E 24 040 P 10

On the Slowing Down of Neutrons
by Collision with Deuterons

Jan. 29, 1937

中性子の最初の energy を E_0 とし. n 回の衝突後のそれを E とすると

$$\overline{x^s \cancel{\left(\frac{E}{E_0}\right)^s}} = \left[\frac{1-(1-\alpha)^{s+1}}{\alpha(s+1)}\right]^n \qquad x = \frac{E}{E_0}.$$

$$\equiv \cancel{\int_0^1 x^s f(x)\,dx} = \int_0^1 x^s f(x,\alpha)\,dx$$

$\cancel{\partial f(x) =}$

$$f(x,\alpha) = \sum a_m x^m.$$

$$\sum \int_0^1 x^s a_m x^n\,dx = \sum_{m=0}^{\infty} \frac{a_m(\alpha)}{m+s+1} = \left[\frac{1-(1-\alpha)^{s+1}}{\alpha(s+1)}\right]^n$$

$n=1$: $\quad a_m = 1. \quad (s+1)$

$$\alpha = \frac{4MM'}{(M+M')^2} = \frac{8}{9}$$

$$\bar{x} = \left(1-\frac{\alpha}{2}\right)^n = \left(\frac{5}{9}\right)^n$$

$$\overline{x^2} = \left\{\frac{1-(1-\alpha)^3}{3\alpha}\right\}^n = \left(1-\alpha+\frac{\alpha^2}{3}\right)^n = \left(\frac{1}{9}+\frac{8^2}{9^2\times 3}\right)^n$$

$$= \left\{\frac{1}{9}\left(1+\frac{64}{27}\right)\right\}^n = \left\{\frac{1}{9^2}\cdot\left(\frac{91}{3}\right)\right\}^n$$

$$(\bar{x})^2 = \left\{\frac{25}{9^2}\right\}^n$$

$$\delta^2 = \frac{\overline{x^2}-(\bar{x})^2}{\bar{x}^2} = \left\{\cancel{9^2}\right\}\left(\frac{\frac{91}{3}-25}{25}\right)^n = \left(\frac{5.33\cdots}{25}\right)^n$$

$$30.3\cdots$$

$$=\{0.21333\cdots\}^n$$

$$25\overline{)5.3333\ldots}$$

$$0.21\ 3333$$

$$\frac{1}{\sqrt{\pi}\,\delta}\int_{-\infty}^{\infty} \cancel{x}\, e^{-\frac{(x-\bar{x})^2}{\delta^2}}\,dx = \cancel{\sqrt{\pi}\,\delta} \qquad \frac{1}{\sqrt{\pi}\,\delta'}\int x^2 e^{-\frac{(x-\bar{x})^2}{\delta'^2}}\,dx$$

$$= \frac{1}{\sqrt{\pi}\,\delta'}\cdot\frac{1}{2}\sqrt{\pi}\,\delta'^3$$

$$= \frac{\delta'^2}{2}$$

$$\overline{(x-\bar{x})^2} = \overline{x^2} - \bar{x}^2 = \bar{x}^2\delta^2.$$

$$\delta'^2 = 2\bar{x}^2\delta^2$$

$$\boxed{\frac{1}{\sqrt{2\pi}\,\bar{x}\,\delta}\int_0^{\infty} e^{-\frac{(x-\bar{x})^2}{2\delta^2\bar{x}^2}}} \cancel{dx}$$

$$\bar{x} =$$

$$E_n = E_0 (1-\alpha x_1)(1-\alpha x_2) \cdots (1-\alpha x_n)$$

$x_i = \dfrac{1-\ln y_i}{2}$ $\qquad 0 \leq x_i \leq 1,$

$$\frac{E_n}{E_0} = \varepsilon_n = (1-\alpha x_1)(1-\alpha x_2)\cdots(1-\alpha x_n) = y_1 y_2 \cdots y_n$$

$\cancel{1-\alpha x_i = y_i \qquad 1-\alpha \leq y_i \leq 1}$

$\cancel{\log(1-\alpha x_i) = u_i \qquad \log\frac{1}{1-\alpha} \geq u_i \geq 0}$

$\cancel{\varepsilon_n = \prod e^{-u_i}} =$ $\qquad n\alpha \leq y_i \leq 1.$

$\cancel{\log \varepsilon_n = \sum_{i=1}^{n} u_i}$

$$\frac{\partial(x_1, x_2 \cdots x_n)}{\partial(\varepsilon_1, x_2 \cdots x_n)} = \begin{vmatrix} 1 & \cdot & \cdot & \cdot & \cdot \\ 0 & 1 & & & \\ 0 & & 1 & & \\ 0 & & & 1 & \\ 0 & & & & 1 \\ 0 & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

$$= -\alpha(1-\alpha x_1)(1-\alpha x_2)\cdots(1-\alpha x_{n-1}).$$

$$\int\!\!\int_{\varepsilon_n \leq \varepsilon_n^{(0)}} dx_1 dx_2 \cdots dx_n = \int\!\!\int \frac{dx_1 dx_2 \cdots dx_{n-1}\, d\varepsilon_n}{\alpha(1-\alpha x_1)\cdots(1-\alpha x_{n-1})}$$

$$= \frac{d\varepsilon_n}{\alpha} \qquad \int_{x_{n-1}}^{\alpha} \frac{dx_{n-1}}{1-\alpha x_{n-1}}$$

$$dx = e^{-u}du$$

$$\int_0^u f_n(u)\,du = \frac{1}{a^n(n-1)!}\left\{\int_0^u u^{n-1}\,du\right.$$

$$\int_a^u e^{-u}(u-a)^{n-1}du$$

$$\int_0^{x} \quad -\binom{n}{1}e^{-a}\int_0^{u-a}$$

$$+\binom{n}{2}e^{-2a}\int_0^{u-2a}$$

$$P_n(u) - \binom{n}{1}e^{-a}P_n(u-a) + \binom{n}{2}e^{-2a}P_n(u-2a)$$

$$+ \cdots$$

$$\int_0^u$$

$$u<a$$

$$\frac{\partial(y_1 y_2 \cdots y_{n-1}\varepsilon_n)}{\partial(x_1, y_2 \cdots y_{n-1} y_n)} = y_1 y_2 \cdots y_{n-1}$$

$$\cancel{\iint dy_1 dy_2 \cdots dy_n =} \int\cdots\int \frac{dy_1 dy_2 \cdots dy_{n-1}}{y_1 y_2 \cdots y_{n-1}}$$

$\varepsilon_n = \mathrm{const.}$

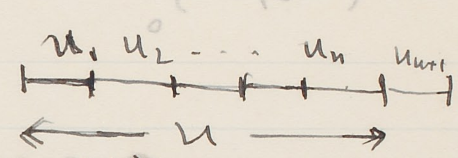
$$1 \geq y_{n-1} \geq \varepsilon_n$$

$$1 \geq \cancel{\frac{\varepsilon_n}{y_{n-1}} \geq}\ y_{n-2} \geq \frac{\varepsilon_n}{y_{n-1}}$$

$$1 \geq y_{n-3} \geq \frac{\varepsilon_n}{y_{n-2}}$$

$$\vdots$$

$$1 \geq y_1 \geq \frac{\varepsilon_n}{y_{n-1} y_{n-2} \cdots y_2}$$



$$I_n^{(n+1)} = I$$

$$= \int \log \frac{y_{n-1} y_{n-2} \cdots y_2}{\varepsilon_n} \cdot \frac{dy_2}{y_2} \cdots \frac{dy_{n-1}}{y_{n-1}}$$

$$= \int \left\{ \log y_2 + \log \frac{y_{n-1} \cdots y_3}{\varepsilon_n} \right\} \frac{dy_2}{y_2} \cdots\cdots$$

$$= \int \left\{ \frac{\log^2 y_2}{2} \Big|_{\frac{\varepsilon_n}{y_{n-1}\cdots y_3}}^{1} + \log \frac{y_{n-1}\cdots y_3}{\varepsilon_n} \cdot \log y_2 \Big|_{\frac{\varepsilon_n}{y_{n-1}\cdots y_3}}^{1} \right\} \frac{dy_3}{y_3} \cdots$$

$$= \int \left\{ \frac{\sum_{i=3}^{n} u_i - \varepsilon_n}{2} \right\} + \cancel{\log}\ \frac{y_{n-1}\cdots y_3}{\varepsilon_n} \cdots$$

$$\frac{\partial(y_1 y_2 \cdots y_{n-1}\,\varepsilon_n)}{\partial(y_1 y_2 \cdots y_{n-1}\,y_n)} = y_1 y_2 \cdots y_{n-1} \qquad a = \log(1-\alpha)^{-1}$$

$$\int\cdots\int \frac{dy_1 dy_2 \cdots dy_{n-1}}{y_1 y_2 \cdots y_{n-1}} = \int\cdots\int du_1 du_2 \cdots du_{n-1}$$

ε_n = const.

