$$\left\{-\frac{\hbar^2}{2M}\{\Delta_1+\Delta_2+\Delta_3\}+K_{12}+J_{13}+J_{23}-E\right\}\psi=0.$$

$$\psi = f(\vec{r}_1,\vec{R}_{23})\frac{e^{-\alpha r_{23}}}{r_{23}} \pm f(\vec{r}_2,\vec{R}_{13})\frac{e^{-\alpha r_{13}}}{r_{13}}$$

$\sqrt{\frac{\alpha}{2\pi}}$

$$\Delta_2+\Delta_3=\frac{1}{2}\Delta_{R_{23}}+2\Delta_{23} \quad \left(-\frac{\hbar^2}{M}\Delta_{23}-W\right)\frac{e^{-\alpha r_{23}}}{r_{23}}=0 \quad (+J_{23})$$

$$\frac{\sqrt{MW}}{\hbar}=\alpha$$

$$\left\{-\frac{\hbar^2}{2M}\{\Delta_1+\frac{1}{2}\Delta_{R_{23}}\}+K_{12}+J_{13}-E+W\right\}=0$$

$$\times\,\psi=0.$$

$$\left\{-\frac{\hbar^2}{2M}\{\Delta_1+\frac{1}{2}\Delta_{R_{23}}\}+J_{13}-E+W\right\}f(\vec{r}_1,\vec{R}_{23}) \pm K f(\vec{r}_1,\vec{R}_{23})$$

$$+\int f(\vec{r}_2,\vec{R}_{13}) \qquad =0$$

$$+\cdots\cdots\cdots\cdots\cdots=0$$

spherical wave = plane wave

$$e^{ikz}=\sum_{n=0}^{\infty}(2n+1)i^n\left(\frac{\pi}{2kr}\right)^{\frac{1}{2}}J_{n+\frac{1}{2}}(kr)\,P_n(\cos\theta)$$

$$\underset{r\to\infty}{\sim}\sum_{n=0}^{\infty}(2n+1)i^n\frac{\sin(kr-\frac{n\pi}{2})}{kr}P_n(\cos\theta),$$

$$\iiint f(\chi,\omega)\,e^{ikr\cos\Theta}\sin\chi\,d\chi\,d\omega \sim \sum_{n=0}^{\infty}(2n+1)i^n\frac{\sin(kr-\frac{n\pi}{2})}{kr}\iint P_n(\cos\Theta)\,f(\chi,\omega)\sin\chi\,d\chi\,d\omega$$

$$\cos\Theta=\cos\chi\cos\theta+\sin\chi\sin\theta\sin(\omega-\varphi)$$

(k,χ,ω),

(r,θ,φ).

$$\iint f(\chi)\, P_n(\cos\Theta) \sin\chi\, d\chi\, d\omega = g_n(\cos\theta).$$

$$P_n(\cos\theta) \sum_{n=0}^{\infty} (2n+1)\, i^n \frac{\sin(kr - \frac{n\pi}{2})}{kr}\, g_n(\Theta)$$

$\sin\theta\, d\theta$

$$= \sum_{n=0}^{\infty} (2n+1)\, i^n \frac{\sin(kr - \frac{n\pi}{2} + \delta_0)}{kr}\, g_n(\Theta)\, P_n(\cos\theta)$$

$$\frac{1}{2ck} \sum_{n=0}^{\infty} (2n+1) [e^{2i\eta_n} - 1]\, P_n(\cos\theta)\, \frac{e^{ikr}}{r}$$

$$= \iint f(\chi,\omega)\, e^{ikr\cos\Theta} \sin\chi\, d\chi\, d\omega$$

(k, χ, ω) $\quad (r, \theta, \varphi)$

$$\cos\Theta = \cos\chi\cos\theta + \sin\chi\sin\theta\sin(\omega - \varphi)$$

$$\left\{ \Delta + \frac{2M}{\hbar^2}(E - V) \right\} \psi = 0.$$

$$\psi = e^{ikz} + \iint f(\chi,\omega)\, e^{ikr\cos\Theta} \sin\chi\, d\chi\, d\omega,$$

