=4.36\times10^{-13}$ cm and the nuclear forces to have a definite range $a=2.32\times10^{-13}$ cm, the interaction between the neutron and the deuteron can be represented roughly by a potential hole with the radius b, which may take a value between $\frac{1}{2\alpha}=2.18\times10^{-13}$cm and $\frac{1}{2\alpha}+a=4.5\times10^{-13}$cm. Thus we can put

$$U_\pm(r)=U_\pm=\text{const. or } 0$$

according as $r<b$ or $r>b$.

A legitimate value for U_+ can be obtained by assuming that $(10)_+$ has a solution with the energy

$$E'=-E_T=-8.3\times10^6eV$$

corresponding to the normal state of ^3H. Such a procedure was already

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used by Massey and Mohr.(1) Thus we find

$$-U_+=13.8\times10^6 eV \quad\text{or}\quad 32.5\times10^6 eV$$

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for the extreme case $b=4.5\times10^{-13}$ cm or 2.18×10^{-13} cm and consequently the cross section of the deuteron at rest for slow neutrons becomes

$$\sigma_+=2.7\times10^{-24}\,\text{cm}^2 \quad\text{or}\quad 1.46\times10^{-24}\,\text{cm}^2.$$

We have calculated further the cross section for several values of the neutron energy, the results being shown in Fig. 1 and 2. In the former case σ_+ reaches to a large maximum and then decreases as the energy increases on account of the presence of the virtual P-level, while in the latter case it decreases steadily with the energy.

On the other hand, U_- can not be determined in like manner, as little is known of the excited states of ^3H, and we can say only that $-U_-$ is much smaller than $-U_+$ or may even be negative. Thus the cross section σ_- in this case can be determined only by making use of further experimental information as follows. Namely, the observed cross section of the deuteron in heavy water is $\sigma_c=4\times10^{-24}$ cm² for slow neutrons of C-group according to Dunning, Pegram, Fink and Mitchell.(2) If we consider the effect of chemical binding of deuterons,(3) the cross section σ_f of the free deuteron for slow neutrons should be about half of the above σ_c, so that we obtain a probable ~~approximate~~ value $\sigma_f=2\times10^{-24}$ cm². This value is to correspond to the average

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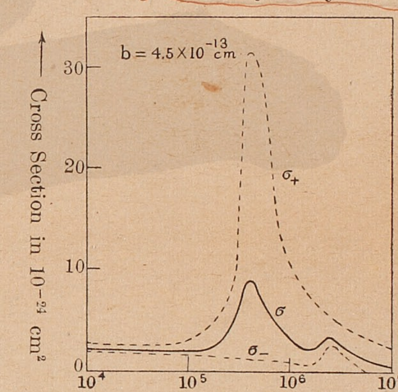
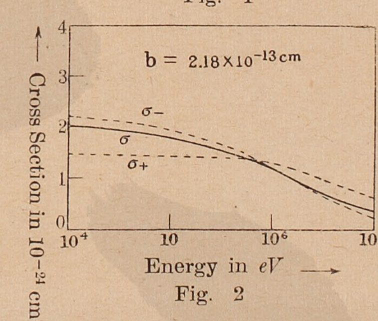


Energy in eV →
Fig. 1



Energy in eV →
Fig. 2

→図ヲ少シ右ヘヨセテ上図ト揃ヘルコト.

$$\sigma=\frac{1}{4}\sigma_+ + \frac{3}{4}\sigma_- \tag{12}$$

of the cross sections of the deuterons for slow neutrons in two cases above considered.

Hence, we can choose U_- with a given valus of b so as to make

(1) Massey and Mohr, Proc. Roy. Soc. A, **148**, 206, 1935.
(2) Dunning, Pegram, Fink and Mitchell, Phys. Rev. **48**. 265, 1935.
(3) Compare § 3.

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σ_- satisfy the relation

$$\frac{1}{4}\sigma_+ + \frac{3}{4}\sigma_- = 2\times 10^{-24}\,\mathrm{cm}^2$$

for slow neutrons, where σ_+ takes the value determined with the same b. We obtain in this manner the results

$$\sigma_- = 1.8\times 10^{-24}\,\mathrm{cm}^2 \qquad -U_- = 1.9\times 10^6\,eV$$

for $b = 4.5\times 10^{-13}$ cm and

$$\sigma_- = 2.18\times 10^{-24}\,\mathrm{cm}^2 \qquad -U_- = 11\times 10^6\,eV$$

for $b = 2.18\times 10^{-13}$ cm. As shown in Fig. 1 and 2, σ_- decreases with the energy in the latter case, while it has a small hump due to the virtual S-level in the former case.