$1 \geq y_{n-1} \geq \varepsilon_n,\ 1-\alpha \qquad 0 \leq u_{n-1} \leq \log \varepsilon_n^{-1} = \delta_n,\ a$

$1 \geq y_{n-2} \geq \dfrac{\varepsilon_n}{y_{n-1}},\ 1-\alpha \qquad 0 \leq u_{n-2} \leq \log \varepsilon_n^{-1} - u_{n-1},\ a$

$0 < \varepsilon_n - \sum_{i=1}^{n-1} u_i < a \qquad 0 \leq u_i \leq a$

$1 \geq y_{n-3} \geq \dfrac{\varepsilon_n}{y_{n-2} y_{n-1}},\ 1-\alpha \quad \to \log y_i = -u_i \qquad 0 \leq u_{n-3} \leq \log \varepsilon_n^{-1} - u_{n-2} - u_{n-1},\ a$

$\cdots$

$1 \geq y_1 \geq \dfrac{\varepsilon_n}{y_2 y_3 \cdots y_{n-1}},\ 1-\alpha \qquad 0 \leq u_1 \leq \log \varepsilon_n^{-1} - u_2 - u_3 \cdots - u_{n-1},\ a$

$\log \varepsilon_n^{-1} < a$

$$\int\cdots\int du_1 du_2 \cdots du_{n-1} = \int\cdots\int_0^{\delta_n - \sum_{i=2}^{n-1} u_i} du_1\, du_2 \cdots$$

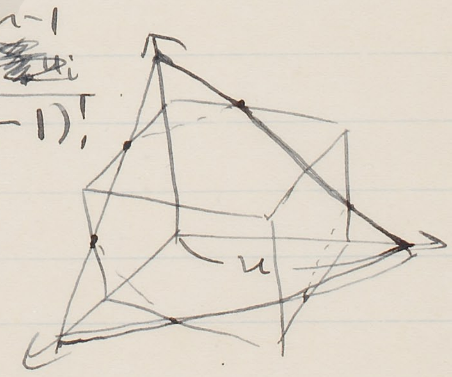
$$= \int\cdots\int \Big(\delta_n - \sum_{i=2}^{n-1} u_i\Big) du_2 \cdots$$

$$= \int\cdots\int \delta_n\Big(\delta_n - \sum_{i=3}^{n-1} u_i\Big) - \sum_{i=3}^{n-1} u_i \Big(\delta_n - \sum_{i=3}^{n-1} u_i\Big) - \frac{\big(\delta_n - \sum_{i=3}^{n-1} u_i\big)^2}{2} \int du_3 \cdots$$

$$= \int\cdots\int \frac{\big(\delta_n - \sum_{i=3}^{n-1} u_i\big)^2}{2}\, du_3\, du_4 \cdots \qquad 0 \leq u_i \leq a$$

$$= \int\cdots\int \frac{\big\{\big(\delta_n - \sum_{i=4}^{n-1} u_i\big) - u_3\big\}^2}{2}\, du_3\, du_4 \cdots$$

$$= \int\cdots\int \frac{\big\{\delta_n - \sum_{i=4}^{n-1} u_i\big\}^3}{2\cdot 3}\, du_4 \cdots = \frac{\delta_n^{\,n-1}}{(n-1)!}$$



$$= \frac{(\log \varepsilon_n^{-1})^{n-1}}{(n-1)!}$$

$a < \log \varepsilon_n^{-1} < 2a$ (δ_n)

$$n\,\frac{(\delta_n - a)^{n-1}}{(n-1)!}$$

$u^{n-1} \qquad (u-a)^{n-1}$

$a < u_1, u_2, \ldots, u_{n-1} < \log \varepsilon_n^{-1}$

$$I(u,p) = \frac{\int_0^u e^{-v} v^p dv}{\int_0^\infty e^{-v} v^p dv} = \int_0^{u\sqrt{p+1}} e^{-v} v^p dv = \int_0$$

$v = \sqrt{p+1}\cdot u$

$v = u$

n回の衝突後のenergyをE_nとし、 $\frac{E_n}{E_0} = x = e^{-u}$ とする.

$\alpha = \frac{4mM}{(m+M)^2}$, $a = \log(1-\alpha)^{-1}$ とおくと,

$1 > x > 1-\alpha$ の $0 < u < a$; $F_n(x)dx = \frac{e^{-u}}{(n-1)!} u^{n-1} du$

$1-\alpha > x > (1-\alpha)^2$ の $a < u < 2a$; $F_n(x)dx = e^{-u} du \{ u^{n-1} - n(u-a)^{n-1} \}$

.....

$(1-k\alpha)^k > x > (1-(k+1)\alpha)^{k+1}$ の $ka < u < (k+1)a$; $F_n(x)dx = e^{-u} du \{ u^{n-1}$
$- n(u-a)^{n-1} + \frac{n(n-1)}{2}(u-2a)^{n-1} \cdots + (-1)^k \binom{n}{k} (u-ka)^{n-1} \}$

$1-k\alpha \geq x > 1-(k+1)\alpha$

$$N(x) = \int_0^x F_n(x)dx \propto P_n(u) - n e^{-a} P_n(u-a) + \frac{n(n-1)}{2} e^{-2a} P_n(u-2a) - \cdots + (-1)^k \frac{n(n-1)\cdots(n-k+1)}{1 \cdot 2 \cdots k} e^{-ka} P_n(u-ka).$$

with $P_n(u) = \int_u^\infty$

$M = 2m$ $\alpha = \frac{8}{9}$ $a = \log 9$. $e^{-a} = \frac{1}{9}$.

$u = \log\frac{1}{x}$

$N(x) = \frac{1}{\Gamma(n)} P_n(u) - \frac{n}{9} P_n(u - \log 9) + \cdots\cdots$

$n = 5$: $P_n(u) = \frac{\cancel{\Gamma(p+1)}}{\Gamma(n)} I\left(\frac{u}{\sqrt{n}}, n-1\right)$

$P_n(u) = \int_u^\infty = \int_0^\infty - \int_0^u \propto 1 - I\left(\frac{u}{\sqrt{n}}, n-1\right)$

$\cancel{\int_0^u e^{-u} u^{n-1} du = \frac{\quad}{n!}}$

$x > \left(\frac{1}{9}\right)^5$ $u < 5(\log 9)$.

$u = \cancel{\log}\, 0$;　$\cancel{N(1) = 1}$

$u = \log 9$;　$N(\frac{1}{9}) = 1 - I(\frac{\log 9}{\sqrt{5}}, 4) - \frac{5}{9} I(1 -$

$$N(x) = \int_{(1-\alpha)^5 > x > (1-\alpha)^{k+1}}^{1} F_n(x)\,dx = \int_0^u e^{-u} u^{n-1} du - n \int_a^u e^{-u} (u-a)^{n-1} du$$

$+ \cdots$

$$= \int_0^u e^{-u} u^{n-1} du - n e^{-a} \int_0^{u-a} e^{-u} u^{n-1} du + \cdots\cdots$$

$x = 1$; $u = 0$.　$N(x) = 0$.

$$\cancel{\ } = I(\tfrac{u}{\sqrt{n}}, n-1) - n e^{-a} I(\tfrac{u-a}{\sqrt{n}}, n-1) + \cdots$$

$n = 5$.　$e^{-a} = \frac{1}{9}$.

$N(x) =$

$$I(\tfrac{u}{\sqrt{5}}, 4) - \tfrac{5}{9} \cdot I(\tfrac{u-\log 9}{\sqrt{5}}, 4) + \tfrac{5 \cdot 4}{2(9)^2} I(\tfrac{u-2\log 9}{\sqrt{5}}, 4)$$

$$- \tfrac{5 \cdot 4 \cdot 3}{2 \cdot 3 \cdot 9^3} I(\tfrac{u-3\log 9}{\sqrt{5}}, 4) + \tfrac{5 \cdot 4 \cdot 3 \cdot 2}{2 \cdot 3 \cdot 4 \cdot 9^4} I(\tfrac{u-4\log 9}{\sqrt{5}}, 4)$$

$$- \cancel{\tfrac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5\ 9^5} I(\tfrac{u-5\log 9}{\sqrt{5}}, 4)}$$

$u = 0$;　$N(1) = 0$.

