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On the Theory of Collision of Neutrons with Deuterons.

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(Read March 13, 1937)

Abstract

The collision of neutrons with deuterons

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§ 1. Reduction of the Collision Problem ~~Introduction~~

~~Whereas~~ The theory of collision of neutrons with ~~the~~ protons ~~has~~ was been developed by many authors and was able to account for ~~sufficient agreement with the theory experiment~~ the experimental results sufficiently well.[1] ~~that the problem~~ The ~~next~~ problem ~~will be that~~ of the collision of neutrons with deuterons which ~~may~~ ~~will be next~~ is the next ~~to it both in~~ simplicity and in ~~importance~~, ~~to be attacked,~~ is ~~already~~ too complicated to be solved rigorously. ~~But~~ ~~a three body problem and is already~~ ~~it will be po~~ It is not impossible, however, to find an approximate solution owing to the simplicity of the structure of the deuteron. ~~But we can find an approximate method solutions in certain cases owing a method reduces~~ ~~It can be reduced, however, to a simpler problem, if the energy of the incident neutron is assumed to be so small~~ We want, however, to reduce the problem to the form as simple as possible and find an approximate solution at least valid for small energies of the incident neutrons.

The wave equation of the ~~three parti~~ system containing a proton and two neutrons has the general form

$$\left\{\Delta_1+\Delta_2+\Delta_3+\frac{2M}{\hbar^2}\left(E-V_{12}+V_{13}+V_{23}\right)\right\}\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\sigma_1,\sigma_2,\sigma_3)=0 \qquad (1)$$

$$\cancel{-V_{12}(\vec{r}_1-\vec{r}_2,\sigma_1,\sigma_2)-V_{13}(\vec{r}_1-\vec{r}_3,\sigma_1,\sigma_3)-V'_{23}(\vec{r}_3-\vec{r}_2,\sigma_2,\sigma_3)}$$

1) See, for example, the summary report of Bethe and Bacher, Rev. Mod. Phys. 8, 82, 1936 and further, Fermi, Ric. Scient. VII, 2, 1, 1936.

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M is the mass of the ~~heavy part~~ proton of ? and M′ ~~the mass of them~~ their mass,

where $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ are the ~~coo~~ position vectors of the proton ~~and~~, ~~the~~ first and the second neutrons respectively, ~~and~~ $\sigma_1, \sigma_2, \sigma_3$ ~~are~~ ~~the~~ and z-components of their spins, ~~respe~~ctively. The potentials V_{12}, V_{13}, and V_{23}' between ~~them~~ any two of them will ~~depend on~~ the coordinates, and the spins and the exchange operators. involve, in general, ~~the exchange~~ of coordinates

~~If we~~ / ~~We assume the~~ consider the operator first terms of the Majorana type alone, they becomes

$$V_{12} = J(r)\, P_{12}^M, \quad V_{13} = J(r')\, P_{13}^M$$
$$V_{23}' = K(r'')\, P_{23}^M$$

respectively, where

$$\vec{r} = \vec{r}_1 - \vec{r}_2, \quad \vec{r}' = \vec{r}_1 - \vec{r}_3 \quad \vec{r}'' = \vec{r}_3 - \vec{r}_2$$

and P^M's are the operators of the ~~coordinates~~ exchange of coordinates.

Further, ~~they~~ if we use the coordinates of the centre of mass

$$\vec{R} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{2},$$

~~the relative coordinates of the proton and the first neutron~~ $\vec{r}$ ~~and~~ the relative coordinates of the second ~~pro~~ neutron with respect to the centre of mass of the ~~proton and~~ the first ~~neutron~~ formed by the

$$\vec{S} = \vec{r}_3 - \frac{\vec{r}_1 + \vec{r}_2}{2}$$

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as independent variables, ~~the wave equation~~ (1) reduces to the ~~form~~ equation