The average cross section σ given by (12) depends on the energy in the manner as indicated by the curve in Fig. 1 or 2. If we change b between the above two extreme values, the shape of the curve will be altered ~~therewith~~, so that the correct value of b will be determined, therewith when the detailed experimental results for various values of the neutron energy will be obtained. The cross section 1.71×10^{-24} cm^2 measured by Dunning and others[1] for fast neutrons from the Rn-Be source is an average over a wide energy range and is insufficient for the determination of b, although it seems to be in favour of the value of b nearer to 2.18×10^{-13} cm than to 4.5×10^{-13} cm.

In any case, the cross section of capture of slow neutrons by deuterons will be much smaller than that by protons, as the above results show that there is no true or virtual $^2S_-$ or $^4S_-$-levels of small energy, while, in the latter case, the existence of the virtual ^1S-level of energy about $12\times 10^4\,eV$ was expected ~~form~~ from theoretical arguments[2]. This seems to be in agreement with the recent experimental results of Kikuchi, Aoki and Takeda,[3] which indicate that the cross section of the γ-ray emission by collision of slow neutrons with deuterons is very small and has an upper limit 0.3×10^{-25} cm^2.

Thus far, we took only Majorana forces into account. The inclusion of small Wigner forces will not give rise to any substantial modification, while the presence of Heisenberg forces will result in the further splitting up and coupling of two set of states above considered,

(1) Loc. cit.
(2) Fermi, loc. cit.
(3) Kikuchi, Aoki and Takeda, Sci. Pap. Inst. Phys. Chem. Res. 31, 195, 1937.

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but our method of estimation is too crude to afford such discussions of finer details.

It shoud be noticed that the angular distribution of scattered neutrons is, of course, spherically symmetric in the relative coordinate system for small energies, so that the probability of it being scattered by an angle between θ and $\theta+d\theta$ in the ordinary coordinate system becomes

$$\frac{1}{2}\left(\frac{3+2\cos^2\theta}{2\sqrt{3+\cos^2\theta}}+\cos\theta\right)\sin\theta d\theta. \qquad (13)$$

Thus, most of the neutrons are scattered into the forward direction as shown in Fig. 3.

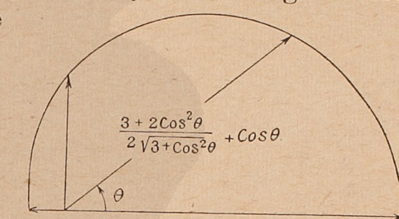

Fig. 3. Angular Distribution of Scattered Neutrons.

§3. Effect of Chemical Binding of Deuterons on their Collision with Slow Neutrons.

In the above calculations, we assumed the deuterons to be free and initially at rest, but we have to consider the effect of chemical binding of heavy hydrogen atoms with other atoms or with one another, if the neutron energy is few volts or smaller. Such an effect in the case of ordinary hydrogen atoms was dealed with in detail by Fermi.(1) The procedure can easily be extended to a more general case, in which the scattering nuclei have the mass M' not equal to that of the neutron, at least in the limit of the strong binding.

Namely, if the energy of the neutron is small compared with that of the chemical binding of heavy hydrogen atoms in the scatterer, the cross section becomes

$$\sigma_b=\left(\frac{M+M'}{M'}\right)^2\sigma_f \qquad (14)$$

and the angular distribution of the scattered neutrons is spherically symmetric in the ordinary coordinate system, where σ_f is the cross section of free deuterons for slow neutrons, as was calculated in §2.

Hence, the ratio is

$$\frac{\sigma_b}{\sigma_f}=2.25 \qquad (M'=2M)$$

for the deuteron in contrast to 4 for the proton $(M'=M)$ and 1 for

(1) Loc. cit.

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the heavy nucleus ($M' \gg M$). Thus, the cross section of deuterons in heavy water for thermal neutrons will be nearly twice as large as that for slow neutrons with the energy large compared with that of the chemical binding, as was pointed out already in §2.

