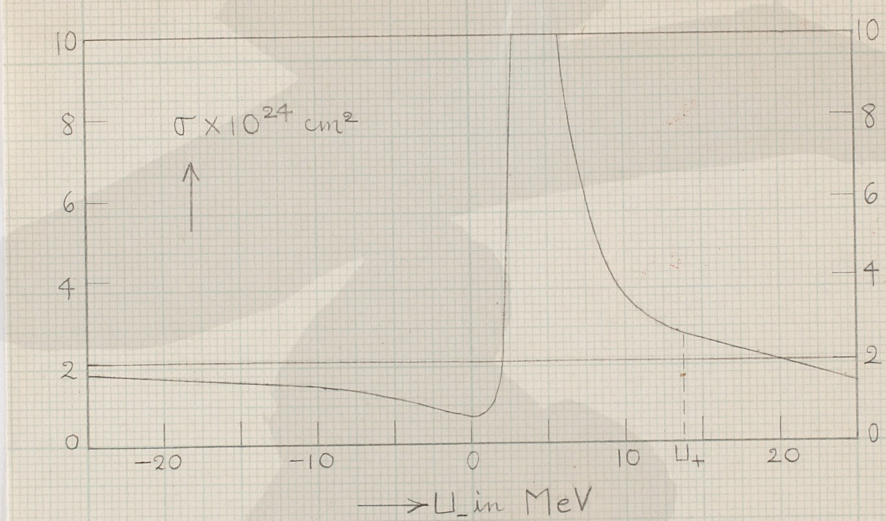


Fig.

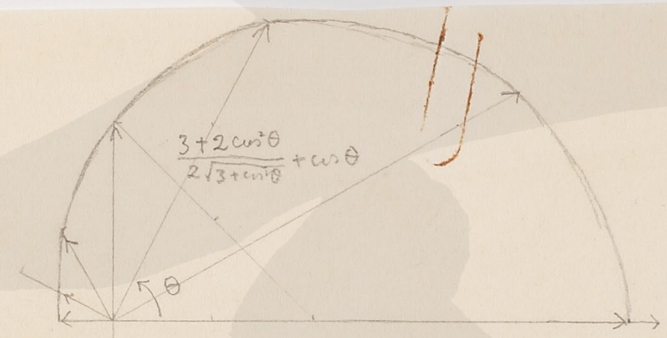
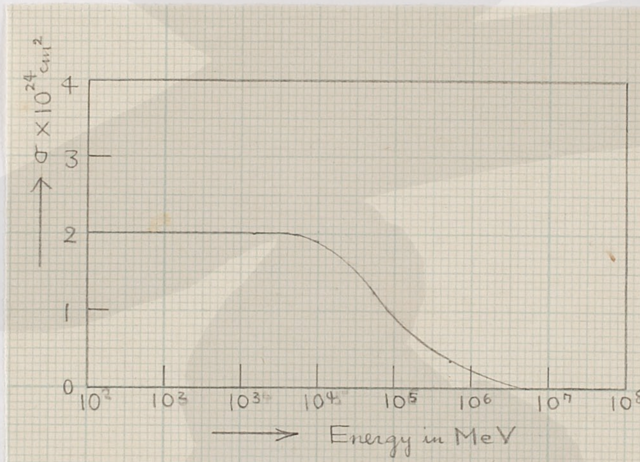
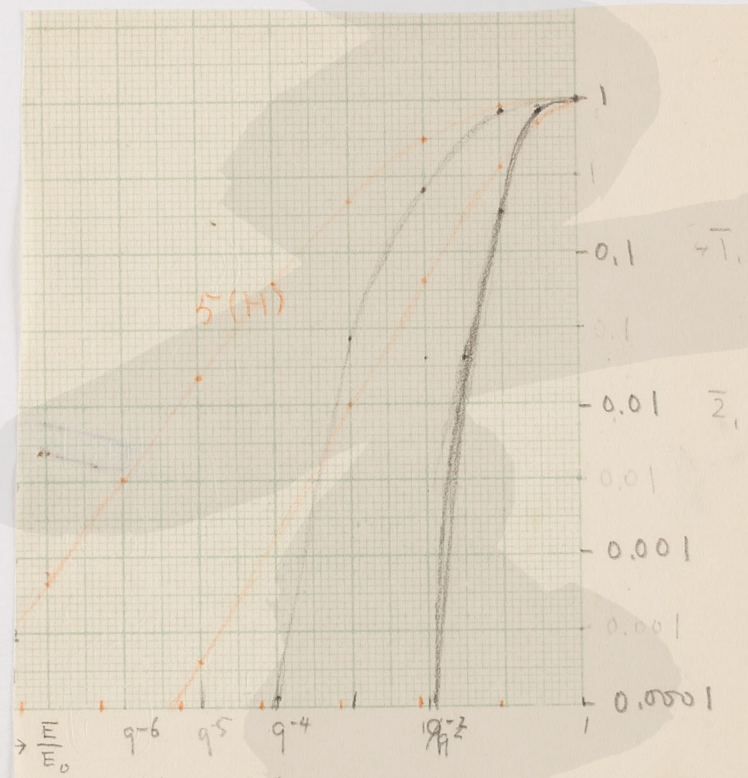