$u = \log 9$;　$N(\frac{1}{9}) = I(\frac{\log 9}{\sqrt{5}}, 4)$

$u = 2\log 9$;　$N(\frac{1}{9^2}) = I(\frac{2\log 9}{\sqrt{5}}, 4) - \frac{5}{9} I(\frac{\log 9}{\sqrt{5}}, 4)$

$u = 3\log 9$;　$N(\frac{1}{9^3}) = I(\frac{3\log 9}{\sqrt{5}}, 4) - \frac{5}{9} I(\frac{2\log 9}{\sqrt{5}}, 4) + \frac{10}{81} I(\frac{\log 9}{\sqrt{5}}, 4)$

$u = 4\log 9$;　$N(\frac{1}{9^4}) = I(\frac{4\log 9}{\sqrt{5}}, 4) - \frac{5}{9} I(\frac{3\log 9}{\sqrt{5}}, 4) + \frac{10}{81} I(\frac{2\log 9}{\sqrt{5}}, 4) - \frac{10}{9^3} I(\frac{\log 9}{\sqrt{5}}, 4)$

$u = 5\log 9$; $N\left(\frac{1}{9^5}\right) = I\left(\frac{5\log 9}{\sqrt{5}}, 4\right) - \frac{5}{9} I\left(\frac{4\log 9}{\sqrt{5}}, 4\right) + \frac{5}{9^4} I\left(\frac{\log 9}{\sqrt{5}}, 4\right)$
$+ \frac{10}{81} I\left(\frac{3\log 9}{\sqrt{5}}, 4\right) - \frac{10}{9^3} I\left(\frac{2\log 9}{\sqrt{5}}, 4\right)$

$\log 9 = 2{,}1972$
$\sqrt{5} = 2{,}236$

$2{,}236\overline{)2{,}1972}$ = 0,9826
20124
18480
17888
5920
4472
14480

$\frac{\log 9}{\sqrt{5}} = 0{,}983$ $I = 0{,}072$
2 〃 $= 1.965$ $I = 0{,}45$
3 〃 $= 2{,}948$ $I = 0{,}78$
4 〃 $= 3.930$ $I = 0{,}937$!!!
5 〃 $= 4.913$ $I = 0.985$

$N(1) = 0$
$N\left(\frac{1}{9}\right) = 0.072$
$N\left(\frac{1}{9^2}\right) = 0{,}41$
$N\left(\frac{1}{9^3}\right) = 0{,}54$
$N\left(\frac{1}{9^4}\right) = 0{,}569$
$N\left(\frac{1}{9^5}\right) = 0{,}554$

0,056
$9)0{,}5$

0,008
$9)0.072$
0,04
0,45
0,04
0.

0,78
−0,25
0.009
0,539

0.0009
$9)0{,}008$

0,78
$9)3{,}90$
0,433

0,937
0,056
0,993
0,434
0.559

0,433
0.001
0,434

0.001
$81)0{,}08$

0,937
$9)4{,}685$
0,521

0,985
0.096
0,00006
1.081
,527
.554

0.521
0.006

0,096
$81)7{,}8$
729
510

0.05
$9)0{,}45$

0.006
$81)0{,}500$
0.008
$9)0{,}072$
0.0001

$\int_0^{(1-\alpha)^n} \Gamma_n(x)\,dx \propto P_n(na) - ne^{-a}P.$

$N\left((1-\alpha)^n\right) = \int_0^{na} e^{-u}u^{n-1}du - ne^{-a}\int_0^{(n-1)a} e^{-u}u^{n-1}du$
$+ \frac{n(n-1)}{2}\int_0^{(n-2)a} e^{-u}u^{n-1}du - \cdots = \int_0^{na}$

$$e^{-u}\left\{u^{n-1}-\binom{n}{1}(u-a)^{n-1}+\cdots\right\}$$

$$\int_0^{na} e^{-u}u^{n-1}du-\binom{n}{1}\int_a^{na} e^{-u}(u-a)^{n-1}du+\binom{n}{2}\int_{2a}^{na} e^{-u}(u-2a)^{n-1}du$$

$$-\cdots$$

$$=\int_0^{na} e^{-u}u^{n-1}du-\binom{n}{1}\left\{e^{-a}\int_0^{(n-1)a} e^{-u}u^{n-1}du+\binom{n}{2}\cdots\right.$$

$$+(-1)^{n-1}e^{-(n-1)a}\binom{n}{n-1}\int_0^{a} e^{-u}u^{n-1}du+0.$$

$$=\int$$

$$0-\binom{n}{n-1}e^{-a}\int_{(n-1)a}^{na} e^{-u}u^{n-1}du+\cdots$$

$$+(-1)^{n-1}\binom{n}{1}e^{-(n-1)a}\int_a^{na} e^{-u}u^{n-1}du$$

$$+(-1)^{n}e^{-na}\int_0^{na} e^{-u}u^{n-1}du.$$

$$=\int_{(n-1)a}^{na} e^{-u}u^{n-1}du+e^{-u}u^{n-1}\Big|_{(n-1)a}^{0}+(n-1)\int_0^{(n-1)a} e^{-u}u^{n-2}du$$

$$-\binom{n}{1}e^{-a}\left(e^{-u}u^{n-1}\Big|_{(n-1)a}^{0}+n(n-1)e^{-a}\int_0^{(n-1)a} e^{-u}u^{n-2}du\right.$$

$$+\cdots$$

$$=\qquad -(n-1)^2e^{-a}\int_0^{(n-1)a}\cdots\qquad e^{-u}$$

$$\frac{[1-e^{-a\frac{\partial}{\partial u}}]^n u^{n-1}}{(n-1)!}+(-1)^n\cdot n!\,(u-(n-1)a)^{n-1}$$

$(-1)^{n-1} \cancel{x} (u - \cancel{\{n\,a\}}\ na)^{n-1} e^{-u}$

$\cancel{u = na} < 0.$ n; even.

$= \underline{(na-u)^{n-1} e^{-u}}$

$n=5.$ $a = \log 9.$

$(5\log 9 - u)^{\cancel{n-1}\,4} e^{-u}$

$u = 4\log 9.$

$\cancel{5}(\log 9)^4 e^{-4\log 9} = \underline{9^4 (\log 9)^{-4}}. = \left(\frac{\log 9}{9}\right)^4.$

$= \left(\frac{2.197}{9}\right)^4$

$\int_{(n-1)a}^{na} (na-u)^{n-1} e^{-u}\, \cancel{du} = e^{-na} \int_0^a v^{n-1} e^{+v} dv$

$na - u = v.$ $\cancel{= e^{-na} I\left(\frac{a}{\sqrt{n}}, n-1\right)}$

$-du = dv$ $= \cancel{e^{-na}} = e^{-na}\{e^a a^{n-1} - (n-1)e^a a^{n-2} + (n-1)(n-2) e^a a^{n-3} - \dots\}$

$n=5.$ $a = \log 9$

$\cancel{9^{-5} I\left(\frac{\log 9}{\sqrt{5}}, 4\right) = 9^{-5} \times 0.072}$

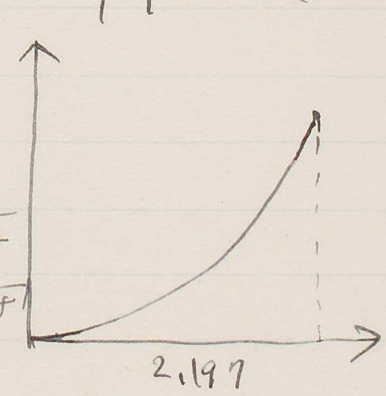
$n=5;$

$e^{-5a} \int_0^a v^4 e^v dv = e^{-\cancel{5}a} \cancel{4} \cdot \{\cancel{a^4 - (n-1)} a^3$

$= e^{-4a} \{a^4 - 4a^3 + 12a^2 - 24a + 24\}$

$a = 2.197$

$a^2 =$



2.2
2.2
4.4
4.4
4.84

2.197
2.197
0

$= 9^{-4}\{(2.197)^4 - 4(2.197)^3 + 12(2.197)^2 - 24(2.197) + 24\}$

$\simeq$

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~~Note~~

Slowing Down by Deuterons

deuteron によって neutron が spherically symmetric に scatter されるとすると，n(回)の scattering の後の energy distribution は

$u = \log \frac{E_0}{E_n}$ とおくと．

$$F_n(u)du \propto \frac{e^{-u}\,du}{\cancel{\alpha^2}\,(n-1)!}\left\{u^{n-1} - \binom{n}{1}(u-a)^{n-1} + \binom{n}{2}(u-2a)^{n-1} - \cdots + (-1)^{n-1}\binom{n}{n-1}\left(u-(n-1)a\right)^{n-1}\right\}$$

$$a = \log(1-\alpha)^{-1} \qquad \alpha = \frac{4MM'}{M+M'} = \frac{8}{9}$$

$$\therefore\ a = \log 9 =$$

$$\cancel{f_1(u)du = \frac{1}{\alpha}} \qquad \text{for } 0<u<a.$$

$$F_1(u)du \propto \frac{e^{-u}}{\cancel{\alpha}}du \qquad \text{for } 0<u<a. \qquad 1-e^{-a}$$

$$\boxed{e^{-u} = \frac{E_n}{E_0} \qquad e^{-u}du = \frac{dE_n}{E_0}}$$

$$f_1(E_1)dE_1 = \frac{dE_1}{\alpha E_0} \qquad \text{for } (1-\alpha) < \frac{E_1}{E_0} < 1$$

$$F_2(u)du \propto \frac{e^{-u}du}{\cancel{\alpha}}\,u \qquad \text{for } 0<u<a$$

$$\text{or } f_2(E_2)dE_2 \propto \frac{dE_2}{E_0}\left(\log\frac{E_0}{E_2}\right) \qquad \text{for } (1-\alpha) < \frac{E_2}{E_0} < 1.$$

$$F_2(u)du \propto e^{-u}du\left(u - 2(u-a)\right) \qquad \text{for } a<u<2a$$

$$\text{or } f_2(E_2)dE_2 \propto \frac{dE_2}{E_0}\left(\cancel{\log\frac{E_0}{E_2}}\ \log\frac{E_0}{E_2(1-\alpha)^2}\right)$$