It should be noticed, on the other hand, that the calculation made by the method of Fermi leads, in the limit of the weak binding, to the result which is not consistent with that of the preceding section. Namely, the cross section thus obtained differs from σ_f of §2 by a factor

$$\frac{1}{2}\left(\frac{M+M'}{M'}\right)^2\left(\frac{M'}{M+M'}+\frac{M'-M}{2M}\log\frac{M'+M}{M'-M}\right), \tag{15}$$

which takes the value 1.368 for the deuteron ($M'=2M$) and is not equal to 1 except for the proton and the heavy nucleus. Correspondingly, the angular distribution of scattered neutrons becomes

$$\text{const.}\times\frac{M\cos\theta+\sqrt{M'^2-M^2\sin^2\theta}}{M+M'}\sin\theta\, d\theta, \tag{16}$$

in contradistinction to

$$\frac{1}{2}\left(\frac{M'^2-M^2+2M^2\cos^2\theta}{M'\sqrt{M'^2-M^2\sin^2\theta}}+\frac{2M}{M'}\cos\theta\right)\sin\theta\, d\theta, \tag{17}$$

both in the ordinary coordinate system, which are also at variance with each other except for $M'=M$ and $M' \gg M$.

Closer examination shows that the origin of this discrepancy is the illegitimate use of Born's approximation, on the part of Fermi's method, in the limit of the weak binding, the agreement of the result of both methods in the case of free protons being only accidental. Nevertheless, Fermi's results ~~seems to be always true in the limit~~ of the strong binding. can be used safely ~~to~~ for the case

§4. Slowing Down of Neutrons by Multiple Collision with Deuterons.

The energy distribution of neutrons with the initial energy E_0 will be constant throughout the interval $\left(\frac{E_0}{9}, E_0\right)$ after the collision with deuterons initially at rest, if we assume the scattering is spherically symmetric in the coordinate system, in which the centre of mass is at rest. Hence, the mean energy after a single collision is $\frac{5}{9}E_0$ and reduces to $\left(\frac{5}{9}\right)^n E_0$ after n collisions, which does not differ very much from

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the corresponding value $\left(\frac{1}{2}\right)^n E_0$ in the case of collisions with proton.[1] Thus, the deuteron seems to have an effect on the slowing down of neutrons comparable with the proton.

There are, however, many reasons which lead us to the opposite conclusion. Firstly, the neutron energy can not be smaller than $\frac{1}{9}E_0$ by a single collision, so that a neutron with the energy of the order of $10^6 eV$ should undergo at least 6 collisions with deuterons before it becomes a slow neutron of several volts, while this can be effected by a single collision with the proton. Secondly, the occurrence of such a multiple collision with the former will be much rarer than that with the latter under similar conditions, because the cross section of the former becomes much smaller than that of the latter as the energy of the neutron decreases. Thirdly, the energy distribution function of neutrons after n collisions with the former, which can be evaluated by using the general formulae of Condon and Breit,[2] has very small value in the neighborhood of the lower limit $\left(\frac{1}{9}\right)^n E_0$ compared with the corresponding value for the latter. The probabilities that the neutron has an energy smaller than $\frac{E_0}{9^4}$ after five collisions, for example, are 1.15×10^{-4} and 0.063 respectively. General state of affairs will be illustrated by Fig. 4. The ordinate shows the probability that the neutron has a fraction of its initial energy less than the abscissa after a number of collisions marked on each curve. Full curves refer to the collisions with deuterons and dashed curves to those with protons. Both the abscissa and the ordinate are in logarithmic scale.

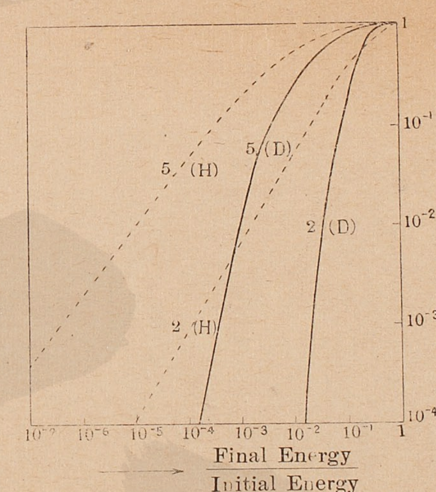

Fig. 4. Probalility that the neutron has an energy smaller than a fraction of the initial energy after multiple collison.

Under these circumstances, we can not expect theoretically any appreciable contribution of the deuteron compound such as heavy water to

(1) Condon and Breit, Phys. Rev. 49, 229, 1936.
(2) Loc. cit.

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the slowing down of neutrons. This conclusion is in good agreement with the recent experimental results of Kikuchi, Aoki and Takeda,[1] which show that the number of neutrons of C-group produced in D_2O by using $D-D$ source is at most 1/27 of that produced in the equal volume of H_2O, although quantitative comparison of the theory and the experiment is very difficult. Previous results of Dunning and others[2] indicating relative efficiencies about 1 : 5.5 of D_2O and H_2O for producing slow neutrons by using $Be-Rn$ source should be attributed to the contribution of B-group as pointed out by Kikuchi and others.[3]

D-D
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Be-Rn
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In conclusion, the authors ~~are indebted~~ want to acknowledge their indebtedness to Prof. S. Kikuchi for valuable discussions.