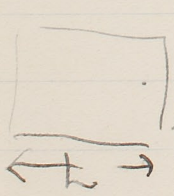
$k^2 = \frac{2ME}{\hbar^2}$. $\quad \alpha^2 = \frac{2MV}{\hbar^2}$ $\quad e^{i(\vec{k}\vec{r})}$

$e^{i(kz)}$

$$-\alpha^2 e^{ikr\cos\Theta} + \iint f(\chi,\omega)\, \alpha^2 e^{ikr\cos\Theta} \sin\chi\, d\chi\, d\omega = 0$$

$e^{-i(\vec{k}_0\vec{r})}$

$$\psi = e^{ikz} + \frac{1}{L^3} \sum_{k_z = 0, \pm 1, \ldots} f_1(k_z)\, e^{i\vec{k}_1\vec{r}}$$



$$k_x, k_y, k_z = \frac{2\pi\, n_x, n_y, n_z}{L}$$

$$e^{-i\vec{k}_0\vec{r}} + \alpha^2 e^{ikz} + \sum_{k_z} f(k_z)\alpha^2 e^{i\vec{k}\vec{r}} = 0$$

$$\sum f(k_z)\iiint \alpha^2 e^{i(\vec{k}-\vec{k}_0)\vec{r}}\, dv = 0.$$

$$\iiint \alpha^2 e^{i(\vec{k}_0-\vec{k}')\vec{r}}\, dv + \sum$$

$$\iiint \alpha^2(r)\, e^{ikr(\cos\chi\cos\theta + \sin\chi\sin\theta\cos(\omega-\varphi) - \cos\chi'\cos\theta - \sin\chi'\sin\theta\cos(\omega'-\varphi))} r^2 dr \sin\theta\, d\theta\, d\varphi$$

$$= \iiint \alpha^2(r)\, e^{ikr(\cos\chi-\cos\chi')\cos\theta} \cdot e^{ikr(\sin\chi\cos(\omega-\varphi) - \sin\chi'\cos(\omega'-\varphi))\sin\theta} \sin\theta\, d\theta\, d\varphi$$

$$\int_0^{2\pi} e^{ikr\{(\sin\chi\cos\omega - \sin\chi'\cos\omega')\cos\varphi - (\sin\chi\sin\omega - \sin\chi'\sin\omega')\sin\varphi\}} d\varphi$$

$$\sin\chi\cos\omega - \sin\chi'\cos\omega' = A\cos\vartheta$$
$$\sin\chi\sin\omega - \sin\chi'\sin\omega' = A\sin\vartheta$$

$$\int_0^{2\pi} e^{iAkr\cos(\varphi+\vartheta)}\, d\varphi =$$

k

$$\iiint \alpha^2(r)\, e^{iKr\cos\Theta} \sin\Theta\, d\Theta\, d\Phi\, r^2 dr$$

$$= 4\pi \int_0^\infty \alpha^2(r) \frac{\sin(Kr)}{Kr} r^2 dr$$

$$4\pi \sum_{k_z} f(k_z) \int_0^\infty \alpha^2(r) \frac{\sin Kr}{Kr} r^2 dr$$

$$+ 4\pi \int_0^\infty \alpha^2(r) \frac{\sin[illegible](k_0 r)}{k_0 r} r^2 dr = 0$$

$$\lim_{K\to 0} \int_0^\infty \alpha^2(r) \frac{\sin(Kr)}{Kr} r^2 dr = \boxed{\int_0^\infty \alpha^2(r) r^2 dr}.$$

$\frac{k_z}{k} = \cos\vartheta.$

$\frac{f(\cos\vartheta)}{d(\cos\vartheta)} \quad g(\cos\vartheta)$

$+ \qquad g(\vartheta_0) = 0.$

$\int g(x, x') f(x')\, dx' + g(x) = 0$

$\qquad \| $

$\qquad P_n(x')$

mass 夫々 M_1, M_2 なる二つの particle の衝突に対して

$\vec{r}_1 - \vec{r}_2 = \vec{r}$ $\qquad \dfrac{M_1\vec{r}_1 + M_2\vec{r}_2}{M_1+M_2} = \vec{R}$