$$\text{for } (1-\alpha)^2 < \frac{E_2}{E_0} < (1-\alpha)$$

$$(n-1)(na)^{n-1} - n(n-1)\alpha^{n-1} + (n(n-1)(n-2)^{n-2}$$

$$\frac{d}{du}\left(e^{-u}\alpha\right) = 0 \qquad \frac{d}{du}\left(e^{-u}(2a-u)\right) = (2a-u)+\ldots$$

$$-u+1=0. \qquad \frac{d}{du}\left(e^{-2u}u^{n-1}\right) = 0 \quad -u+(n-1)=0.$$

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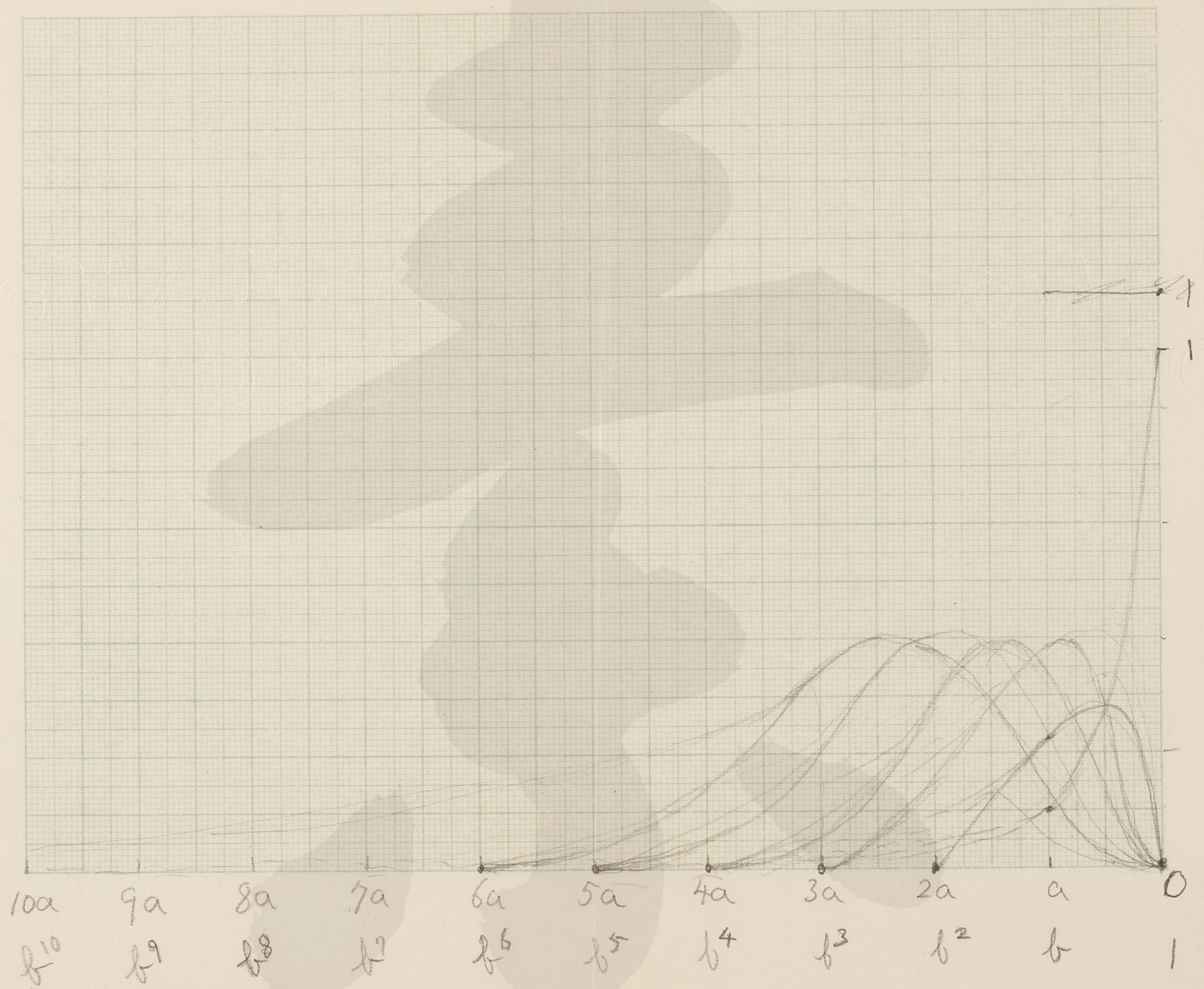

$$b = 1 - \frac{4MM'}{M+M'}$$
$$= \frac{1}{9},$$
$$a = \log b^{-1}$$
$$= \log 9$$
$$= 2.19722$$

$$u = \log \frac{E_0}{E}$$

$$\frac{E}{E_0}$$

$N(x) = \int_0^{x}$ $\qquad \alpha = \frac{4mM}{(m+M)^2} \qquad a = \log(1-\alpha)^{-1}$

$\frac{E_n}{E_0} = x = e^{-u}$

$1 > x > 1-\alpha$ or $0 < u < a$; $F_n(x)dx = e^{-u} u^{n-1} du$.

$1-\alpha > x > (1-\alpha)^2$ or $a < u < 2a$; $F_n(x)dx = e^{-u}\{u^{n-1} - n(u-a)^{n-1}\} du$.

$\vdots$

$(1-\alpha)^k > x > (1-\alpha)^{k+1}$ or $ka < u < (k+1)a$; $F_n(x)dx = e^{-u}\{u^{n-1} - n(u-a)^{n-1}$
$+ \frac{n(n-1)}{2}(u-2a)^{n-1} - \cdots + (-1)^k \binom{n}{k}(u-ka)^{n-1}\}$

$N(x) = \int_0^{(1-\alpha)^k > x > (1-\alpha)^{k+1}} F_n(x)dx = \int e^{-u} u^{n-1} du - n \int^{na} e^{-u} (u-a)^{n-1} du$

$+ \cdots (-1)^k \binom{n}{k} \int e^{-u} (u-ka)^{n-1} du + \cdots$

$= \int_0^{na} e^{-u} u^{n-1} du - n e^{-a} \int_0^{(n-1)a} e^{-u} u^{n-1} du$

$+ \cdots (-1)^k \binom{n}{k} e^{-ka} \int_0^{(n-k)a} e^{-u} u^{n-1} du + \cdots$

$N(1) = \int_0^{na} e^{-u} u^{n-1} du - n e^{-a} \int_a^{na} e^{-u} (u-a)^{n-1} du$

$+ \cdots + (-1)^k \binom{n}{k} \int_{ka}^{na} e^{-u} (u-ka)^{n-1} du$

$+ \cdots (-1)^{(n-1)} \binom{n}{n-1} \int^{na} e^{-u} (u-(n-1)a)^{n-1} du$

$= \int_0^{na} e^{-u} u^{n-1} du - n e^{-a} \int_0^{(n-1)a} e^{-u} u^{n-1} du$

$+ \cdots + (-1)^k \binom{n}{k} e^{-ka} \int_0^{(n-k)a} e^{-u} u^{n-1} du + \cdots (-1)^{(n-1)} \binom{n}{n-1} e^{-(n-1)a} \int_0^{a} e^{-u} u^{n-1} du$

$x(1-\alpha)^k > x(1-\alpha)^{k+1}$

$$N(x) = \int_0^{na} e^{-u} u^{n-1} du - \int_0^u e^{-u} u^{n-1} du$$

$$- n e^{-a} \int_{0}^{(n-1)a} \qquad + n e^{-a} \int_{0}^{u-a} e^{-u} u^{n-1} du$$

$$\cdots$$

$$+ (-1)^k e^{-ka} \binom{n}{k} \int_0^{(n-k)a} \qquad + (-1)^k e^{-ka} \binom{n}{k} \int_{0}^{u-ka} e^{-u} u^{n-1} du$$

$$+ \cdots \qquad + 0$$

$$= N(1) - \int_0^u e^{-u} u^{n-1} du + n e^{-a} \int_0^{u-a} e^{-u} u^{n-1} du$$

$$+ \cdots + (-1)^k e^{-ka} \binom{n}{k} \int_0^{u-ka} e^{-u} u^{n-1} du$$

$$I(u, p) = \frac{\int_0^{u\sqrt{p+1}} e^{-v} v^p dv}{\Gamma(p+1)}$$

$p = n-1$

$$= \frac{\int_0^{\sqrt{n}\cdot u} e^{-v} v^{n-1} dv}{\Gamma(n)}$$

$$\cancel{= \overline{(n-1)!}}$$

$$\int_0^u e^{-u} u^{n-1} du = (n-1)!\, I\left(\frac{u}{\sqrt{n}}, n-1\right)$$

$a = \log 9 = 2{,}1972 \qquad e^{-a} = \frac{1}{9}$

$n = 5. \qquad 1-\alpha = \frac{1}{9}$

$$N(1) = \int_0^{5a} e^{-u} u^4 du - \left(\frac{5}{9}\right)\int_0^{4a} e^{-u} u^4 du$$

$$+ \cdots + \left(\frac{1}{9}\right)^4 \cdot \frac{5\cdot 4\cdot 3\cdot 2}{1\cdot 2\cdot 3\cdot 4}\int_0^a e^{-u} u^4 du$$