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Institute of Physics,
Osaka Imperial University.

(1) Loc. cit.
(2) Loc. cit.
(3) Loc. cit.

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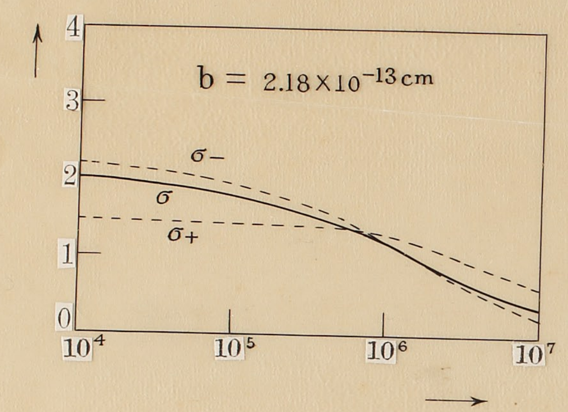

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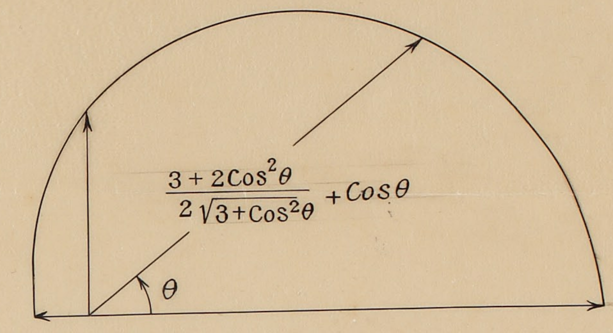

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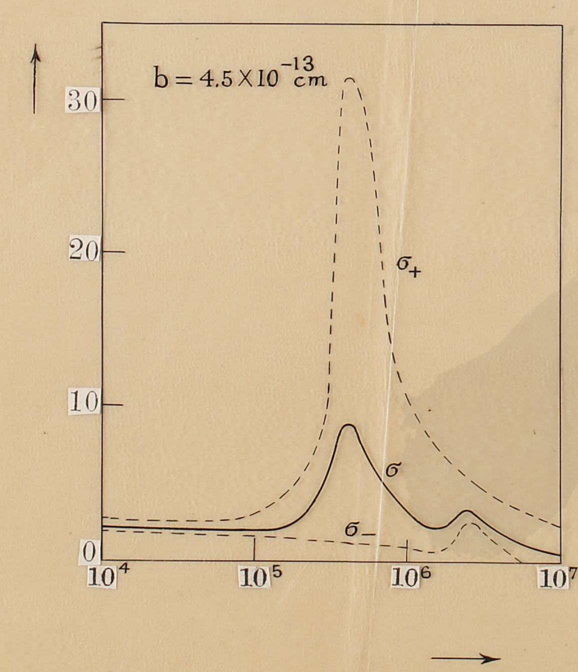

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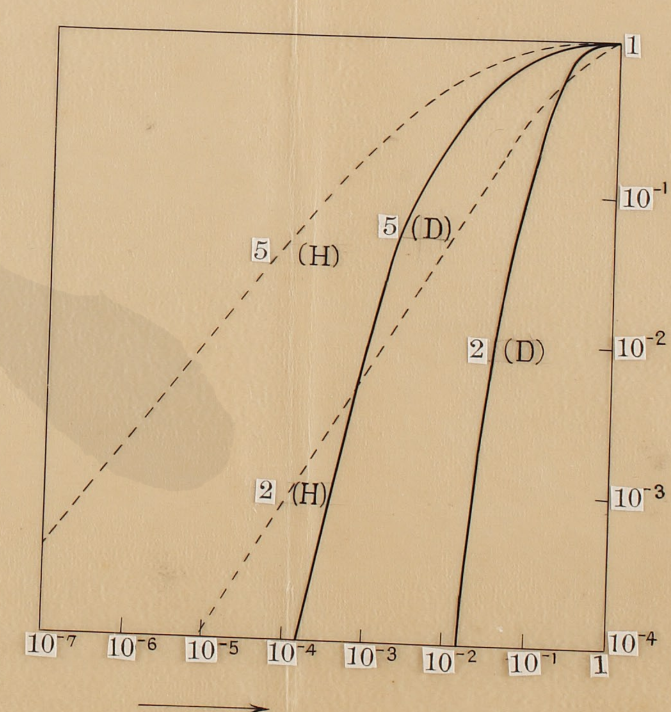

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