E24 050 P10

Stationary State of the System ~~Containing~~ Three Particle
n n p

$$\{\Delta_1 + \Delta_2 + \Delta_3 + \frac{2M}{\hbar^2}(E - V)\}\psi = 0$$

$$\psi = \chi_{\alpha\alpha\alpha}\alpha(1)\alpha(2)\alpha(3) + \chi_{\alpha\alpha\beta}\alpha\alpha\beta + \chi_{\alpha\beta\alpha}\alpha\beta\alpha + \chi_{\beta\alpha\alpha}\beta\alpha\alpha$$
$$+ \chi_{\alpha\beta\beta}\alpha\beta\beta + \chi_{\beta\alpha\beta}\beta\alpha\beta + \chi_{\beta\beta\alpha}\beta\beta\alpha + \chi_{\beta\beta\beta}\beta\beta\beta.$$

$(\chi_{\alpha\alpha\alpha}\alpha(1)\alpha(2)\alpha(3),\ \chi_{\alpha\alpha\beta}\alpha\alpha\beta,\ \chi_{\alpha\beta\alpha}(\alpha\beta\alpha + \beta\alpha\alpha),$
$\chi_{\alpha\beta\beta}(\alpha\beta\beta + \beta\alpha\beta),\ \chi_{\beta\beta\alpha}\beta\beta\alpha,\ \chi_{\beta\beta\beta}\beta\beta\beta\ )$

$(\chi_{\alpha\beta\alpha}(\alpha\beta\alpha - \beta\alpha\alpha)\quad \chi_{\alpha\beta\beta}(\alpha\beta\beta - \beta\alpha\beta).)$
$(\chi_{\alpha\alpha\alpha}\alpha\alpha\alpha,\ \chi_{\alpha\beta\alpha}(\alpha\beta + \beta\alpha)\alpha,\ \chi_{\beta\beta\alpha}\beta\beta\alpha\ )$
$(\chi_{\alpha\alpha\beta}\alpha\alpha\beta,\ \chi_{\beta\beta}(\alpha\beta + \beta\alpha)\alpha\quad \chi_{\beta\beta\beta}\beta\beta\beta\ )$

$\psi = \chi_{\alpha}((12)_+, 3)(\alpha\beta\alpha - \beta\alpha)\alpha;\ \chi_{\beta}((12)_+, 3)(\alpha\beta - \beta\alpha)\beta$
$\chi_{\alpha}^{(1)}((12)_-, 3)\alpha\alpha\alpha,\ \chi_{\alpha}^{(0)}((12)_-, 3)(\alpha\beta + \beta\alpha)\alpha,$
$\chi_{\alpha}^{(-1)}((12)_-, 3)\beta\beta\alpha,$
$\chi_{\beta}^{(1)}((12)_-, 3)\alpha\alpha\beta,\ \chi_{\beta}^{0}$
$\chi_{\beta}^{(-1)},$

$\alpha + P_{13}$
$\{a_{123} + b_{12}P_{12} + b_{13}P_{13} + b_{23}P_{23} + c_{12}P_{12} + c_{13}Q_{13} + c_{23}Q_{23}$
$a_{12} + a_{13} + a_{23}\quad + \quad d_{12}P_{12}Q_{12} + d_{13}P_{13}Q_{13} + d_{23}P_{23}Q_{23}\}$
$\times \chi_{\alpha}((12)_+, 3)(\alpha\beta - \beta\alpha)\alpha = \{a_{12} + a_{13} + a_{23} + b_{12} + b_{13}P_{13}$
$+ b_{23}P_{23} \pm c_{12} \pm d_{12}\}\chi_{\alpha}((12)_+, 3)(\alpha\beta - \beta\alpha)\alpha.$
$+ c_{13}$

$(\alpha\beta\alpha - \alpha\alpha\beta. = \alpha\beta\alpha - \beta\alpha\alpha + \beta\alpha\alpha - \alpha\alpha\beta.$

一般には 8個の spin state の内 ααα, βββ は夫々他と combine しない

が ααβ, αβα, βαα; αββ, βαβ, ββα の三個宛

が互いに combine する.

$$\{ a_{12}+a_{13}+a_{23} + b_{12}P_{12} + b_{13}P_{13} + b_{23}P_{23} + c_{12}Q_{12} + c_{13}Q_{13} + c_{23}Q_{23}$$

$$+ d_{12}P_{12}Q_{12} + d_{13}P_{13}Q_{13} + d_{23}P_{23}Q_{23} \} \alpha\alpha\beta$$

$$= \{ a_{12}+a_{13}+a_{23} + b_{12}P_{12} + b_{13}P_{13} + b_{23}P_{23} + c_{12} + d_{12}P_{12} \} \alpha\alpha\beta$$

$$+ \{ c_{13} + d_{13}P_{13} \} \beta\alpha\alpha$$

$$+ \{ c_{23} + d_{23}P_{23} \} \alpha\beta\alpha$$

$$\begin{pmatrix} A + c_{12} + d_{12}P_{12} & c_{23}+d_{23}P_{23} & c_{13}+d_{13}P_{13} \\ c_{23}+d_{23}P_{23} & A+c_{13}+d_{13}P_{13} & c_{12}+d_{12}P_{12} \\ c_{13}+d_{13}P_{13} & c_{12}+d_{12}P_{12} & A+c_{23}+d_{23}P_{23} \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

$$= \begin{pmatrix} A+B_3-E & B_1 & B_2 \\ B_1 & A+B_2-E & B_3 \\ B_2 & B_3 & A+B_1-E \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = 0.$$

$$\begin{vmatrix} A+B_3-E & B_1 & B_2 \\ B_1 & A+B_2-E & B_3 \\ B_2 & B_3 & A+B_1-E \end{vmatrix} = 0$$

$$(A+B_1+B_2+B_3-E) \begin{vmatrix} 1 & B_1 & B_2 \\ 1 & A+B_2-E & B_3 \\ 1 & B_3 & A+B_1-E \end{vmatrix} = 0$$

$$(A+B_1+B_2+B_3-E)\{ (A+B_2-E)(A+B_1-E) - B_3^2$$

$$- B_1(A+B_1-E) + B_2B_3 + B_1B_3 - B_2(A+B_2-E) \} = 0$$

$$\longleftrightarrow \quad E^2 - 2AE + (A^2 - B_1^2 - B_2^2 - B_3^2 + B_1B_2 + B_2B_3 + B_1B_3) = 0$$

$$E = A \pm \sqrt{B_1^2+B_2^2+B_3^2 - B_1B_2 - B_1B_3 - B_2B_3} =$$

$$E = A \pm \sqrt{\tfrac{1}{2}\{(B_1-B_2)^2+(B_3-B_1)^2+(B_2-B_3)^2\}}$$

or $A + B_1 + B_2 + B_3$.