$$\left\{\Delta_r+\Delta_s + \frac{2\hbar^2}{M}\left(E' - J(r)P^M_{12} - J(r')P^M_{13} + K(r'')P^M_{23}\right)\right\} \times \psi(\vec{r},\vec{s}) = 0 \qquad (2)$$

if we separate ~~the spins~~ relative coordinates from the variables $\sigma_1, \sigma_2, \sigma_3$ a.s., the coordinates ~~of the centre~~ of ~~the~~ mass $\vec{R}$, (2) can be written alternatively in the form, where Δ_r, Δ_s are the Laplacians ~~for~~ with respect to $\vec{r}$ and $\vec{s}$ respectively.

$$\left\{\Delta_r + \frac{3}{4}\Delta_s + \frac{2M}{\hbar^2}\left(E' - J(r)\right)\right\}\psi(\vec{r},\vec{s}) - \frac{2M}{\hbar^2}J(r')\psi(\vec{r}',\vec{s}') - \frac{2M}{\hbar}K(r'')\psi(\vec{r}'',\vec{s}'') = 0,$$

where

$$\vec{s}' = \vec{r}_2 - \frac{\vec{r}_1+\vec{r}_3}{2}, \qquad \vec{s}'' = \vec{r}_1 - \frac{\vec{r}_3+\vec{r}_2}{2}.$$

Now, ~~when the incident neutron~~ when the energy E of the incident neutron is ~~too small~~ not large enough to disintegrate ~~(or excite)~~ the deuteron, the wave function can be assumed to have the form

$$\psi(\vec{r},\vec{s}) = \varphi(\vec{s})\chi(r) \pm \varphi(\vec{s}')\chi(r'), \qquad (\quad)$$

i.e. ~~$E' < 0$~~ smaller than the binding energy of the deuteron $E_D = 2.2 \times 10^6$ eV provided that it ~~the latter~~ has no true excited levels,

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where $\chi(r)$ is the normalized wave function of the deuteron, satisfying

$$\left\{\Delta_r + \frac{M}{\hbar^2}\left(E_D - J(r)\right)\right\}\chi(r) = 0. \qquad (\ )$$

The above function ( ) with the sign + or − should be multiplied by ~~sign + or − sign~~ ~~corresponds to~~ the antisymmetric or symmetric state ~~of H^3~~ with respect to the exchange of spins of two neutrons, i.e. either one of two functions

~~$\alpha(1)\alpha(2)\beta(3)$~~

$$\left.\begin{array}{l} \alpha(\sigma_1)\{\alpha(\sigma_2)\beta(\sigma_3) - \beta(\sigma_2)\alpha(\sigma_3)\} \\ \beta(\sigma_1)\{\alpha(\sigma_2)\beta(\sigma_3) - \beta(\sigma_2)\alpha(\sigma_3)\} \end{array}\right\}$$

or ~~either~~ $\alpha(\sigma_1)\{\alpha(\sigma_2)\beta(\sigma_3) -$ of six functions

$$\begin{array}{lll} \alpha(1)\alpha(2)\alpha(3) & \alpha(\sigma_1)\{\alpha(\sigma_2)\beta(\sigma_3) + \beta(\sigma_2)\alpha(\sigma_3)\}, & \alpha(\sigma_1)\beta(\sigma_2)\beta(\sigma_3) \\ \beta(\sigma_1)\alpha(\sigma_2)\alpha(\sigma_3) & \beta(\sigma_1)\{\quad " \quad\} & \beta(\sigma_1)\beta(\sigma_2)\beta(\sigma_3), \end{array}$$

where $\left.\begin{array}{ll} \alpha(\sigma) = 1 & \text{or } 0 \\ \beta(\sigma) = 0 & \text{or } 1 \end{array}\right\}$ for $\sigma = 1$ or -1.

The ~~former states~~ former correspond to ~~the~~ ~~doublet~~ ^2S states of H^3 and the latter to the mixture of ^2S and ^4S states.