$$= 4!\left\{ I\left(\frac{5a}{\sqrt{5}}, 4\right) - \frac{5}{9} I\left(\frac{4a}{\sqrt{5}}, 4\right) + \frac{10}{9^2} I\left(\frac{3a}{\sqrt{5}}, 4\right) \right.$$

$$\left. - \frac{10}{9^3} I\left(\frac{2a}{\sqrt{5}}, 4\right) + \frac{5}{9^4} I\left(\frac{a}{\sqrt{5}}, 4\right)\right\}$$

$$N\left(\frac{1}{9}\right) = N(1-\alpha) = N(1) - \int_0^a e^{-u} u^{n-1} du$$

$$= N(1) - 4!\left\{ I\left(\frac{a}{\sqrt{5}}, 4\right)\right\}$$

$$N\left(\frac{1}{9}\right)^2 = N\{(1-\alpha)^2\} = N(1) - \int_0^{2a} e^{-u} u^{n-1} du + \frac{5}{9}\int_0^a e^{-u} u^{n-1} du$$

$$= N(1) - 4!\left\{ I\left(\frac{2a}{\sqrt{5}}, 4\right) - \frac{5}{9} I\left(\frac{a}{\sqrt{5}}, 4\right)\right\}$$

$$N\left(\frac{1}{9}\right)^3 = N\{(1-\alpha)^3\} = N(1) - 4!\left\{ I\left(\frac{3a}{\sqrt{5}}, 4\right) - \frac{5}{9} I\left(\frac{2a}{\sqrt{5}}, 4\right) \right.$$

$$\left. + \frac{10}{9^2} I\left(\frac{a}{\sqrt{5}}, 4\right)\right\}$$

$$N\{(1-\alpha)^4\} = N(1) - 4!\left\{ I\left(\frac{4a}{\sqrt{5}}, 4\right) - \frac{5}{9} I\left(\frac{3a}{\sqrt{5}}, 4\right) \right.$$

$$\left. + \frac{10}{9^2} I\left(\frac{2a}{\sqrt{5}}, 4\right) - \frac{10}{9^3} I\left(\frac{a}{\sqrt{5}}, 4\right)\right\}$$

$$\cancel{= I\left(\frac{5a}{\sqrt{5}}, 4\right)}$$

$$= \int_{4a}^{5a} e^{-u} u^4 du - \frac{5}{9}\int_{3a}^{4a} e^{-u} u^4 du$$

$$+ \cdots + \frac{5}{9^4}\int_0^a e^{-u} u^4 du$$

$=$

$n=5$

$\frac{u}{\sqrt{n}}$	n	$I\left(\frac{u}{\sqrt{n}}, n-1\right)$
0		
.1		
.2		
.3		
.4		
.5		
.6		
.7		
.8		
.9		
1.0		

$\sqrt{5} = 2{,}236$

$= (n-1)a + v.$

$u = na - v.$

$$F_n(x)\,dx = e^{-(na-v)}\left\{(na-v)^{n-1} - n(na-v)^{n-1} + \frac{n(n-1)}{2}((n-2)a-v)^{n-1} - \cdots + (-1)^{n-1} n(a-v)^{n-1}\right\}$$

$$= e^{-(na-v)}\left\{n(v-a)^{n-1} - \frac{n(n-1)}{2}(v-2a)^{n-1} + \cdots\right\}$$

$$= \cancel{e^{-na}}\{$$

$$\int \cancel{F_n(x)\,dx = e^{-(n-1)a}\, e^{-v}\{(n-1)a+v}$$

$(1-a)^{n-1} > x$

$$\int_{(1-a)^n} F_n(x)\,dx = \int_u^{na} e^{-u}\{u^{n-1} - n(u-a)^{n-1} + \cdots + (-1)^{n-1}\binom{n}{n-1}\{u-(n-1)a\}^{n-1}\}$$

$$u^{n-1} - n(u-a)^{n-1} + \cdots + (-1)^{n-1}\binom{n}{n-1}\{u-(n-1)a\}^{n-1} + (-1)^n (u-na)^{n-1} = a_0 + a_1 u' + \cdots + a_{n-1} u^{n-1}$$

$\cancel{u = 0. \quad 1 - n(a)^{n-1} + \cdots + (-1)^n (na)^{n-1}}$

$$1 - n\, e^{-a\frac{\partial}{\partial x}} + \cdots + (-1)^n e^{-na\frac{\partial}{\partial x}}$$

$$= e^{-u}\left(1 - e^{-a\frac{\partial}{\partial x}}\right)^n (u^{n-1}) = 0.$$

$$\int F_n(x)\,dx = -\int_u^{na} (-1)^n (u-na)^{n-1} e^{-u}\,du = \int_{u}^{na} (na-u)^{n-1} e^{-u}\,du$$

$na - u = v,$

$$= e^{-na}\int_0^{na-u} e^{+u} u^{n-1}\,du = \cancel{e^{-na}\int e^{z}} = e^{-na}(-1)^{n-1}\int_{u-na}^{0} e^{-v} v^{n-1}\,dv$$

$u = (n-1)a.$

$$e^{-na}\int_0^a e^{u} u^{n-1}\,du = e^{-na}\left\{e^{a} a^{n-1} - (n-1)\int_0^a e^{u} u^{n-2}\,du\right\}$$

$$= e^{-na}\left\{e^{a} a^{n-1} - (n-1)e^{a} a^{n-2} + \cdots (-1)^{n-1}(n-1)!\, e^{a}\right\} = e^{-(n-1)a}\left(\cancel{(1-a)^{n-1}} \ldots\right)$$

C^{12}: $\alpha = \dfrac{4MM'}{(M+M')^2} = \dfrac{4\times 12}{(13)^2}$

$$
\begin{array}{r}
0.716 \\
169\,\overline{)\,121} \\
1183 \\
\hline
270 \\
169 \\
\hline
1010
\end{array}
$$

$\log_{10} (0.716)^{-1} = -(\bar{1}.85491)$

$= \underline{0.14509}$

$\log_e\left(\dfrac{1}{0.716}\right) =$

$(1-\alpha)^{n-1} \geq x$ ならば、 $u \geq (n-1)a$ なる neutron の percentage

は $\dfrac{N((1-\alpha)^{n-1})}{N(1)} = \dfrac{e^{-(n-1)a}\,\cancel{(a-1)^{n-1}}\{a^{n-1}-(n-1)a^{n-2}+\cdots(-1)^{n-1}(n-1)!\}}{\int_0^{na} e^{-u}u^{n-1}du - \cdots\cdots + (-1)^{n-1} n \int_0^{a} e^{-u}u^{n-1}du}$

$1-\alpha = \frac{1}{9}$

$n = 5.$

$$\frac{N(\frac{1}{9^4})}{N(1)} = \frac{e^{-4a}(a^4-4a^3+12a^2-\cancel{10a}\,24a+24)}{4!\{I(\frac{5a}{\sqrt{5}},4)-\frac{5}{9}I(\frac{4a}{\sqrt{5}},4)+\frac{10}{9^2}I(\frac{3a}{\sqrt{5}},4) - \frac{10}{9^3}I(\frac{2a}{\sqrt{5}},4)+\frac{5}{9^4}I(\frac{a}{\sqrt{5}},4)\}}$$

		I
$\frac{a}{\sqrt{5}} = \frac{2.1972}{\sqrt{5}}$	$=0.983$	0.075
$\frac{2a}{\sqrt{5}} =$	1.965	0.45
$\frac{3a}{\sqrt{5}} =$	2.948	0.785
$\frac{4a}{\sqrt{5}} =$	3.930	0.937
$\frac{5a}{\sqrt{5}} =$	4.913	0.985

0.985
0.097
1.082
−0.527
0.555
12
1.0900
555
6.660
13.32

$81)\,0.785 = 0.097$; 729; 560

$18)\,0.937 = 0.521$; 90; 37; 36; 10

0.075

$9)\,0.45 = 0.5$

$81)\,0.500 = 0.006$; 488

$N(1) = \cancel{6.66 \cdot 4!}\ 13.08\ \ 13.32$

$\boxed{\frac{N(1)}{4!} = 0.555}$

$$e^{-4\log 9} = \frac{1}{9^4}\cdot\{(2.1972)^4 - 4(2.1972)^3 + 12(2.1972)^2 - 24(2.1972) + 24\}$$

	log	
2.1972 :	0.34183	2.1972
$(\ \ )^2$:	0.68374	4.826
$(\ \ )^3$:	1.02561	10.607
$(\ \ )^4$:	1.36748	23.307

23.307
57.917
24.
105.224
95.161
10.063

4.826
12
9652
4826
57.912

10.607
4
42428
52.7328
95.161

2.1972
24
87888
43944
52.7328

$= \frac{81}{81}$
81
648
6561

0.000115
13.32) 0.001534
1332
2020
1332
6880

0.0015 34
6561) 10.063
6561
35020
32805
22150
19683
24670

$$\boxed{\frac{N(\frac{1}{q^4})}{N(1)} = 1.15 \times 10^{-4}}$$

$\bar{4}.06070$

0.261
111) 29
222
680
666
140
111
29

$$\boxed{\frac{N(\frac{1}{q^2})}{N(1)} = \frac{0.555-0.41}{0.555} = \frac{145}{555} = 0.261}$$