$$A = a_{12}+a_{13}+a_{23}+b_{12}P_{12}+b_{13}P_{13}+b_{23}P_{23}$$

$$B_1 = c_{23}+d_{23}P_{23} \quad \text{etc.}$$

$E_1 = A + B_1 + B_2 + B_3$, $\quad E_2 = A + \sqrt{B_1^2+B_2^2+B_3^2-B_1B_2-B_1B_3-B_2B_3}$ (C_n)

$E_3 = A - \sqrt{\quad}$ " C

$$\left.\begin{aligned}(A+B_3-E_1)e_1 + B_1e_2 + B_2e_3 &= 0\\ B_1e_1 + (A+B_2-E_1)e_2 + B_3e_3 &= 0\end{aligned}\right\}$$

$B_3 \mid B_1 \; (B_1+B_2)e_1 + B_1e_2 + B_2e_3 = 0$

$-B_2 \mid (B_1+B_2)\; B_1e_1 + (B_1+B_3)e_2 + B_3e_3 = 0$

$$(B_3B_1 + B_3B_2 - B_1B_2)e_1 + (B_1B_3 + B_1B_2 + B_2B_3)e_2 = 0$$

$e_1 : e_2 : e_3 = (-B_1B_2 + B_3B_1 + B_2B_3)$ $\quad e_1 = e_2$

$\{B_1^2 - (B_1+B_2)(B_1+B_3)\}e_1 + \{B_1B_2 + B_1B_3 + B_2B_3\}e_3 = 0$

$(+B_1B_2 + B_1B_3 + B_2B_3)e_2 + (-B_1B_2 + B_1B_3 + B_2B_3)e_3 = 0.$

$$\frac{e_1}{B_1B_3 - B_1B_2 - B_2B_3} = \frac{e_2}{\quad}$$

$e_1 = e_2 = e_3 = X_1$ $\quad A + B_3 \quad X_1$

E_1 : $\alpha\alpha\beta + \alpha\beta\alpha + \beta\alpha\alpha$.

E_2 : $(B_3 - C)e_1 + B_1e_2 + B_2e_3 = 0 \qquad B_3 \mid B_1$

$B_1e_1 + (B_2 - C)e_2 + B_3e_3 = 0 \qquad -B_2 \mid (B_3 - C)$

$\{B_3(B_3-C) - B_1B_2\}e_1 + \{B_1B_3 - B_2(B_2-C)\}e_2 = 0.$

$\{B_1^2 - (B_2-C)(B_3-C)\}e_2 + \{B_1B_2 - B_3(B_3-C)\}e_3 = 0.$

$B_1^2 + \ldots - B_2B_3 + B_2C + B_3C - B_1^2 + B_2^2 - B_3^2 + B_1B_2 + B_2B_3 + B_3B_1,$

$= \{B_1B_2 - B_3(B_3-C) + B_1B_3 - B_2(B_2-C)\}$

$C_3e_1 + C_2e_2 = 0$

$-(C_3+C_2)e_2 - C_3e_3 = 0.$

$$\frac{e_1}{C_2} = \frac{e_2}{C_3} = \frac{e_3}{-(C_3+C_2)}$$

$B_2 e_1 + B_3 e_2 + (B_1 - C) e_3 = 0$ $\quad B_2 \quad -B_1$

~~$B_1 B_2$ + $B_1 B_2 - B_1$~~ $\{B_2(B_2 - C) - B_1 B_3\} e_2 + \{B_2 B_3 - B_1(B_1 - C)\} e_3 = 0.$

$\{B_1 B_2 - B_3(B_3 - C)\} e_1 = \{$~~$B_2(B_2-C)$~~$ + B_1 B_3 - B_2(B_2 - C)\} e_2$

$= \{B_2 B_3 - B_1(B_1 - C)\} e_3 = 0.$

$E_2: \dfrac{\alpha\alpha\beta}{B_1 B_2 - B_3(B_3 - C)} + \dfrac{\alpha\beta\alpha}{B_1 B_3 \pm B_2(B_2 - C)} + \dfrac{\beta\alpha\alpha}{B_2 B_3 - B_1(B_1 - C)}$

$E_3: \dfrac{\alpha\alpha\beta}{B_1 B_2 - B_3(B_3 + C)} + \cdot \quad \cdot \quad - \quad -$

Quartet ~~Triplet~~ State (Symmetric State)

E_1: $(\alpha\alpha\alpha, \ \alpha\alpha\beta + \alpha\beta\alpha + \beta\alpha\alpha, \ \alpha\beta\beta + \beta\alpha\beta + \beta\beta\alpha, \ \beta\beta\beta.)$

~~Singlet~~

E_2: Doublet state $(\frac{1}{2}, -\frac{1}{2})$ $\quad \dfrac{\alpha\alpha\beta}{B_1 B_2 - B_3(B_3 - C)} + \dfrac{\alpha\beta\alpha}{\quad} + \cdot \cdot$

$\dfrac{\beta\beta\alpha}{B_1 B_2 - B_3(B_3 - C)} + \cdot \cdot$

E_3: Doublet state $(\frac{1}{2}, -\frac{1}{2})$

$\dfrac{\alpha\alpha\beta}{B_1 B_2 - B_3(B_3 + C)} + \cdot \cdot$

$\dfrac{\beta\beta\alpha}{B_1 B_2 - B_3(B_3 + C)} + \cdot \cdot$

$Q_{12}\left\{\dfrac{\alpha\alpha\beta}{B_1 B_2 - B_3(B_3 - C)} + \dfrac{\alpha\beta\alpha}{B_1 B_2 - B_2(B_2 - C)} + \dfrac{\beta\alpha\alpha}{B_2 B_3 - B_1(B_1 - C)}\right\}$