If we multiply both sides of ( ) by $\chi^*(r)$ and integrate with respect to $\vec{r}$, we obtain ~~the relation~~ an integrodifferential equation

$$\frac{3}{4}\Delta_\rho\varphi + \frac{4M}{3\hbar^2}\left(E' + E_D\right)\varphi$$
$$= \frac{4M}{3\hbar^2}\iiint\left\{\chi^*(r)\{J(r') \pm J(r)\}\varphi(\vec{\rho}')\chi(r'') + K(r'')\left(\varphi(\vec{\rho}')\chi(r') \pm \varphi(\vec{\rho})\chi(r)\right)\right\}d\vec{r}$$

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on account of the equation ( ) and the assumption ~~of~~,
~~approximate orthogonality of~~

$$\iiint \chi^*(r)\,\chi(r)\,d\vec{r} \simeq 0.$$

This can be transformed into an integral equation with the asymptotic form ~~for large~~

$$\cancel{\varphi(\vec{s}) = F(\vec{s}) + \frac{e^{iks}}{4\pi s}\int G}$$

$$\varphi(\vec{s}) \simeq F(\vec{s}) + \frac{e^{iks}}{4\pi s}\iiint G(\vec{t}')\,F(\vec{t}')\,d\vec{t}$$

~~for large~~ s, ~~where~~ t'

$$\varphi(\vec{s},\theta) \simeq F(\vec{s},\theta) \pm \frac{e^{iks}}{4\pi s}\iiint G(\vec{t})\,F(t,\pi-\varphi)\,d\vec{t}$$

for large s, ( ) where, (s,θ,φ) are the polar coordinates of the vector $\vec{s}$, and

$$G(\vec{s}) = \frac{4M}{3\hbar^2}\iiint \chi^*(r)\{J(r)\pm J(r)\}\varphi(\vec{s}'')\chi(r'') + K(r'')\{\varphi(\vec{s}')\chi(r')\pm\varphi(\vec{s})\chi(r)\}\,d\vec{r} \quad (\ )$$

$\frac{4M}{3\hbar^2}U_\pm(s)$

$F(s,\theta)$ is an ~~solution~~ axially symmetric solution of ~~function~~ independent of φ sati the homogeneous equation

$$\Delta_s F(s,\theta) + \frac{4M}{3\hbar^2}(E' + E_D - U_\pm(s))\,F(s,\theta) = 0 \quad (\ )$$

with the asymptotic form

$$F(s,\theta) \sim e^{iks_z} + \frac{e^{iks}}{s}f(\theta), \quad (\ )$$

( ) Mott, Theory of Atomic Collisions, Oxford, 1933, chapter VI.

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In order to ~~avoid the explicit~~ estimate the ~~theoretical~~ scattering cross section without the use of the complicated solution of the integral equation ( ),

§2. Estimation of the Scattering Cross Section

where

$$k = \frac{2}{\hbar}\sqrt{\frac{M(E'+E_D)}{3}}$$

the double sign ~~the ± signs~~ corresponds to that in ( ).

and $U_\pm(s)$ is ~~a suitably~~ ~~an~~ most convenient ~~auxiliary~~ ~~suitably chosen~~ auxiliary potential, ~~so~~ It is ~~which~~ desirable to choose $U(s)$ ~~can be chosen~~ such that $F(s,\theta)$ of the above type ~~the above~~ is already an approximate solution of the integral equation ( ), i.e.

$$\varphi(s,\theta) \cong F(s,\theta).$$

~~In the case of ¹S(+) state, it will be legitimate~~ ~~Since the ne~~ If we consider the deuteron ~~to be~~ as a sphere of diameter ~~radius~~ $\frac{1}{\alpha} = 4.36\times10^{-13}$ cm and the range of forces $a = 3\times10^{-13}$ cm (2.33), ~~the maximum and~~ ~~take~~ equal to ~~the distance~~ interaction between ~~the deuteron it~~ ~~and~~ the neutron and the deuteron ~~becomes~~ reduces to zero for $s > \frac{1}{2\alpha} + a =$ ~~7.36~~ ~~6.78×10⁻¹³~~ cm.