とおけば、

$$\mathrm{grad}_1 = \mathrm{grad}_r + \frac{M_1}{M_1+M_2}\,\mathrm{grad}_{\vec{R}}$$

$$\mathrm{grad}_2 = -\mathrm{grad}_r + \frac{M_2}{M_1+M_2}\,\mathrm{grad}_{\vec{R}}$$

$$\frac{\hbar^2}{2M_1}\Delta_1 + \frac{\hbar^2}{2M_2}\Delta_2 = \frac{\hbar^2}{2}\left\{\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\Delta_r + \frac{1}{(M_1+M_2)^2}\Delta_R\right\}$$

$$\mu = \frac{M_1M_2}{M_1+M_2} \qquad = \frac{\hbar^2}{2\mu}\Delta + \frac{\hbar^2}{2(M_1+M_2)}\Delta_R$$

$$\left\{\frac{\hbar^2}{2\mu}\Delta + \{E' - V(r)\}\right\}\psi = 0. \qquad k^2 = \frac{2\mu E'}{\hbar^2}. \quad \alpha^2 = \frac{2MV}{\hbar^2}$$

$$\psi = e^{ikz}$$

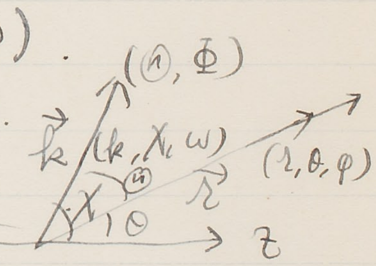
$$\{\Delta + k^2 - \alpha^2(r)\}\psi = 0.$$

$$\psi = e^{ikz} + \iint e^{ikr\cos\Theta} f(\chi,\omega)\sin\chi\, d\chi\, d\omega.$$

$$\cos\Theta = \cos\theta\cos\chi + \sin\theta\sin\chi\cos(\varphi-\omega).$$

$$-\alpha^2 e^{ikr\cos\theta} \pm \iint f(\chi,\omega)\,\alpha^2 e^{ikr\cos\theta}\sin\chi\, d\chi\, d\omega. = 0.$$

$$\cancel{-\alpha^2 e^{ikr\cos\theta}} + \iint g(\Theta,\Phi)\,\alpha^2 e^{ikr\cos\Theta}\sin\Theta\, d\Theta\, d\Phi$$



$$g(\Theta,\Phi) = f(\chi,\omega)$$

$$e^{-i\vec{k}\vec{r}}\ \cancel{e^{-ikr\cos\Theta'}} \quad -\alpha^2 e^{i\vec{k}_0\vec{r}} \pm \iint f(\vec{k})\,\alpha^2 e^{i\vec{k}\vec{r}}\sin\chi\, d\chi\, d\omega. = 0$$

$$\iint \sin\chi\, d\chi\, d\omega\, f(\chi\omega) \iiint \alpha^2 e^{i(\vec{k}-\vec{k}')\vec{r}}\, r^2 dr\, \sin\theta\, d\theta\, d\varphi$$

$$= \iint \sin\chi\, d\chi\, d\omega\, f(\chi,\omega) \iiint \alpha^4(r)\, e^{ikr\cos\Theta^*} \sin\Theta^*\, d\Theta^*\, d\Phi^*\, r^2 dr$$

$\vec{k}-\vec{k}' : (K, \Theta^*, \Phi^*)$

$$= \iint \sin\chi\, d\chi\, d\omega\, f(\chi,\omega) \cdot 4\pi \int_0^\infty \alpha^2(r) \frac{\sin Kr}{Kr}\, r^2 dr$$

$$= \int_0^\infty \alpha^2(r) \frac{\sin K_0 r}{K_0 r}\, r^2 dr + \iint f(\chi,\omega) \sin\chi\, d\chi\, d\omega \int_0 \alpha^2(r) \frac{\sin Kr}{Kr} \ldots$$