$= Q_{12}\left\{ \ \text{''} \ + \dfrac{\alpha\beta\alpha}{B_2 B_3 - B_1(B_1 - C)} + \dfrac{\beta\alpha\alpha}{B_1 B_2 - B_2(B_2 - C)}\right\}$

$$\{a_{12}+a_{13}+a_{23}+b_{12}P_{12}+b_{13}P_{13}+b_{23}P_{23}+c_{12}Q_{12}+c_{13}Q_{13}+c_{23}Q_{23}$$
$$+d_{12}P_{12}Q_{12}+d_{13}P_{13}Q_{13}+d_{23}P_{23}Q_{23}\}\chi_1(\ \ )$$

$$= E_1\chi_1(\ ,\ ,\ ) \qquad \chi_1(\ ,\ ,\ )=\psi_1,$$

$$(A+B_1+B_2+B_3)\chi_1=0,$$

χ_1 としては $(\psi_1 - P_{12}Q_{12}\psi_1)$ $P_{(23)}$ に関して anti-sym. なものをとる。

$$(A+\sqrt{\quad})\chi_2 = 0 \quad (\ ,\ ,\ ) = 0$$
$$(A-\sqrt{\quad})\chi_3 = 0 \quad (\chi_3, \chi_3, \chi_3) = 0$$

$$(1-P_{12}Q_{12})\chi_2\left\{\frac{\alpha\alpha\beta}{B_1B_2-B_3(B_3-C)}+\cdots\right\}$$
$$(1-P_{12}Q_{12})\chi_3\left\{\frac{\alpha\alpha\beta}{B_1B_2-B_3(B_3+C)}+\cdots\right\}$$

$$\chi_2$$

$$(A+B_1+B_2+B_3-E_1)\chi_1(\alpha\alpha\beta+\alpha\beta\alpha+\beta\alpha\alpha)=0.$$

$$\begin{cases}(A+B_3-E)\chi_1+B_1\chi_2+B_2\chi_3=0 & B_3\\ (\ \ B_1)\chi_1+(A+B_2-E)\chi_2+B_3\chi_3=0 & -B_2\\ (\ \ B_2)\chi_1+(\ B_3\ )\chi_2+(A+B_1-E)\chi_1=0\end{cases}$$

$$(A\pm\sqrt{\ ,\ ,}-E_2)\chi_2=0.$$

$$(A+B_3-E_2)\frac{\chi_2}{B_1B_2-B_3(B_3-C)}+\frac{B_1\chi_2}{(\quad)}+\frac{B_2\chi_2}{(\quad)}=0$$

$$(\ \ )\chi_1+\frac{(B_3-C)\chi_2}{B_1B_2-B_3(B_3-C)}+\frac{B_1}{(\quad)}+\frac{B_2}{(\quad)}$$

$$H = T + \sum_{i<j} V_{ij}$$

$$\sout{(T + \sum_{i<j} V_{ij}) X_1 = E_0 X_1}$$

$$\sout{(T + \sum_{i<j} V}$$

$$\Psi = w(\vec{r}_1, \vec{r}_2, \vec{r}_3)(\alpha_1\alpha_2\alpha_3,\ \alpha_1\alpha_2\beta_3 + \alpha_1\beta_2\alpha_3 + \beta_1\alpha_2\alpha_3,\ \cdots,\ \beta_1\beta_2\beta_3)$$

$$u(\vec{r}_1, \vec{r}_2, r_3)(\alpha_1\beta_2 - \beta_2\alpha_1)\alpha_3 + v(\vec{r}_1, \vec{r}_2, r_3)\{\alpha_1\beta_2\alpha_3 + \beta_1\alpha_2\alpha_3 - 2\alpha_1\alpha_2\beta_3\}$$

etc

$$H \begin{pmatrix} u \\ v \end{pmatrix} = E \begin{pmatrix} u \\ v \end{pmatrix}$$

$$H = T + \begin{pmatrix} J_{12} + J_{23}P_{23} + J_{13}P_{13} & 0 \\ 0 & -J_{12} + J_{23}P_{23} + J_{13}P_{13} \end{pmatrix}$$

$$(T + J_{12} + J_{23}P_{23} + J_{13}P_{13})u = Eu$$

$$(T - J_{12} + J_{23}P_{23} + J_{13}P_{13})v = E'v$$

$$(T + J_{12} + J_{23}P_{23} + J_{13}P_{13})u = Eu$$

$$(T + J_{12} + J_{23}P_{23} + J_{13}P_{13})\varphi u = (E + \varepsilon)\varphi u.$$

$m\vec{v} = M\vec{V}' + m\vec{v}'$

$mv^2 = MV'^2 + mv'^2$

$(m+M)\vec{U} = m\vec{v}$ $\qquad (m+M)\vec{U}' = m\vec{v}' + M\vec{V}'$

$\therefore \vec{U} = \vec{U}'$

$\vec{u} = \vec{v}$ $\qquad \vec{u}' = \vec{v}' - \vec{V}'$

~~$\vec{u}\vec{u}' = \vec{v}\vec{v}' - \vec{v}\vec{V}'$~~

$M^2V'^2 = m^2v^2 + m^2v'^2 - 2\cos\varphi \cdot m^2\vec{v}\vec{v}'$

$= mMv^2 - mMv'^2$

$1 - \alpha x = 1 - \frac{\alpha(1-\cos\varphi)}{2}$

$(m+M)v'^2 - 2m\,\vec{v}\vec{v}' - (M-m)v^2 = 0.$

$$v' = \frac{m\frac{\vec{v}\vec{v}'}{v} \pm \sqrt{\frac{m^2(\vec{v}\vec{v}')^2}{v^2} + (M^2-m^2)v^2}}{m+M}$$

$$v' = \frac{m\,\vec{v}\vec{v}' \pm \sqrt{m^2(\vec{v}\vec{v}')^2 + (M^2-m^2)v^2v'^2}}{(m+M)v'}$$

$\vec{V}' = \frac{m\vec{v} - m\vec{v}'}{M}.$ $\qquad M\vec{u}' = M\vec{v}' - (m\vec{v} - m\vec{v}')$

$= (M+m)\vec{v}' - m\vec{v}$

$M\vec{u}\vec{u}' = (M+m)\vec{v}\vec{v}' - mv^2$

$Muu' = (M+m)vv' - mv^2.$

$$\cos\varphi = \frac{\vec{u}\vec{u}'}{uu'} = \frac{M+m}{M}\frac{\vec{v}\vec{v}'}{v^2} - \frac{m}{M}.,$$

$$(m+M)\left(\frac{v'}{v}\right)^2 - \frac{2mM}{M+m}\left(\cos\varphi + \frac{m}{M}\right) - (M-m) = 0,$$

$$\left(\frac{v'}{v}\right)^2 = \frac{M^2 - m^2 + 2mM\cos\varphi + 2m^2}{(M+m)^2}$$