$\bar{1}.41664$

$n=10$;

$$\frac{N(\frac{1}{q})}{N(1)} = \frac{0.555-0.072}{0.555} = \frac{0.483}{0.555} \simeq 0.870$$

$\bar{1}.93952$

0.870
555) 483
4440
3900
3885
150

$$\frac{N(\frac{1}{q^3})}{N(1)} = \frac{0.555-0.54}{0.555} = \frac{0.015}{0.555}$$

$$= \frac{3}{111} = 0.027$$

$\bar{2}.43136$

0.027
111) 300
222
780

$$g_n\left(\frac{E}{E_0}\right) d\left(\frac{E}{E_0}\right) = u_n\left(\log\left(\frac{E}{E_0}\right)\right)$$

$$f_n(E) dE \qquad 0 \le E \le E_0.$$

$$f_{2n}(E) dE = dE \int_E^{E_0} f_n(E') f_n\left(\frac{E}{E'}\right) dE'$$

$$= \frac{dE}{E_0} \int_E^{E_0} g_n\left(\frac{E}{E'}\right) g_n\left(\frac{E'}{E_0}\right) \frac{dE'}{E'}$$

$$= \frac{dE}{E_0} \int_E^{E_0} g_n\left(\frac{E}{E'}\right) g_n\left(\frac{E'}{E_0}\right) d\left(\frac{E'}{E}\right)$$

$$\frac{8\frac{1}{3}}{243)\,400} \quad 0.1 \quad \& \quad \frac{40}{3\times 9^2}$$

$n=10.$ $$N(1) = 9!\Big\{ I\Big(\frac{10a}{\sqrt{10}}, 9\Big) - \frac{10}{9} I\Big(\frac{9a}{\sqrt{10}}, 9\Big)$$

$$+ \frac{10\cdot 9}{1\cdot 2\cdot 9^2} I\Big(\frac{8a}{\sqrt{10}}, 9\Big) - \frac{10\cdot 9\cdot 8}{1\cdot 2\cdot 3\, 9^3} I\Big(\frac{7a}{\sqrt{10}}, 9\Big) + \frac{10\cdot 9\cdot 8\cdot 7}{1\cdot 2\cdot 3\cdot 4\, 9^4} I\Big(\frac{6a}{\sqrt{10}}, 9\Big)$$

$$- \underbrace{\frac{\cancel{10}^{2}\cdot \cancel{9}\cdot \cancel{8}\cdot 7\cdot \cancel{6}^{2}}{1\cdot 2\cdot 3\cdot 4\cdot 5}}_{=28\times 9} \frac{1}{9^5} I\Big(\frac{5a}{\sqrt{10}}, 9\Big) + \cancel{28\times 9}\; \underbrace{28\times 9\times \frac{5}{6}}_{=14\times 3\times 5 = 210} \frac{1}{9^6} I\Big(\frac{4a}{\sqrt{10}}, 9\Big)$$

$$- \underbrace{\cancel{210}^{30}\times \frac{4}{\cancel{7}}}_{=120} \frac{1}{9^7} I\Big(\frac{3a}{\sqrt{10}}, 9\Big) + \underbrace{\cancel{120}^{15}\times \frac{3}{8}}_{=45} \frac{1}{9^8} I\Big(\frac{2a}{\sqrt{10}}, 9\Big)$$

$$- \underbrace{\cancel{45}^{5}\times \frac{2}{\cancel{9}}}_{=10} \frac{1}{9^9} I\Big(\frac{a}{\sqrt{10}}, 9\Big).$$

$$= 9!\Big\{ I\Big(\frac{10a}{\sqrt{10}}, 9\Big) - \frac{10}{9} I\Big(\frac{9a}{\sqrt{10}}, 9\Big) + \frac{5}{9} I\Big(\frac{8a}{\sqrt{10}}\, 9\Big)$$

$$- \frac{400}{243} I\Big(\frac{7a}{\sqrt{10}}, 9\Big) + \frac{210}{9^4} I\Big(\frac{6a}{\sqrt{10}}, 9\Big) - \frac{28}{9^4} I\Big(\frac{5a}{\sqrt{10}}, 9\Big)$$

$$+ \frac{210}{9^6} I\Big(\frac{4a}{\sqrt{10}}, 9\Big) - \frac{120}{9^7} I\Big(\frac{3a}{\sqrt{10}}, 9\Big) + \frac{5}{\cancel{4}9^7} I\Big(\frac{2a}{\sqrt{10}}, 9\Big)$$

$$- \frac{10}{9^9} I\Big(\frac{a}{\sqrt{10}}, 9\Big)$$

$$\frac{a}{\sqrt{10}} \cdot a = (3.16\overset{23)}{\times})^{-1} 2.1972 = 0.6948$$

$$\begin{array}{lll} \log 2.1972 & & .34187 \\ -\log 3.1623\ (\sqrt{10}) & & \bar{1}.50000 \\ \hline & & \bar{1}.84187 \\ & & 0.6948 \end{array}$$

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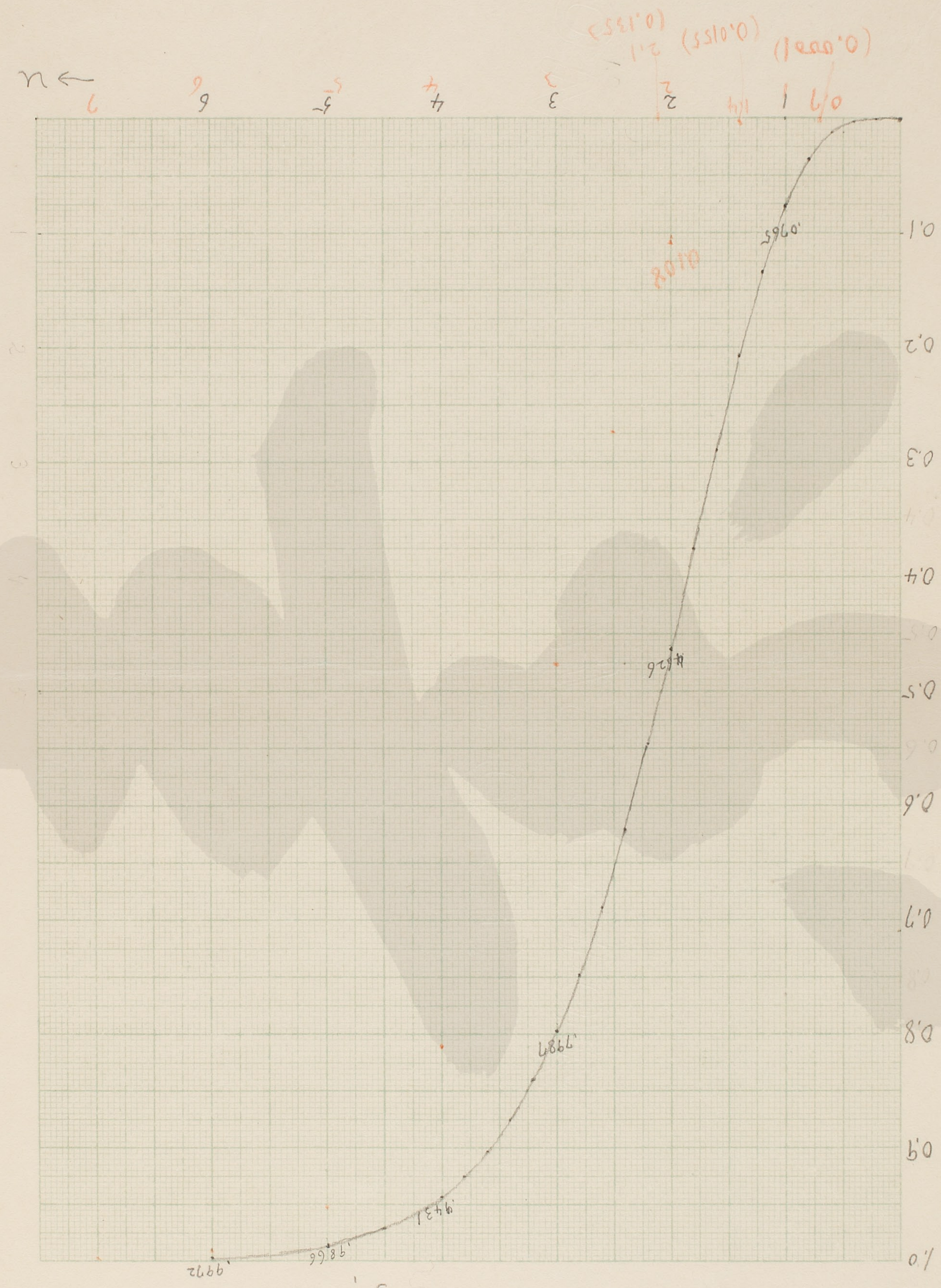
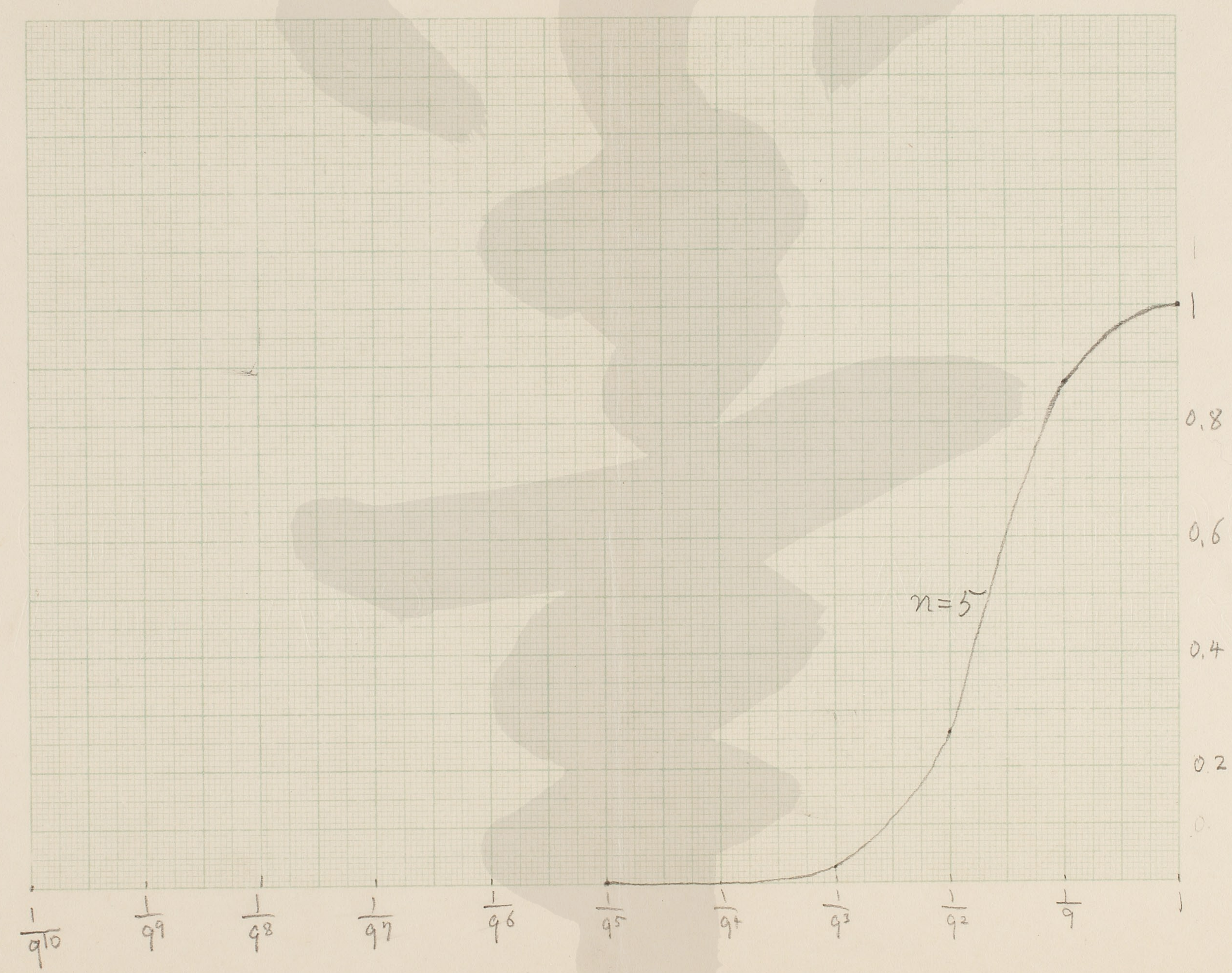