Fig 3. Angular Distribution of Scattered Neutrons
~~Scattered by Free Deuterons~~


$\sigma \times 10^{24}\ cm^2$
0
1
2
3
4
10^2
10^3
10^3
10^4
10^5
10^6
10^7
10^8
Energy in MeV



Probability that the neutron has an energy smaller than a certain fraction of the initial energy after multiple collision

E 24 090 P 10

* was found to be twice as large as that for neutrons of several volts.

~~Applicability of applicability~~ [about] of Fermi's method was discussed.

§4. ~~The reason~~ Theoretical ~~reasons~~ for ~~small contribution~~ [theory of the effect of chemical binding on the neutron scattering] of the deuteron to the slowing down were pointed out.

in the limit of the weak binding, the agreement of the results of both methods in the case of free protons being only accidental. Nevertheless, Fermi s results seems to be always true in the limit of the strong binding.

Fermi's theory of the chemical binding [concerning effect of] on the ~~neut~~ scattering of slow neutrons was extended to more general case and the limitation [a necessary] of ~~its applicability~~ this theory was discussed. The cross . . . . .

4. Slowing Down of Neutrons by Multiple Collision with Deuterons

If we ~~assume~~ consider the angular distribution of neutrons scattered to by deuterons initially at rest to be given by ( ), the ~~m~~ean energy after a single collision will be only

Since the distribution of ~~the~~ scattered ~~neutrons are~~ is spherically symmetric in the coordinate system, in which the centre of mass is at rest, their energy distribution after the collision with ~~the~~ deuterons ~~at~~ initially at rest ~~becomes~~ will be uniform between $\left(\frac{E_0}{9}\right.$ and $\left.E_0\right)$, ~~if~~ where E_0 is the initial energy of the neutron throughout the interval. Hence, ~~Thus~~ the mean energy after a single collision is $\frac{5}{9}E_0$ ~~and~~, so that it will be

E 24 090 P 10

On the Theory of Collision of Neutrons with Deuterons

By Hideki Yukawa and Shoichi Sakata

(Read March 13, 1937)

Abstract

The cross section of ~~the~~ deuterons for slow neutrons was ~~calculated~~ estimated by assuming ~~a~~ suitable ~~centre~~ potential holes in place of for the interaction complicate exchange force (simple forms for the wave functions and ~~considering~~ taking ~~the~~ the structures of ^2H and ^3H into account. The ~~and the observed~~ ~~dependence~~ dependence of the cross section ~~of~~ on the energy of the neutron was discussed

§1. The problem of the neutron-deuteron collision was ~~~~ reduced to a simple form by taking the structures of ^2H and ^3H and neglecting the forces ~~depending~~ on the spin. ~~The~~ cross section of scattering of ~~deuterons~~ ~~for~~ slow neutrons by deuterons ~~with the experimental value~~ ~~and the~~ was ~~discussed~~ determined. §2. By comparing the estimated potential holes with the experimental value, ~~^3H the~~ the excited levels of ^3H and the ~~forms~~ of the effective cross section of capture was found to be small in ~~agreed~~ agreement with the experiment. The energy dependence of the scattering cross section ~~seems to be~~ does not contradict with the existing data. §3. ~~Effect of the~~ The cross section for thermal neutrons *

E24 090

§1. Reduction of the Collision Problem

The theory of collision of neutrons with protons was developed by many authors and was able to account for the existing experimental results ~~of the experiment~~ sufficiently well.[1] The problem of the neuton-~~proton~~ deuteron collision, which will be the next to be attacked, ~~but~~ is a three body problem already too complicated to be solved rigorously. ~~It will not be impossible, however, to find an approximate solution owing to the simplicity of the structure of the deut/eron~~ But, It will however, be worth while to reduce the problem to the form as simple as possible by taking the ~~theoretical informations~~ interactions in the structures of ~~the deuteron and~~ the ^2H and ^3H into account ~~and~~ to find an approximate solution at least valid for neutrons of small energies.

The wave equation of the system containing a proton and two neutrons with the coordinates $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$ and the z components of the spins $\sigma_1, \sigma_2, \sigma_3$ respectively has the general form

$$\left\{\Delta_1+\Delta_2+\Delta_3+\frac{2M}{\hbar^2}(E-V_{12}-V_{13}-V'_{23})\right\}\psi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\sigma_1,\sigma_2,\sigma_3)=0, \quad (1)$$

where M is the common mass and V_{12}, V_{13}, V'_{23} are the potentials of forces, ~~between any two of them~~ each of two particles involving, in general, the coordinates, spins and exchange operators. (in general)

If we neglect the terms other than those of Majorana type, they become

$$V_{12}=J(\Delta)P^M_{12}, \quad V_{13}=J(\Delta')P^M_{13}, \quad V'_{23}=K(\Delta'')P^M_{23}$$

respectively, where

(1°) See, for example, the summary report of Bethe and Bacher, Rev. Mod. Phys. 8, 82, 1936 and further, Fermi, Ric. Scient. VII-2, 1, 1936.