~~$\frac{1-\cos\varphi}{2} = x$~~ $\qquad = 1 - \frac{2mM(1-\cos\varphi)}{(M+m)^2}$

$\varphi = 0 \quad x = 0$

$\varphi = \pi \quad x = 1.$ $\qquad = 1 - \frac{4mM}{(M+m)^2}\frac{1-\cos\varphi}{2}$

$$\int_{ka}^{(k+1)a} e^{-u} du \left\{ u^{n-1} - \binom{n}{1}(u-a)^{n-1} + \binom{n}{2}(u-a)^{n-1} - \cdots + \cdots + (-1)^k \binom{n}{k}(u-ka)^{n-1} \right\}$$

$$= e^{-u} u^{n-1} \Big|_{ka}^{(k+1)a} + (n-1)\int_{ka}^{(k+1)} e^{-u} du\, u^{n-2}$$

$$- \binom{n}{1} e^{-u} (u-a)^{n-1} \Big|_{ka}^{(k+1)a} - (n-1)\binom{n}{1} \int_{ka}^{(k+1)a} e^{-u} du (u-a)^{n-2}$$

$$+ \cdots - (n-1)\frac{n(n-1)}{2} \int \cdots$$

$$\cos\varphi = \frac{M+m}{M}\left(\frac{v'}{v}\right)\cos\Theta - \frac{m}{M}.$$

$$= \frac{M+m}{M} \frac{\sqrt{M^2+m^2+2Mm\cos\varphi}}{(M+m)} \cos\Theta - \frac{m}{M}.$$

$$M^2\left(\cos\varphi + \frac{m}{M}\right)^2 = \left(M^2+m^2+2Mm\cos\varphi\right)\cos^2\Theta.$$

$$M^2\cos^2\varphi + 2Mm\cos\varphi + m^2 = \left(M^2+m^2+2Mm\cos\varphi\right)\cos^2\Theta$$

$$M^2\cos^2\varphi + 2Mm\sin^2\Theta\cos\varphi + \left(M^2\cos^2\Theta - m^2\sin^2\Theta\right) = 0.$$

$$\cos\varphi = \frac{-Mm\sin^2\Theta \pm \sqrt{M^2m^2\sin^4\Theta + M^2m^2\sin^2\Theta \cdots}}{M^2}$$

$$= \frac{-m\sin^2\Theta \pm \sqrt{M^2\cos^2\Theta - m^2\sin^2\Theta\cos^2\Theta \cdots}}{M}$$

$$\cos\varphi = \frac{-m\sin^2\Theta \pm \cos\Theta\sqrt{M^2 - m^2\sin^2\Theta}}{M}$$

$$= \frac{-m + m\cos^2\Theta \pm \cos\Theta\sqrt{M^2 - m^2 + m^2\cos^2\Theta}}{M}$$

$$d(\cos\varphi) = \frac{2m\sin\Theta\cos\Theta + \cdots}{M} d\Theta$$

~~$m\vec{v} + mv = MV'$~~,

$$(m\vec{v} - m\vec{v}')^2 = M(mv^2 - mv'^2)$$

$$(m+M)v'^2 - 2m\vec{v}\vec{v}' - (M-m)v^2 = 0$$

$$2m\vec{v}\vec{v}' = (m+M)v'^2 - (M-m)v^2$$

$$2m\frac{\vec{v}\vec{v}'}{vv'} = (m+M)\frac{v'}{v} - (M-m)\frac{v}{v'}$$

$$(M+m)x^2 - 2m\cos\Theta\, x - (M-m) = 0.$$

$$x = \frac{m\cos\Theta \pm \sqrt{m^2\cos^2\Theta + M^2 - m^2}}{M+m}$$

(+)

$2m = M$ $\quad$ $$x = \frac{\cos\Theta \pm \sqrt{3+\cos^2\Theta}}{3}$$

$$x = \frac{\cos\Theta + \sqrt{3+\cos^2\Theta}}{3}.$$

$$\cos\varphi = \frac{\cos\Theta + \sqrt{3+\cos^2\Theta}}{2}\cos\Theta - \frac{1}{2}.$$

$$d(\cos\varphi) = \left\{\cos\Theta + \frac{\sqrt{3+\cos^2\Theta}}{2} + \frac{\cos^2\Theta}{2\sqrt{3+\cos^2\Theta}}\right\} d(\cos\Theta)$$

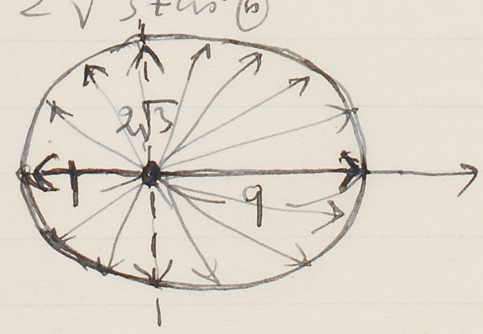
$$= \frac{2\cos\Theta\sqrt{3+\cos^2\Theta} + 3 + \cos^2\Theta + \cos^2\Theta}{2\sqrt{3+\cos^2\Theta}}\, d(\cos\Theta)$$

$$= \frac{3 + 2\cos^2\Theta + 2\cos\Theta\sqrt{3+\cos^2\Theta}}{2\sqrt{3+\cos^2\Theta}}\, d(\cos\Theta)$$

$$\frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\frac{3+2+4}{4}$$

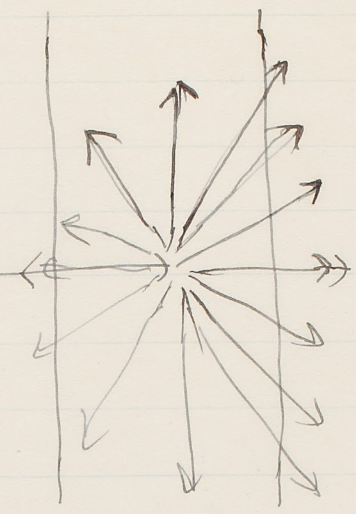
$$\frac{3+2-4}{4}$$



Dunning, Segram, Fink and Mitchell, Phys. Rev. 48, 265, 1935.