$b = 4.5\times10^{-13}$ cm,

2.18×10^{-13} cm
3.8×10^{-13} cm
16.4×10^{6} eV

~~and the least~~ so that we can ~~take~~ ~~replace~~ put ~~assume~~

$$U(s) = -U_\pm = \text{const.} \quad \text{for } s < b$$
$$= 0 \quad \text{for } s > b$$

according as ~~the~~ + or − sign is ~~was~~ taken in ( ).

~~The depth U_+ can be~~ A legitimate value for U_+ can be obtained by assuming that $(\ )_+$ has a solution with the

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$2.6 \times \frac{3}{4} = 2.?$

$3.1 \times \frac{3}{4} = 2.3$

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8.3 MEV

energy value

$$E' = \cancel{E_D} - E_T$$

where $E_T = 8.3 \times 10^6$ eV ~~are~~ the binding energy of ~~the~~ H^3. Thus for $b = 4.5 \times 10^{-13}$ cm, we obtain

$$U_+ = 13.8 \times 10^6 \text{ eV},$$

and the cross section

$$\sigma_+ = \overset{2.7 \times 10^{-24}}{3.6 \times 10^{-24}} \text{ cm}^2$$

$E_0 \ll \frac{3}{2}(E' - E_D)$

for ~~the~~ energies of the incident neutrons small compared with E_D ~~and i.e.~~ and

$$\sigma_+ = \overset{2.3 \times 10^{-24}}{\cancel{3.1 \times 10^{-24}}} \text{ cm}^2$$

for $E_0 = 0.75 \times 10^6$ eV.

~~Although we U_- can not be determined in a similar manner, it will be much smaller than~~ In order to decide ~~the whether such a procedure is correct or not~~ accuracy of ~~such a proce~~ this result, one has to insert the function $F(\vec{r})$ thus obtained in place of $\varphi(\vec{r})$ in the expression for $G(\vec{r})$ and ~~calculate~~ perform the integration of the second term of the right hand side of ( ).

* into account as will be considered in §.

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1.4 3 4.2 1.4   318   10.6 3

$\frac{33}{4} = 8.2$

8.

On the other hand, although U_- can not be determined in a ~~similar~~ manner as U_+, it will be much smaller than U_+ and ~~even negative~~ may be even negative. Thus, if we take $U_- = \frac{U_+}{2}$ with (or $U_- = -U_+$)
$b = 4.5 \times 10^{-13}$ cm, we obtain

$$\sigma_- = 8.1 \times 10^{-24}\ \mathrm{cm}^2 \qquad (\ )$$

or

$$\sigma_- = 1.1 \times 10^{-24}\ \mathrm{cm}^2 \qquad (\ )$$

for ~~small~~ small energies of the neutrons.

The ~~average value~~ observed cross section ~~will be the~~ is to corresponds to the average value

$$\sigma = \frac{1}{4}\sigma_+ + \frac{3}{4}\sigma_-,$$

which becomes ~~which is~~

$$\doteq 6.8 \times 10^{-24} \text{ or } 1.5 \times 10^{-24}\ \mathrm{cm}^2$$

for ~~small energy,~~ ~~according~~ as $U_- = \frac{U_+}{2}$ or $U_- = -U_+$. ~~The experiment~~