$= 0$

$\vec{k}_0 - \vec{k}' : (K_0, \ldots, \ldots)$

$$f(\chi,\omega) = \sum_{n=0}^{\infty} \sum_{m=-n}^{m=+n} A_n^m P_n^m(\cos\theta) e^{im\varphi}.$$

$$\iint e^{ikr\cos\Theta} f(\chi,\omega) \sin\chi\, d\chi\, d\omega$$

$$e^{ikr} f(\theta) \sin\theta\, d\theta.$$

$$e^{ikr\cos\Theta} = \sum_{l=0}^{\infty} (2l+1)\, i^l P_l(\cos\Theta) f_l(r).$$

$$f(\chi,\omega) = \sum_n C_n P_n(\cos\chi)$$

$$\sum_{l,n} (2l+1)\, i^l f_l(r)\, C_n \iint P_l(\cos\Theta) P_n(\cos\chi) \sin\chi\, d\chi.$$

$$= \sum_{l,n} (2l+1)\, i^l f_l(r)\, C_n \iint P_l(\cos\Theta) P_n(\cos\chi) \sin\chi\, d\chi.$$

$$P_l(\cos\Theta) = P_l(\cos\theta) P_l(\cos\chi) + 2 \sum_{m=1}^{l} \frac{(l-m)!}{(l+m)!} P_l^m(\cos\theta) P_l^m(\cos\chi) \cos m(\varphi-\omega)$$

$$= \sum_l (2l+1)\, i^l f_l(r)\, C_l \frac{2}{2l+1} P_l(\cos\theta)$$

$$= 2\sum_l f_l(r)\, C_l\, i^l P_l(\cos\theta)$$

$$J_{l+\frac{1}{2}}(x) \sim \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{l+1}{2}\pi\right)$$

$$f_l = \left(\frac{\pi}{2kr}\right)^{\frac{1}{2}} J_{l+\frac{1}{2}}(kr)$$

$$\sim \frac{1}{kr}\cos\left(kr - \frac{(l+1)\pi}{2}\right)$$

$$\sim \frac{2}{kr}\sum_l \cos\left(kr - \frac{(l+1)\pi}{2}\right) C_l\, i^l P_l(\cos\theta)$$

$$= \frac{1}{kr}\sum_l e^{ikr} C_l\, (-i)\, P_l(\cos\theta)$$

$$(i)^{2l+2} =$$

$$+ \sum_l e^{-ikr} C_l\, (i)^{2l+1} P_l(\cos\theta).$$

$$= \left[\frac{e^{ikr}}{kr}(-i) f(\theta) + \frac{e^{-ikr}}{kr}(i) f(\pi-\theta)\right]$$

$$e^{ikr} + \frac{e^{ikr}}{kr}(-i) f(\theta) + \frac{e^{-ikr}}{r}(i) f(\pi-\theta).$$

$$A_n L_n \sim (2l+1)\, i^l f_l \sim C_l (-i) \frac{e^{ikr}}{r}$$

$$A_n e^{i\eta_l} - (2l+1)\, i^l = C(-1)^{l+1}$$

$$(A_n e^{-i\eta_l} - (2l+1)\, i^l)$$

$$A_n (-1)^{l+1} (1 - (-1)^{l+1}) \cdot A_l = (2l+1)(1 - (-1)^{l+1})\, i^l$$

$$e^{i\eta_l} \quad e^{-i\eta_l} \quad e^{i\eta_l}$$

$$(e^{i\eta_l} - (-1)^l e^{-i\eta_l}) A_l = -(2l+1)(1 - (-1)^l)\, i^l$$

$$l = 0: \quad (e^{i\eta_l} - e^{-i\eta_l}) A_l =$$

$$\psi = \sum A_l P_l(\cos\theta) L_l(r)$$

$$\frac{e^{ikr}}{2ikr}\left(A_l e^{i\eta_l} - (2l+1) i^l\right) - e^{-ikr}\left(A_l e^{-i\eta_l} - (2l+1) i^l\right)$$

$$= e^{ikr}\, C_l \, P_l(\cos\theta) - e^{-ikr} (-1)^l C_l P_l(\cos\theta)$$

$$A_l e^{i\eta_l} - (2l+1) i^l = (-1)^l \{A_l e^{-i\eta_l} - (2l+1) i^l\}$$

$$A_l (e^{i\eta_l} - (-1) e^{-i\eta_l}) = (2l+1) i^l (1 - (-1)^l)$$

$A_l = 0$ for l: even.