cross section $\times 10^{+24}cm^2$	slow	fast
	4.0	1.71

D_2O $\begin{matrix} g/cm^2 \\ 1.36 \end{matrix}$: fraction transmitted 0.631 (slow)



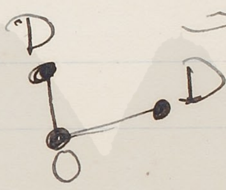
Deuteron, strong binding.

$$-\frac{h}{2\pi i}\frac{\partial\Psi}{\partial t} = -\frac{h^2}{8\pi^2 M}\left(\frac{\partial^2}{\partial x_1^2}+\cdots + \frac{1}{2}\frac{\partial}{\partial x_2^2}+\cdots\right)\Psi + U\Psi + V\Psi$$

$$M' = \frac{M(2M)}{M+2M} = \frac{2}{3}M$$

$$\gamma = \frac{2n}{1+n}$$

$$\chi''(r) + \frac{2}{r}\chi'(r) = \frac{4\pi^2 M}{3h^2}V(r)\chi(r)$$



$$-\frac{h}{2\pi i}\dot{\Psi} = -\frac{h^2\rho}{\pi M \gamma} - \frac{h^2}{8\pi^2 M}\left(\frac{\partial^2\Psi}{\partial x_1^2}+\cdots - \frac{1}{2}\frac{\partial}{\partial x_2^2}+\cdots\right)$$

$$+U(x_2, y_2, z_2)\Psi - \frac{h^2\rho}{\pi M \gamma}\delta_R(r)\Psi.$$

$$\sigma_{mn}\,d\omega = \frac{4\rho^2}{\gamma^2}\frac{p'}{p}\,d\omega \left|\iint_{r\le R_0} u_m u_n\, e^{\frac{2\pi i}{h}(\vec{p}-\vec{p}',\,\vec{r}_2)} d\vec{r}_1 d\vec{r}_2\right|^2$$

$$\left(\frac{3}{4\pi R_0^3}\right)^2 \times \left|\iint u_m u_n\, e^{\frac{2\pi i}{h}(\vec{p}-\vec{p}',\,\vec{r}_2)} d\vec{r}_2\right|^2$$

~~$r \le R_0$~~

st. bound:

$$\frac{4\rho^2}{\gamma^2}\,d\omega.$$

free:

$$\frac{4\rho^2}{\gamma^2}\int\frac{\frac{p'}{p}\,d\omega}{\sqrt{1-\gamma(1-\cos\varphi)}}\,d\omega$$

$$\left(\frac{p'}{p}\right)^2 = 1 - \frac{4n}{1+n}\cdot\frac{1-\cos\varphi}{2}, \quad 2\gamma = \frac{4n}{1+n}$$

$$\frac{1}{2}\int^{\gamma}(1+\gamma\cos\varphi)\sin\Theta\, d\Theta$$

$$\cos\varphi = -1+1\cos^2\Theta$$

$$\cos\varphi = \frac{\cos\Theta+\sqrt{5+\cos^2\Theta}}{2}\cos\Theta - \frac{1}{2}.$$

$$\gamma = \frac{2}{3}$$

$$\frac{1}{2\pi}\int\sqrt{\frac{1}{3}+\frac{2}{3}\left\{\frac{\cos\theta+\sqrt{3+\cos^2\theta}}{2}\cos\theta-\frac{1}{2}\right\}}\,d(\cos\theta)$$

$$=\frac{1}{3}+\frac{1}{6}\int_{-1}^{+1}(x^2+\sqrt{3+x^2})\,dx \qquad x^2=y,\quad 2x\,dx=y$$

$$=\frac{1}{6}\left(\frac{2}{3}+\int_0^1\sqrt{3+y}\,dy\right)$$

$$=\frac{1}{6}\left(\frac{2}{3}+\frac{2}{3}\left.(3+y)^{\frac{3}{2}}\right|_0^1\right)$$

$$=\frac{1}{9}\left(1+8-(3)^{\frac{3}{2}}\right)$$

$$=1-\frac{(3)^{\frac{3}{2}}}{9}=1-\frac{1}{\sqrt{3}} \qquad (3)^{\frac{3}{2}}(3)^{\frac{1}{2}}$$

$$\frac{\sigma_b}{\sigma_f}=\frac{1}{1-\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}}{\sqrt{3}-1}=\frac{\sqrt{3}(\sqrt{3}+1)}{2}=\frac{3+\sqrt{3}}{2}$$

$$=\frac{4.732}{2}=2.37$$

(2)

$$\frac{1}{2}\int_{-1}^{+1}\sqrt{\frac{1}{3}+\frac{2}{3}\left\{\frac{x+\sqrt{3+x^2}}{2}x-\frac{1}{2}\right\}}\,dx$$

$$=\frac{1}{2}\int_{-1}^{+1}\sqrt{\frac{1}{3}\left\{x^2+x\sqrt{3+x^2}\right\}}\,dx \qquad 1-2$$

$$(3+x^2) \quad (a+b\sqrt{3+x^2})^2 = 2ab\sqrt{3+x^2}+a^2+b^2(3+x^2)$$

$$1+2\sqrt{3}$$

$$=\frac{1}{2\sqrt{3}}\int_{-1}^{+1}\sqrt{x+\sqrt{3+x^2}}\cdot\sqrt{x}\,dx$$

$x=0$: 0

$x=+1$: $\sqrt{3}$

$x=-1$:

$\left(\frac{p'}{p}\right) = \sqrt{1 + \frac{\alpha(1-\cos\varphi)}{2}} = \left(\frac{v'}{v}\right) = x$ $\Big|^{1}_{\sqrt{1-\alpha}}$

~~$\cos\varphi = \frac{M+m}{M}\left(\frac{v'}{v}\right)\cos\Theta - \frac{m}{M}$~~

$\frac{v'}{v} = x$

$\frac{3}{2}\cdot\frac{1}{3} - \frac{3}{2} = -1.$

$\cos\varphi = \frac{M+m}{M}$

$2m\cos\Theta = (m+M)\, x - (M-m)\frac{1}{x}$

$\alpha = \frac{4mM}{(m+M)^2} = \frac{4n}{(1+n)^2}$

$\cos\Theta = \frac{m+M}{2m}x - \frac{M-m}{2mx}$

$d(\cos\Theta) = \frac{m+M}{2m}dx + \frac{M-m}{2mx^2}dx$

$\sqrt{1-\alpha} = \sqrt{\frac{(1-n)^2}{(1+n)^2}} = \frac{n-1}{1+n}$

$= \frac{dx}{2m}\left\{(m+M) + \frac{M-m}{x^2}\right\}$

$= \frac{dx}{2}\left\{(1+n) + \frac{(n-1)}{x^2}\right\}$

$\frac{1}{2}x\,d(\cos\Theta) = \frac{dx}{4}\left\{(1+n)x + \frac{(n-1)}{x}\right\}$