2.7 24.3 4)27. 6.8   2.7 3.3 2.6 1.5

If we compare this with the experimental values of Dunning and others() $1.71 \times 10^{-24}\ \mathrm{cm}^2$ and $4.0 \times 10^{-24}\ \mathrm{cm}^2$ for fast ~~and~~ slow neutrons, we ~~reach~~ ~~the~~ ~~arrive at~~ reasonable conclusion of the correct cross section for σ_- lies ~~between~~ we can ~~guess~~ conclude that (\ ) and (\ ), so that U_- ~~is~~ has ~~a~~ some value somewhere between $\frac{U_+}{2}$ and $-U_+$ ().
Thus, ~~there should be no resonance~~ we can not expect except any appreciable resonance for ^2S ~~no~~ ~~for the scattering~~ for the scattering of slow neutrons due to the ~~presence~~ virtual $^2S(-)$ or $^4S(-)$ level, in contradistinction to the case of neutron-proton scattering, ~~by~~ in which the large cross section for slow

() Dunning, Pegram, Fink and Mitchell, Phys. Rev. 48, 265, 1935.

() We ~~should~~ ~~can neglect~~ For ~~very~~ case of ~~very~~ slow neutrons we have to take the effect of chemical binding of deuterons*

*, in the ordinary coordinate system, the ~~neutrons are scattering~~ ~~scattered to the forward direction~~ distribution function becomes ~~inclines~~

, in which the deuteron is initially at rest

probability ~~fraction~~ of neutrons in the being scattered ~~scattered~~ by an angle between θ and $\theta+d\theta$

~~$2\cos\theta$~~

$$\frac{1}{2}\left\{\frac{3+2\cos^2\theta}{2\sqrt{3+\cos^2\theta}}+\cos\theta\right\}\sin\theta\,d\theta .$$

~~where θ is the angle between~~

Thus, ~~most of~~ the ~~neu~~ ~~scattering~~ ~~neutrons~~ ~~angle is smaller~~ distribution function inclines ~~markedly~~ conspicuously to the forward direction as shown in Fig. 1.

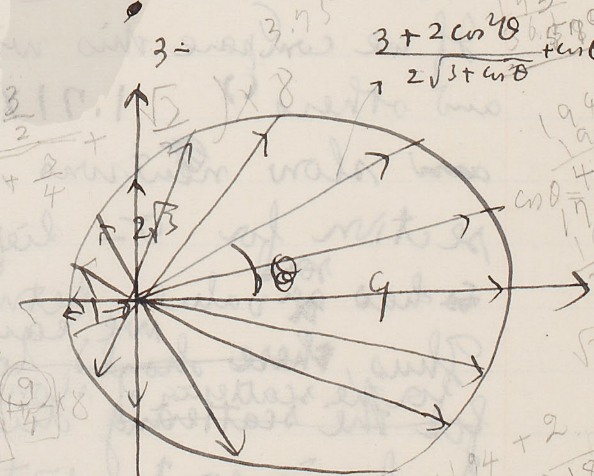

Fig. 1

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neutrons was considered to be due to the presence of the virtual 'S level'.[ ]
~~At the same time~~, This ~~conclusion~~ in turn ~~draw~~ leads us to ~~another that~~ further conclusion that the ~~capture~~ cross section ~~of~~ of the capture of slow neutrons by deutrons ~~will~~ should be small compared with that by protons in accordance with the ~~result~~ experimental ~~experimental~~ result of Kikuchi, Aoki and Takeda,[ ] which ~~shows that~~ the ~~that~~ former ~~cross section~~ upper ~~limit of~~ the former cross section gives ~~the upper limit~~ 0.3 x 10^-25 cm^2 ~~for~~ the upper...
The angular distribution of ~~scattering~~ neutrons is, of course, a value ~~approximately~~ spherically symmetric ~~with respect to the relative~~ in the coordinate system, in which the centre of mass is at rest, so that *

§ 3. Effect of Chemical Binding of Deuterons on their ~~Scattering of Neutrons~~ Collision with Slow Neutrons.