$$I(u,4) = \frac{\int_0^{\sqrt{5}u} e^{-u} u^4 \, du}{5!}$$



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$$F_n(u)\,du \propto \frac{e^{-u}du}{(n-1)!}\left\{u^{n-1} - \binom{n}{1}(u-a)^{n-1} + \binom{n}{2}(u-2a)^{n-1} - \cdots + (-1)^{n-1}(u-(n-1)a)^{n-1}\right\}$$

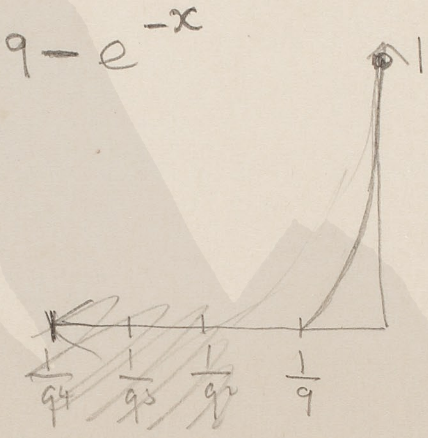
$n=1$:

$$F_1(x)\,du \propto e^{-u}du$$

$$\int_{\log\frac{E_0}{E}}^{\log 9} e^{-u}\,du = 9 - \frac{E_0}{E} = \qquad \log\frac{E_0}{E} = x$$

$$= 9 - e^{-x}$$

$$\frac{9-e^{-x}}{8}$$



$$+\int_0^a e^{-u}du$$

$n=2$:

$$F_2(u)\,du \propto u e^{-u} du \quad \text{for } 0<u<a$$

$$\propto \{u - 2(u-a)\}e^{-u}du \quad \text{for } a<u<2a$$

$$1-(1+a)e^{-a}$$

$$C\left\{\int_0^a u e^{-u} du + \int_a^{2a}(2a-u)e^{-u}du\right\} = C\left\{-ae^{-a}+1-e^{-a}+2a(e^{-a}-e^{-2a})\right.$$

$$-\int_a^{2a} u e^{-u}du = -(ae^{-a} - 2ae^{-2a}) - \int_a^{2a} e^{-u}du$$

$$-ae^{-a} + 2ae^{-2a} - e^{-a} + e^{-2a}\}$$

$$= C\{1 - (1+a+2a+a+1)e^{-a} - (2a-2a-1)e^{-2a}\}$$

$$= C\{1 - 2e^{-a} + e^{-2a}\} \qquad (1-e^{-a})^2$$

$\log 9 = 2.1972$

$$1 - \frac{2}{9}\log 9 + \frac{1}{81} =$$

1.0123
-0.4883
0.5240

$9)\,4.3944$
0.4883

$$\left(1-\frac{1}{9}\right)^2 \quad \left(\frac{8}{9}\right)^2$$

$81)\,64$

$81)\,1.00 = 0.0123$
81
190
162
280

$$C = \frac{1}{0.524} \qquad C = \frac{81}{64}$$

$$\int_0^a u e^{-u} du = 1-(1+a)e^{-a}$$

$$= 1-(1+\log 9)\frac{1}{9} = 1-\frac{3.1972}{9} = 0.645$$

$9)\overline{3.1972}$ 0.35524

$$\int_a^{2a}(2a-u)e^{-u}du = 2a(e^{-a}-e^{-2a})-ae^{-a}+2ae^{-2a} - e^{-a}+e^{-2a}$$

$$= (a-1)e^{-a}+e^{-2a}$$

$$= \frac{(\log 9-1)}{9}+\frac{1}{81} = $$

$9)\overline{2.1972}$ 0.1330 0.0123

$81)\overline{64}$ 0.79 567 730 729

0.1453 ~~0.3552~~ 005 0.645 0.790

$\boxed{u=a}$ $\boxed{0.184}$

$0.79)\overline{0.1453}$ 79 663 632 380

$$\int_{\frac{3}{2}a}^{2a}(2a-u)e^{-u}du = 2a\cdot(e^{-\frac{3}{2}a}-e^{-2a})+2ae^{-2a}-\frac{3}{2}ae^{-\frac{3}{2}a}$$

$$-e^{-\frac{3}{2}a}+e^{-2a} = \left(\frac{a}{2}-1\right)e^{-\frac{3}{2}a}+e^{-2a}$$

$$= \left(\frac{2.1972}{2}-1\right)\frac{1}{27}+\frac{1}{81}($$

$\times\frac{81}{64}$

$$\left(\frac{2.1972}{2}-1\right)\frac{3}{64}+\frac{1}{64}$$

0.0986 $\times 3$ 1.2958

$\boxed{0.0202}$ $64)\overline{1.2958}$ 128 150

$u=\frac{3}{2}a$

~~3.0936~~

3.2958 3.1972

$$\int_{\frac{a}{2}}^{a} u e^{-u} du = \frac{a}{2}e^{-\frac{a}{2}} - ae^{-a} + e^{-\frac{a}{2}} - e^{-a}$$

$$= \left(\frac{a}{2}+1\right)e^{-\frac{a}{2}} - (a+1)e^{-a} = \frac{1.0986}{3} - \frac{3.1972}{9} = \frac{0.0986}{9}$$

$0.0986\times\frac{9}{64} = $

$$\int_0^{\frac{a}{2}} u e^{-u} du = 1 - \left(1+\frac{a}{2}\right) e^{-\frac{a}{2}}$$

$$= 1 - \frac{1.0986}{3} = \frac{1.9814}{3}$$

$1 - \left(1+\frac{x}{2}\right)e^{-\frac{x}{2}}$

$\left(1+\frac{x}{2}\right)\left(1-\frac{x}{2}+\cdots\right)$

$\times \frac{81}{64}$

$1.9814 \times 27 = 13\,8698 + 39628 = 53.4978$

$u = \frac{a}{2}$

$64\,)\,53.4978 = 0.836$; 512, 229, 192, 377

	E/E_0	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$
$n=2$	u	$\frac{a}{2}$	a	$\frac{3a}{2}$	$2a$
	f	0.836	0.184	0.02	0
		$\bar{1}.9221$	$\bar{1}.26482$	$\bar{2}.30103$	