$k = \int_{\sqrt{1-\alpha}}^{1}\left\{(1+n)x + \frac{(n-1)}{x}\right\}\frac{dx}{4} = \frac{1}{2}\left\{\frac{(1+n)x^2}{4} + \frac{(n-1)}{2}\log x\right\}\Big|_{\sqrt{1-\alpha}}^{1}$

$= \frac{1}{2}\left\{\frac{(1+n)}{4}\left\{1 - \frac{(n-1)^2}{(1+n)^2}\right\} + \frac{(n-1)}{2}\log\frac{1+n}{n-1}.\right\}$

$= \frac{1}{2}\left\{\frac{n}{(1+n)} + \frac{(n-1)}{2}\log\frac{n+1}{n-1}\right\}$

$\frac{3+1}{3-1} = \log 2$ $\frac{2}{2}$

$n=1:\quad k_1 = \frac{1}{4}$

$n=2:\quad k_2 = \frac{1}{3} + \frac{1}{4}\log 3\}$

$= 0.3333\cdots + \frac{1}{4}(1.0986)$

$\quad\quad = 0.27465$

$= 0.608$

$\frac{1}{k_2} =$

$k_3 = \frac{1}{2}\left\{\frac{3}{4} + \log 2\right\} = \frac{1}{2}\{0.75 + 0.6931\}$

$= \frac{1}{2}\{1.4431\} = 0.7215$

$n = 12$; $k_{12} = \frac{1}{2}\left\{\frac{12}{13} + \frac{11}{2}\log\frac{13}{11}\right\}$

$= 0.4615 + ~~0.91960~~\ 0.4598$

$= ~~1.877~~ = 0.9213$

$\log 13 =$ 1.182 ; 0.1655 × 17 = 0.1672 ; 0.1672 × 55 = 9.1960 ; 0.91960 / 2 = 0.4598

$\log\frac{1+\frac{1}{n}}{1-\frac{1}{n}} \cong \frac{2}{n}$

$n \gg 1$: $\frac{1}{2}\left\{1 - \frac{1}{n} + \frac{n-1}{2}\cdot\frac{2}{n}\right\} = \frac{1}{2}\left\{2 - \frac{2}{n}\right)$

$= 1 - \frac{1}{n}$!!!

$\frac{1}{0.608} \cong 1.67.$

	slow	C-group
D_2 :	8.0×10^{-24}	~~6.8×10^{-24}~~ 13.3
O :	3.3×10^{-24}	3.3
D_2O :	11.3×10^{-24}	16.6×10^{-24}.

$\frac{16.6}{11.3} = 1.5$

$$\left\{ \Delta_1 + \tfrac{1}{2}\Delta_2 + \frac{2M}{\hbar^2}(E - U - V) \right\} \psi = 0.$$

$$\chi''(r) + \frac{2}{r}\chi'(r) = \frac{2\gamma M}{\hbar^2} V(r)\chi(r) \qquad \gamma = \frac{n}{1+n}$$

$$\chi = \frac{v}{r}. \qquad \chi' = \frac{v'}{r} - \frac{v}{r^2} \qquad \chi'' = \frac{v''}{r} - \frac{2v'}{r^2} + \frac{2v}{r^3}$$

$$v'' = \frac{2\gamma M}{\hbar^2} V(r) v.$$

$$\psi = \Psi \chi$$

$$\overline{V\psi} = \frac{3}{4\pi R_0^3} \iiint V(r') \Psi(\vec{r}_1', \vec{r}_2) \chi(r') \, d\vec{r}_1'$$

$$= \frac{3}{4\pi R_0^3} \Psi(\vec{r}_2 \vec{r}_2) \iiint V(r') \chi(r') d\vec{r}_1' \quad \text{for } r < R_0$$

$$= \frac{3}{4\pi R_0^3} \Psi(\vec{r}_2 \vec{r}_2) \, 4\pi \int V v r dr$$

$$\frac{3}{4\pi R_0^3} \int \cdots \int \tilde{u}_n(\vec{r}_2) e^{-\frac{2\pi i}{h}(p'\vec{r}_2)} u_m(\vec{r}_2) e^{\frac{2\pi i}{h}(p\vec{r}_2)} d\vec{r}_1 d\vec{r}_2$$
$r \le R_0$

$$= \int \tilde{u}_n(\vec{r}_2) e^{-\frac{2\pi i}{h}(p'\vec{r}_2)} u_m(\vec{r}_2) e^{\frac{2\pi i}{h}(p\vec{r}_2)} d\vec{r}_2$$

$$\propto \rho^2 (2\gamma)^2 \frac{p'}{p} d\omega.$$

$$\boxed{\frac{e^{ikr}}{r}\sin\varphi\, d\varphi}$$

$$\left.\begin{aligned}\vec{r} &= \vec{r}_1 - \vec{r}_2\\ \vec{R} &= \frac{\vec{r}_1 + 2\vec{r}_2}{3}\end{aligned}\right\}$$

$$kz + KZ = \left(k + \frac{K}{3}\right) z_1 + \left(\frac{2}{3}K - k\right) z_2$$

$$k = \frac{2}{3}K \;!!!$$

$$\left\{e^{ikz} + \frac{e^{ikr}}{r} f(\theta)\right\} e^{iKZ}$$

$$= e^{i K z_1} + \frac{e^{ik|\vec{r}_1 - \vec{r}_2| + KZ}}{|\vec{r}_1 - \vec{r}_2|} \int f_0 \sin\varphi\, d\varphi$$

$$= e^{iKz_1} + \frac{1}{r_1}\left\{1 + \frac{r_2\cos\varphi}{r_1} + \cdots\right\} e^{i\left(kr_1 - kr_2\cos\varphi + \frac{K}{3}z_1 + \frac{2}{3}z_2\right)} f(\theta)$$