~~We can expect~~ In the above calculations, we assumed the deuterons ~~always~~ alwa to be free and at rest initially, but we ~~have to can expect an appreciable~~ are not sure whether the effect of chemical binding of deuterons on their collision with ~~slo~~ neutrons of energy below few volts is small or not. ~~The~~ Such an effect for neutron-proton ~~collision~~ scattering was dealed with by Fermi[ ] in detail and the calculation can be easily extended to a more general case, in which the scattering nucleus have a mass M' not equal to ~~the~~ the mass M of the neutron. ~~In the limit of strongly bound~~ ~~The ratio~~ In the

( ) Compare Bethe and Bacher, loc. cit. and Fermi, loc. cit.
( ) Sci. Pap. Inst. Phys. Chem. Res. 31, 195, 1937
( ) loc. cit.

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limit of strong binding, the scattering cross section σ_b becomes

$$\sigma_b = (1+\frac{M}{M'})^2 \sigma_f ,$$

where σ_f is the scattering cross section ~~for free deu~~ of scattering ~~by free deutrons~~ calculated in the preceding section for free deuterons.

~~for d~~ Thus ~~for deuteron~~ $M' = 2M$

$$\sigma_b = \cancel{\tfrac{9}{4}} \sigma_f \; 2.25\, \sigma_f$$

~~Thus~~ Hence, the ratio becomes

$$\frac{\sigma_b}{\sigma_f} = 2.25$$

for the deuteron, i.e. $M' = 2M$, which is midway between 4 for the proton, i.e. $M' = M$ in contrast to and 1 for ~~the~~ heavy nuclei, i.e. $M' \gg M$. Thus, ~~we~~

~~The angle distribution of the scattered neutrons in the limit of~~ ~~Thus~~ ~~can expect~~ ~~the~~ the scattering cross section of deutrons for thermal neutrons by deuterons in the heavy water will ~~about~~ be ~~twice~~ ~~about~~ nearly twice as large as that for ~~neutrons~~ those with the energy of several volts.

~~The difference between the cross~~ If ~~we the cross section~~ observe we assume the cross section $4.0 \times 10^{-24}\,\mathrm{cm}^2$ ~~observed by~~ measured by Dunning and others to be attributed to ~~due~~ the thermal neutrons, the cross section for neutrons ~~wit~~ of several volts

E24 080 P10

$$1.4431 \times 8$$

$$9\overline{)11.7448}$$

$$1.305$$

$$1+\frac{}{\left(\frac{4}{3}\right)^2\frac{1}{2}\left\{\frac{3}{4}+\cancel{\frac{2}{2}}\log\frac{4}{2}\right\}} \times \frac{8}{9}$$

$$\begin{array}{r} 0.6931 \\ 0.75 \\ \hline 1.4431 \end{array}$$

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will be about $2\times10^{-24}\,cm^2$, which ~~does not~~ differs little from the value $9.71\times10^{-24}\,cm^2$ for fast neutrons. This result is in sufficient agreement with the expectation of the theory that the cross section will be approximately constant throughout the wide energy range ~~unless~~ in the ~~intersection of~~ absence of the resonance. In the limit of the strong binding ~~It will be~~ the angular distribution of the neutron is of course spherically symmetric in the ordinary coordinate system.

It should be noticed, that ~~the method of Fermi~~ the ~~result~~ cross section (~~for in the limit~~ calculated in the limit of the free deutron according to ~~by the method of Fermi~~. ~~become~~ is

in the limit of the free deuteron

(in the limiting case of ~~weak~~ the weak binding ~~the free deuteron~~ can also be calculated by the method of Fermi ~~with that the result~~ but the value ~~the result~~ is not ~~...~~ in agreement with that of the ~~beat~~ preceding section, the former being ~~differing~~ differs from the latter by a factor ~~$\left(\frac{1+n}{n}\right)^2$~~ $\left(1+\frac{M}{M'}\right)^2 \frac{1}{2}\left\{\frac{1}{1+\frac{M}{M'}} + \frac{\frac{M'}{M}-1}{2}\log\right.$

$$\left(1+\frac{M}{M'}\right)^2 \frac{1}{2}\left\{\frac{M'}{M+M'} + \frac{M'-M}{2M}\log\frac{M'+M}{M'-M}\right\} \quad (\ )$$