e

$$e^{ikr} \frac{1}{2}\int_0^{\pi} e^{ikr\cos\theta} \sin\theta \, d\theta$$

$$= \frac{e^{ikr} - e^{-ikr}}{2ikr} = \qquad \frac{x^2}{2}$$

$$\frac{1}{2}\int_0^{\pi} e^{ikr\cos\theta} \cos\theta \sin\theta \, d\theta \qquad = \frac{1}{2}\int_{-1}^{1} e^{ikrx}\, x\, dx$$

$$= \left.\frac{e^{ikrx}\, x}{2ikr}\right|_{-1}^{1} - \frac{1}{2ikr}\int e^{ikrx}\, dx$$

$$= \frac{e^{ikr} + e^{-ikr}}{2ikr} - \frac{1}{2ikr}\, \frac{e^{ikr} - e^{-ikr}}{ikr}$$

$$\int ik(1+\cos\theta) \sin\theta \, d\theta \, . \; e^{ikr\cos\theta}$$

$$= \frac{e^{ikr}}{r}$$

$(-1)^n$

$$e^{-\frac{in\pi}{2}}\{A_n e^{i\eta_n}-(2n+1)i^n\} = (-1)^n \left(A_n e^{-i\eta_n}-(2n+1)i^n\right) \quad \left(e^{+\frac{in\pi}{2}}\right)$$

$$A_n e^{i\eta_n}-(2n+1)i^n = A_n e^{-i\eta_n}-(2n+1)i^n.$$

$f(\theta)$ $\quad e^{-\frac{n\pi i}{2}}$ $\quad \frac{\sin(kr-\frac{1}{2}n\pi+\eta_n)}{kr} = \frac{e^{ikr-\frac{n\pi i}{2}+i\eta_n}}{2ikr}$

plane + out going wave (wave)

$$\psi_0 = \sum_{n=0}^{\infty}(2n+1)i^n e^{i\eta_n} L_n(r) P_n(\cos\theta)$$

$$f_1(\theta) = \frac{1}{2ik}\sum_{n=0}^{\infty}(2n+1)(e^{2i\eta_n}-1)P_n(\cos\theta)$$

plane + ingoing wave

$$\psi_1 = \sum_{n=0}^{\infty}(2n+1)i^n e^{-i\eta_n} L_n(r) P_n(\cos\theta),$$

$$f_1(\theta) = \frac{1}{2ik}\sum_{n=0}^{\infty}(2n+1)(e^{-2i\eta_n}-1)P_n(\cos\theta)$$

asym. $\psi_0 = \sum_{n=0}^{\infty}(2n+1)\frac{e^{ikr}}{2ikr}\cdot e^{2i\eta_n}P_n(\cos\theta).$

~~$\psi_1 = \sum_{n=0}^{\infty}(2n+1)\frac{e^{ikr}}{2ikr}$~~ $+\sum_{n=0}^{\infty}(-1)^{(n+1)}(2n+1)\frac{e^{-ikr}}{2ikr}P_n(\cos\theta)$

$$e^{ikz} = \sum_{n=0}^{\infty}(2n+1)i^n P_n(\cos\theta)\left(\frac{e^{ikr}}{2ikr}(i)^{n+1}+\frac{e^{-ikr}}{2ikr}(i)^{n+2}\right)$$

$$= \sum_{n=0}^{\infty}(2n+1)\frac{e^{ikr}}{2ikr}P_n(\cos\theta)$$

$$+\sum_{n=0}^{\infty}(2n+1)\frac{e^{-ikr}}{2ikr}(-1)^{n+1}P_n(\cos\theta).$$