$r_1 > r_2$

$$\frac{1}{|\vec{r}_1 - \vec{r}_2|} = \frac{1}{(r_1^2 - 2r_1 r_2\cos\varphi + r_2^2)^{\frac{1}{2}}}$$

$$= \frac{1}{r_1}\left\{1 + \frac{r_2\cos\varphi}{r_1} + \cdots\cdots\right\}$$

$$|\vec{r}_1 - \vec{r}_2| = r_1\left(1 - \frac{r_2}{r_1}\cos\varphi + \cdots\right)$$

$$\simeq e^{iKz_1} + \frac{e^{i(kr_1 + Kz_1)}}{r_1} f(\theta)$$

$$\cos\theta = \frac{\cos\Theta + \sqrt{3 + \cos^2\Theta}}{2}\cos\Theta, \quad -\frac{1}{2}$$

$$z_1 = r_1\cos\Theta.$$

$$\frac{\partial}{\partial r_1} e^{i\left(kr_1 + \frac{K}{3}z_1\right)} = ik\left(1 + \frac{1}{2}\cos\Theta\right) e^{i\left(kr_1 + \frac{K}{3}z_1\right)}$$

$$= iK\left(\frac{2}{3} + \frac{1}{3}\cos\Theta\right)$$

$4\pi a^2$

$$\frac{1}{2}\left(\frac{2}{3} + \frac{1}{3}\cos\Theta\right) d(\cos\theta) = \frac{1}{2}\left(\frac{2}{3} + \frac{1}{3}\cos\Theta\right)\left(\cos\Theta + \frac{3 + 2\cos^2\Theta}{2\sqrt{3 + \cos^2\Theta}}\right) d(\cos\Theta)$$

$$= \frac{1}{6}(2 + x)\left(x + \frac{3 + 2x^2}{2\sqrt{3 + x^2}}\right) dx$$

$$= \frac{1}{6}\left\{2x + x^2 + \frac{3 + 2x^2}{\sqrt{3 + x^2}} + x\left(\frac{3 + 2x^2}{2\sqrt{\cdots}}\right)\right\} dx$$

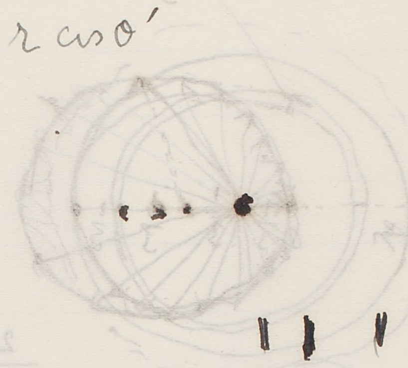
$$\frac{1}{6}\int_{-1}^{+1}\left\{x^2 + \frac{3 + 2x^2}{\sqrt{3 + x^2}}\right\} dx = \frac{1}{6}\left[\frac{2}{3} - 3\int_{-1}^{+1}\frac{dx}{\sqrt{3 + x^2}} + 2\int_{-1}^{+1}\sqrt{3 + x^2}\, dx\right.$$

$$3\sinh^{-1}\frac{2}{2\sqrt{3}} \qquad 4 + 3\log 3$$

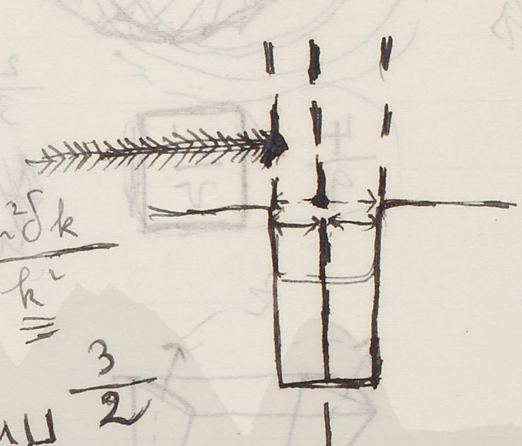
$e^{-ikz'}\left\{e^{ikz}+\frac{e^{ikr}}{r}f(\theta)\right\}\left\{e^{-ikz'}+\frac{e^{ikr}}{r}f(\theta)\right\}$ Theory

$r\cos\varphi$ $\qquad z'=r\cos\theta'$

$$=\int\frac{e^{ikr(1-\cos\theta')}}{r}4\pi r^2dr$$



$$=4\pi k\int e^{ikr(1-\cos\theta')}r\,dr$$



$\frac{\sin^2\delta k}{k^2}$

$$=4\pi k\frac{\sin\theta'd\theta'}{\{k(1-\cos\theta')\}^2}$$

$k'=\mu\sqcup^{\frac{3}{2}}$

$4\pi a^2\times\frac{1}{[illegible]}$

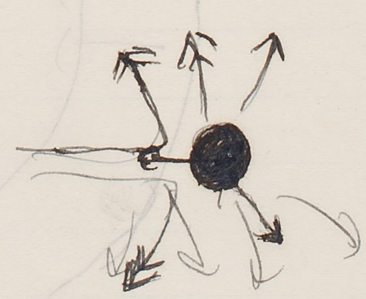
$\left(k\cos\varphi+\frac{K}{3}r_1\cos\theta\right)$

$\cos\varphi=\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\varphi_1\omega-\omega')$ $\quad(\sin\omega\cos\omega'+\sin\omega\sin\omega')$

$(k\cos\theta'+\frac{k}{3}\cos\theta)^2+k^2\sin^2\theta'\,\cos^2(\omega-\omega')$

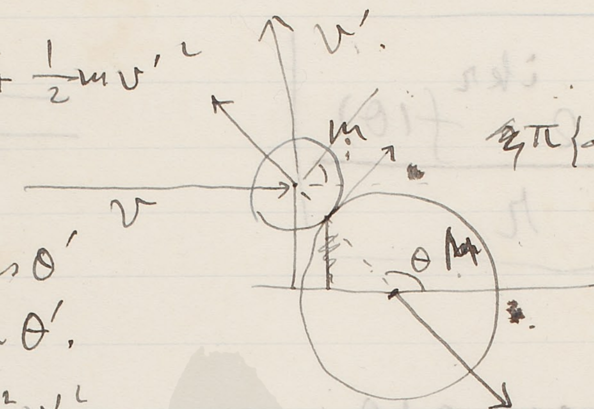
$$=k^2\left\{\left(\cos\theta'+\frac{1}{2}\right)^2+\sin^2\theta'\cos^2(\omega-\omega')\right\}$$

$$=k^2\left(\frac{5}{4}+\cos\theta'\right)$$

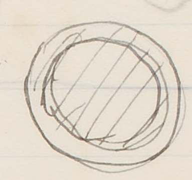


f $\qquad\sin\theta$

$\frac{1}{2}mv^2 = \frac{1}{2}MV^2 + \frac{1}{2}mv'^2$



$\frac{3}{2}\pi\{a\ a\cos\theta\}^2$ ($\sin\theta$)