Deuteron

Proton. ~~n=2~~

$a = \infty$. $\int_u^\infty \frac{u^{n-1}e^{-u}du}{(n-1)!}$

$\int_0^u \frac{u^{n-1}e^{-u}du}{(n-1)!} = I\left(\frac{u}{\sqrt{n}}, n-1\right)$

$\int_u^\infty \frac{u^{n-1}e^{-u}}{(n-1)!} du = 1 - I\left(\frac{u}{\sqrt{n}}, n-1\right)$

$n=2$ $\int_u^\infty u e^{-u} du = u e^{-u} + e^{-u}$

$= (1+u)e^{-u}$

$m=\frac{1}{2}$ $\left(u = \frac{1}{2}\log 9\right)$

$\left(1+\frac{2.1972}{2}\right)\frac{1}{3}$

$3)\,2.0986 = \boxed{0.6995}$; 18, 29

$\bar{1}.845$

$(1+u)(1-u)$

$u = m\log 9$ $\qquad \frac{1+m\log 9}{9^m} = \frac{1+m\times 2.1972}{9^m}$

$m=1$ $\qquad 9)\,3.1972 = \boxed{0.3552}$; 27, 49 $\qquad \bar{1}.550$

$m=2$ $\qquad 2.1972 \times 2 = 4.3944$

$81\,)\,5.3944 = \boxed{0.0666}$; 486, 534, 486, 484 $\qquad \bar{2}.8235$

$2.1972 \times 3 = 6.5916$

$729\,)\,7.5916 = 0.010$; 729, 301 $\qquad = 1.05$

$m=3$ $\qquad 0.01$

$m=4$ $\log(1+4\log 9) - 4\log 9$

$m=5$

$\bar{2}.$

0.95424

$\log_{10}(1+5\log_e 9) - 5\log_{10} 9$

$\log 11.9860$ $\qquad 1.0788$

$2.1972 \times 5 = 10.9860$

95424, 38, 1.0788

$0.95424 \times 5 = 4.7712$

14.7712, 4.3076

$n=5$; $\frac{1}{4!}\int_u^\infty u^4 e^{-u} du = \frac{1}{4!}\{u^4 + 4u^3 + 12u^2 + 24u + 24\} e^{-u}$

$u = \log 9$; $= \frac{1}{4!}\{(\log 9)^4 + 4(\log 9)^3 + 12(\log 9)^2 + 24(\log 9) + 24\}\frac{1}{9}$ (24×9)

$\log 9 = 2.1972$ $\log(2.1972) = 0.34187$

$(\log 9)^2 = 4.82877$ 2 ″ $= 0.68374$

$(\log 9)^3 = 10.609$ 3 ″ $= 1.02561$

$(\log 9)^4 = 23.305$ 4 ″ $= 1.36748$

23.305
42.436
57.932
52.733
24.
200.406

216) 200.406 = $\boxed{0.928}$
1944
6006
432
1686

24
9
216

4.8277
12
96554
48277
57.9324

2.1972
24
87888
43944
52.7328

$\bar{1}.96755$

$u = 2\log 9$ $\frac{1}{9\times 216}(\cancel{23.30}\,16\times(\log 9)^4 + 32(\log 9)^3 + 48(\log 9)^2 + 48(\log 9) + 24)$

$= \frac{1}{9\times 27}(2(\log 9)^4 + 4(\log 9)^3 + 6(\log 9)^2 + 6(\log 9) + 3)$

46.610
42.436
28.966
13.183
3.
134.195

27
9
243

243) 134.195 = $\boxed{0.552}$
1215
1269
1215
545

4.8277
6
28.9662

2.1972
6
13.1832

$\bar{1}.74194$

$u = 3\log 9$.

$\frac{1}{24\times 9^3}\{3^4(\log 9)^4 + 4\times 3^3(\log 9)^3 + 4\times 3^3(\log 9)^2 + 8\times 3^2(\log 9) + 8\times 3\}$

$= \frac{1}{24\times 3^3}\{3(\log 9)^4 + 4(\log 9)^3 + 4(\log 9)^2 + \frac{8}{3}\log 9 + \frac{8}{9}\}$

24
3
216
3
648

648) 138.410 = $\boxed{0.214}$
1296
881
648
2330

69.915
42.436
19.311
5.859
0.889
138.410

4.8277
4
19.3108

2.1972
8
3) 17.5776
5.859

9) 8 = 0.88
72

$\bar{1}.33041$

$u = 4\log 9.$

$$\frac{1}{24\times 9^4}\left\{4^4(\log 9)^4 + 4^4(\log 9)^3 + 3\times 4^3(\log 9)^2 + 6\times 4^2(\log 9) + 6\times 4\right\}$$

$$= \frac{8\,4^3}{24\times 9^4}\left\{4(\log 9)^4 + 4(\log 9)^3 + 3(\log 9)^2 + 1.5(\log 9) + \frac{3}{8}\right\}$$

$$\begin{array}{r} \cancel{216} \\ \cancel{81} \\ \underline{\cancel{81}} \\ 243 \\ \underline{81} \\ 243 \\ \underline{1944} \\ 19683 \end{array} \qquad \begin{array}{r} 1\,1\,3\,2 \\ 93.220 \\ 42.436 \\ 14.483 \\ 33.296 \\ \underline{0.375} \\ 153.810 \\ \underline{8} \\ 1230.49 \end{array} \qquad \begin{array}{r} 4.8277 \\ \underline{3} \\ 14.4831 \\ 2.1972 \\ \underline{1.5} \\ 109860 \\ \underline{21972} \\ 3.29580 \end{array}$$

$$\begin{array}{r} \boxed{0.06252} \\ 19683\overline{)1230.49} \\ \underline{118098} \\ 49510 \\ \underline{39366} \\ 101440 \\ \underline{98415} \\ 30250 \end{array} \qquad \begin{array}{l} \log \\ \overline{2}.79602 \\ \overline{2}.79602 \end{array} \qquad \begin{array}{r} 0.375 \\ 8\overline{)3} \\ 24 \\ 60 \end{array}$$

$u = 5\log 9$

$$\frac{1}{24\times 9^5}\left\{5^4(\log 9)^4 + 5^3\times 4(\log 9)^3 + 12\cdot 5^2(\log 9)^2 + 24\times 5(\log 9) + 24\right\}$$

$$= \frac{1}{24\times 4^2\times 9^5}\left\{10^4(\log 9)^4 + 8\times 10^3(\log 9)^3 + 48\times 10^2(\log 9)^2 + 16\times 120(\log 9) + 24\times 16\right\}$$

$\boxed{3^{11}\times 2^7}$

$$\begin{array}{r} 233050. \\ 84872. \\ 23173. \\ 4219. \\ \underline{384} \\ 345700. \\ 5.53870 \\ \underline{7.35553} \\ \overline{2}.18317 \end{array} \qquad \begin{array}{r} 10.609 \\ 8 \\ 0.30103 \\ \underline{7} \\ 2.10721 \\ 0.47712 \\ \underline{11} \\ 47712 \\ \underline{47712} \\ 5.24832 \\ \underline{2.10721} \\ 7.35553 \end{array} \qquad \begin{array}{r} 482.77 \\ \underline{48} \\ 386216 \\ \underline{193108} \\ 23172.96 \end{array} \qquad \begin{array}{r} 16 \\ \underline{120} \\ 320 \\ \underline{16} \\ 1920 \\ \underline{384} \\ 2.1972 \\ \underline{192} \\ 43944 \\ 197748 \\ \underline{21972} \\ 4218.624 \end{array}$$

$\boxed{0.0152}$

$\overline{2}.18317$

$u = 6\log 9.$

$$\frac{1}{24\times 9^6}\left\{6^4(\log 9)^4 + 4\times 6^3(\log 9)^3 + 12\times 6^2(\log 9)^2 + 24(\log 9) + 24\right\}$$

$$= \frac{6^3\times 2}{24\times 9^6}\left\{3(\log 9)^4 + \frac{12}{9}(\log 9)^3 + (\log 9)^2 + \frac{1}{3}(\log 9) + \frac{1}{18}\right\}$$

$$\frac{3^3\times 2^4}{2^3\times 3\times 3^{12}} = \frac{2}{3^{10}}$$

69.915
21.218
4.828
0.732
0.056
96.749
2
193.498

$3)2.1972 = 0.7324$

$18)100 = 0.05555$

2.28668
4.7712
$\bar{3}.5155$

0.00

$u = 7\log 9$

$$1 - I\left(\frac{7\log 9}{\sqrt{5}}, 4\right)$$

2.1972
7
15.3804

$2.236)15.3804 = 6.88$

$I(6.88, 4)$
$= 0.9992663$
0.9993807
0.99935
0.00065

$\bar{4}.81291$