~~result~~ of the preceding section

~~This factor~~ ~~$2.25\times0.608 = 1.368$~~ ~~is~~ reduces to 1 for the cases $M'=M$ and $M'\gg M$, whereas it is 1.368 for

2.25
$\times\ 608$
1800
1350
1.368

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This factor is not equal to 1 in general ~~in general~~, except for the cases $M' = M^2$ and $M' \gg M$, ~~being equa~~ and is 1.368 for case of the deuteron, i.e. $M' = 2M$. The origin of this discrepancy seems to be due ~~the use of~~ illegitimate Born's approximation in the part method of Fermi's method

Closer examinations show that the origin of this discrepancy is due to the inlegitimate use of Born's approximation ~~in~~ on the part of Fermi. The angular distribution of the scattered neutrons, which ~~being given~~ by besides, becomes in the similar manner ~~can be calculated~~ and ~~is proportional~~

becomes ~~is proportional to~~

$$\left(\frac{M'}{M+M'} + \frac{M^{*}-1}{2}\log\frac{M+M'}{M'-M}\right)^{-1}\left(\frac{M\cos\theta + \sqrt{M'^2 - \cancel{M^2+M^2\cos} - M^2\sin^2\theta}}{M+M'}\right)\sin\theta\, d\theta, \quad (\ )$$

~~which~~ and differs also from the corresponding expression (3) ~~in the preceding section~~, except for the cases $M' = M$ and $M' \gg M$. , the

$$\cancel{\frac{\cos\theta + \sqrt{\left(\frac{M'}{M}\right)^2 - 1} - }{1 + \frac{M'}{M^2}}}$$

$$\cancel{\frac{2M\cos\theta + M'^2 - M^2\sin^2\theta - M}{1}}$$

, which can be obtained by the ~~usual~~ ordinary method used in the preceding section,

$$\cancel{M}\ \frac{1}{2}\left(\frac{M'^2 - M^2 + 2M^2\cos^2\theta}{M'\sqrt{M'^2 - M^2\sin^2\theta}} + \frac{2M}{M'}\cos\theta\right)\sin\theta\, d\theta$$

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for the case of scattering of ~~cross sections~~ neutrons by a free protons, the agreement of ~~the~~ ~~both~~ the results of two methods for free nuclei being only accidental ~~ly~~.

§4. Slowing Down of Neutrons ~~in Heavy H₂O Water~~ by ~~Multiple~~ Collision with Deuterons

On account of the comparatively small mass ~~of~~ the deuteron ~~We may~~ At first sight, the deuterons seems to ~~be effective~~ have a considerable effect on the slowing down of neutrons on account of their comparative masses. ~~Namely,~~ The meaning energy of the latter, for instance, ~~after a single collision with the former~~ is ~~being~~ ~~$\frac{4}{9}$~~ reduced to $\frac{5}{9}$ of the initial energy E_0 by a single collision with the deuteron ~~at rest~~ initially at rest, if we assume the angular distribution ~~( )~~ of scattered neutrons to be given by ( ), ~~But~~ so that, after n collisions, the meaning energy becomes $\left(\frac{5}{9}\right)^n E_0$, which does not differ appreciably from the corresponding value $\left(\frac{1}{2}\right)^n E_0$ in the case of collisions with protons. There are, however, many reasons, which ~~in~~ leads to the opposite conclusion. Firstly, the neutron energy ~~after~~ can not ~~become~~ smaller than $\frac{E_0}{9}$ by a single collision, so that a neutron with initial energy of the order of 10^6 eV