$-mv\cos\theta$

$= MV\cos\theta : mv'\cos\theta'$

$v\sin\theta = v'\sin\theta'$,

$(mv - MV)^2\cos^2\theta = m^2v'^2$

$+ m^2v^2\sin^2\theta$

$d\theta : a^2\pi\{a\sin\theta d\theta$ ($\cos\theta$)

$\pi a^2\, 2\sin\theta\cos\theta\, d\theta$

$m^2v^2 - 2m^2v^2\cos^2\theta + M^2V^2\cos^2\theta$

$= \frac{1}{2}m^2v^2$

$(mv - MV)^2\cos^2\theta + m^2v^2\sin^2\theta = m^2v'^2$

~~$mv - MV$~~

~~$mv - MV =$~~ $v' =$

$V^2 = \frac{m}{M}v^2 - \frac{m}{M}v'^2$

$M \gg m$. ~~$\frac{1}{2}mv^2$~~ $v \fallingdotseq v'$,

$\sin\theta = \sin\theta'$

$\Theta = \theta + \theta' = 2\theta$.

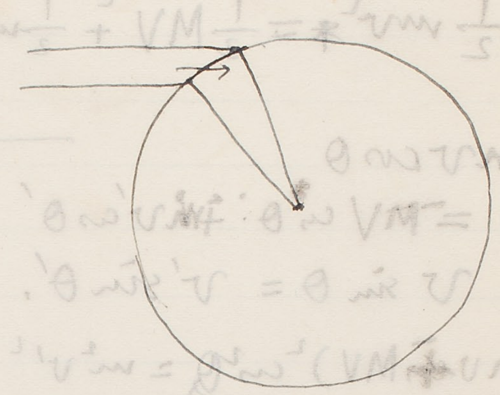
$\pi a^2 \sin\frac{\Theta}{2} d\Theta$

$M = m$. $v^2 = V^2 + v'^2$

$-v\cos\theta = -V\cos\Theta + v'\cos\theta'$

$v\sin\theta = v'\sin\theta'$

$$e^{i\vec{k}\vec{r}} + \frac{e^{ikr}}{r} f(\theta)$$



$\vec{k}(k, \theta, \varphi)$. $f(\theta) \sin\theta \, d\theta$

$k_0 = \frac{1}{2} k$

$\vec{k}_0 + \vec{k}$ ($k \sin\theta$, $k_0 + k\cos\theta$)

$$\frac{\sqrt{k_0^2 + k^2 + 2k_0 k \cos\theta}}{k_0 + k} = \frac{\sqrt{k_0^2 + 4k_0^2 + 4k_0^2\cos\theta}}{3k_0}$$

$$= \frac{1}{3} \int \sqrt{5 + 4\cos\theta} \, \sin\theta \, d\theta.$$

$$\frac{1}{3} \int (5 + 4x)^{\frac{1}{2}} dx = \frac{1}{3} \, 2 \, (5 + 4x)^{\frac{3}{2}} \Big|_{0}^{1} \Big|_{-1}^{0}$$

$$= \frac{2}{3} (9^{\frac{3}{2}} + 1^{\frac{3}{2}}) = \frac{2}{3} (26) = \frac{4 \times 14}{3}.$$

3^3

$$= \frac{2}{3} \times 14 \ !!!$$

$$\int_{-1}^{+1} \frac{1}{\sqrt{3+x^2}}\,dx = \log\left(x+\sqrt{3+x^2}\right)\Big|_{-1}^{1}$$

$$= \log 3$$

$$\frac{1}{6}\left[\frac{2}{3} - 3\log 3 + 4 + 3\log 3\right]$$

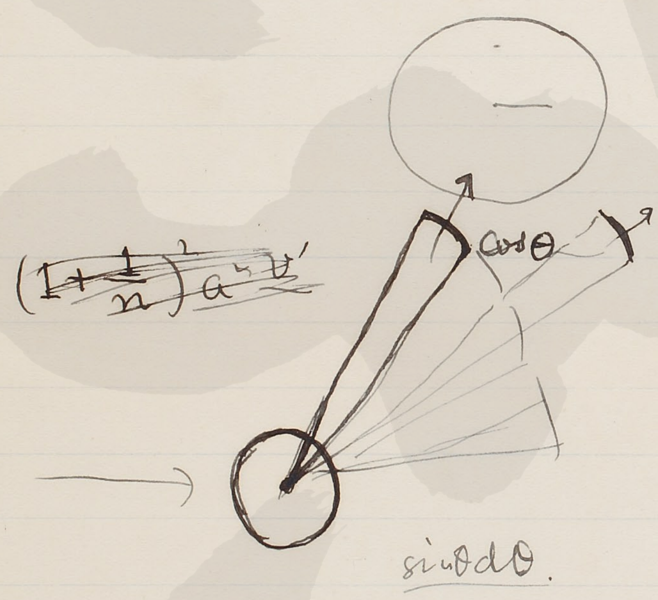
$$= \frac{1}{6}\,8\,\frac{}{3}$$

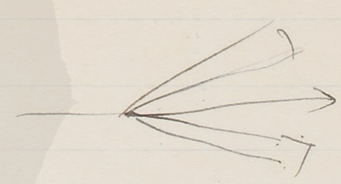
$$\int_{-1}^{+1}\left(x + \frac{3+2x^2}{2\sqrt{3+x^2}}\right)dx$$

$$= \int_{-1}^{+1}\sqrt{3+x^2}\,dx - \frac{3}{2}\int_{-1}^{+1}\frac{dx}{\sqrt{3+x^2}}$$

$$= 2 + \frac{3}{2}\log 3 - \frac{3}{2}\log 3.$$

$d\omega$





$$\left(\frac{x+\sqrt{3+x^2}}{3}\right)\left(x+\frac{3+2x^2}{2\sqrt{3+x^2}}\right) x + \frac{}{}$$

$$= \frac{x^2}{3} + \frac{x\sqrt{3+x^2}}{3} + \frac{}{2\sqrt{3+x^2}} \quad \frac{3+2x^2}{2\cdot 3} +$$

$$\frac{2}{9} + 1 + \frac{2}{9} = \frac{13}{9}$$