$$g^6 = \frac{0.95424^6}{5.72544}$$

0.53×10^6

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should undergo at least ~~6 or 7~~ collisions with deuterons before ~~the energy~~ it ~~slowed~~ becomes a slow neutron of several volts, ~~whereas the it in contrast to~~ whereas ~~it can be reduced to~~ any this can be effected by a single collision with the proton. Secondly, ~~the scattering cross section of the slow neutrons for is small for the deuteron~~ ~~the small cross section of for the scattering of slows neutrons by by deuterons compared with that by protons~~ ~~the probability of~~ The ~~multip~~ occurrence of the multiple collision ~~in this case~~ with deuterons will be much ~~smaller than~~ rarer than ~~it~~ that ~~in the case of~~ with protons ~~proton~~ on account of ~~the relatively small~~ the fact that the scattering ~~cross~~ section ~~of scattering of slow~~ in the former case becomes smaller ~~a~~ compared with that in the latter case as the neutron energy decreases. Thirdly, the ~~shape of~~ energy distribution functions, after n ~~several a fix equal number of collisions with~~ ~~corresponding to the same number of collisions are very much different from each other~~ ~~for a fixed number of times~~ after ~~several~~ n collisions with the deuterons, which ~~cannot~~ ~~can be calculated computed easily~~ ~~was~~ can be evaluated by using the general formula of Condon and Breit,[1] ~~have very much different shapes in two case similar,~~ ~~are~~ has very small value in the neighborhood of the minimum ~~small~~ energy ~~in the case of the deuterons compared with~~ ~~in comparison with the corresponding~~ value as ~~illustrated in~~ Fig. . For example, ~~In the case of n = 2~~, For example, for n = 2 and 5, the probability that

[1] Condon and Breit, Phys. Rev. 49, 229, 1936

$\frac{1}{x_a} = 9^4$　　$x = \frac{E}{E_0}$

$\boxed{u = 4\log 9}$

$$\int_0^{x} \left(\log \frac{1}{x}\right)^{n-1} \frac{dx}{n!}$$

$\log \frac{1}{x} = u.$

$$= \int_{u_0}^{\infty} \frac{e^{-u} u^{n-1} du}{(n-1)!}$$

$$\int_{4\log 9}^{\infty} \frac{e^{-u} u^4}{4!} du = 1 - I\left(\frac{4\log 9}{\sqrt{5}}, 4\right)$$

$= I$

$= 0.0652$

$\frac{0.065}{1.15 \times 10^{-4}} = 0.05 \times 10^4 = 500$

$$\begin{array}{r} 2.1972 \\ \times\ 4 \\ \hline 8.7888 \end{array}$$

$$\begin{array}{r} 3.9 \\ 2.236 \overline{)\,8.7888} \\ 6\,708 \\ \hline 20808 \\ .9\,348487 \end{array}$$

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the neutron ~~of energy E_0~~ has an energy smaller than $\frac{E_0}{94}$ after 5 collisions with deuterons is only 1.15×10^{-4}, whereas it is about 0.065 in the case of collisions with protons.

~~Thus, the efficiency of heavy water, for example,~~ for slowing down ~~Considering~~ these circumstances, then we can not expect theoretically any ~~under~~ appreciable contribution of the deuteron compound such as heavy water to the slowing down of neutrons, in good agreement ~~accordance~~ with the recent experimental results[1] of Kikuchi, Aoki and Takeda, which shows ~~that~~ that ~~a~~ the number ~~amount~~ of ~~neutrons~~ neutrons of C-group produced ~~although we should quantitative~~ in ~~heavy~~ D_2O by using D-D source is at most 1/27 of that produced in equal amount of H_2O, although quantitative comparison of the theory and the experiment is very difficult. ~~on account of the~~ According to previous results of Dunning and others[2], ~~which shows indicating~~ which indicate ~~rather~~ relatively ~~large~~ efficiency of ~~heavy about~~ about 1 : 5.5 of heavy waters and light for producing slow neutrons, ~~seems~~ to be due to / ~~will~~ can not be attributed the presence of B-group in Be-Rn source as pointed out by Kikuchi and others, ~~also~~ should be ~~attributed to the presence~~ be attributed to the ~~effect~~ contribution of B-group already present in Be-Rn source.

) loc. cit.

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