~~r = r~~ $\vec{s} = \vec{r}_1 - \vec{r}_2$, $\vec{s}' = \vec{r}_1 - \vec{r}_3$, $\vec{s}'' = \vec{r}_3 - \vec{r}_2$

and P^{M}'s are the operators of the coordinates exchange.

Further, if we adopt the coordinates of the centre of mass

$$\vec{R} = \frac{\vec{r}_1+\vec{r}_2+\vec{r}_3}{3},$$

~~the~~ the ~~relative~~ coordinates of those ~~of the second neutrons relative to that~~ of the protn relative ~~with respect~~ to the first neutron $\vec{r}$ and those of the second neutron relative to the centre of mass of them

$$\vec{r} = \vec{r}_3 - \frac{\vec{r}_1+\vec{r}_2}{2}$$

as independent variables, (1) reduces to

$$\Big\{\Delta_s + \Delta_r + \frac{2M}{\hbar^2}(E' - J(s)P^M_{12} - J(s')P^M_{13} + K(s'')P^M_{23})\Big\}\psi(\vec{s},\vec{r}) = 0 \quad (2)$$

after separating $\vec{s}$ and $\vec{s}$ from $\vec{R}$ and $\sigma_1\sigma_2$'s , where

$$\vec{r}' = \vec{r}_2 - \frac{\vec{r}_1+\vec{r}_3}{2} \qquad \vec{r}'' = \vec{r}_1 - \frac{\vec{r}_3+\vec{r}_2}{2}$$

or

$$\Big\{2\Delta_s + \frac{3}{2}\Delta_r + \frac{2M}{\hbar^2}(E' - J(s))\Big\}\psi(\vec{s},\vec{r})$$
$$-\frac{2M}{\hbar^2}J(s')\psi(\vec{s}',\vec{r}') - \frac{2M}{\hbar^2}K(s'')\psi(\vec{s}'',\vec{r}'') = 0, \quad (3)$$

Now, when the energy E_0 of the incident neutron is not large enough to disintegrate the deuteron, i.e. E_0 is smaller than ~~the~~ its binding energy ~~of the deuteron~~ (latter) $E_D = 2.2\times10^6$ eV, provided that it has no true exited levels, the wave ~~fe~~ function can be assumed to have the approximate form[1]

$$\psi_\pm(\vec{s}\,\vec{r}) = \varphi(\vec{r})\chi(s) \pm \varphi(\vec{r}')\chi(s') \quad (4)$$

(1) ~~In~~ the following paragraphs, ~~the~~ each functions $\varphi(\vec{r})$, $F(\vec{r})$, $G(\vec{r})$, ~~in the following~~ we should add the double signs to each of which should be ~~added~~ suffixed ~~we omit~~ ~~we will omit~~ ~~we omit~~ want to omit hereafter In order to avoid the complication of the ~~expression~~ formula,

$\vec{r}'$ $E' = \frac{2}{3}E_0$ $3\times\left(\frac{1}{3}\right)^2 = \frac{1}{3}$

E' is the total energy other ~~than~~ minus the kinetic energy of the centre of mas and the approximate

where $\chi(s)$ is the normalized wave function of the deuteron, satisfying the equation

$$\left\{\Delta_s - \frac{M}{\hbar^2}(E_D + J(s))\right\}\chi(s) = 0, \qquad (6)$$

The ~~above~~ wave function Ψ_+ ~~with the sign = +~~ should be multiplied by ~~the~~ either of two anti-symmetric ~~wave~~ spin ~~wave~~ functions

$$\left.\begin{matrix}\alpha(\sigma_1)\\ \beta(\sigma_1)\end{matrix}\right\}\{\alpha(\sigma_2)\beta(\sigma_3) - \beta(\sigma_2)\alpha(\sigma_3)\}$$

corresponding to ^2S states of ^3H, ~~whereas~~ while ~~that with the sign~~ Ψ_- should be mutiplied by either of six wave functions

$$\left.\begin{matrix}\alpha(\sigma_1)\\ \beta(\sigma_1)\end{matrix}\right\}\alpha(\sigma_2)\alpha(\sigma_3) \quad \left.\begin{matrix}\alpha(\sigma_1)\\ \beta(\sigma_1)\end{matrix}\right\}\{\alpha(\sigma_2)\beta(\sigma_3)+\beta(\sigma_2)\alpha(\sigma_3)\} \quad \left.\begin{matrix}\alpha(\sigma_1)\\ \beta(\sigma_1)\end{matrix}\right\}\beta(\sigma_2)\beta(\sigma_3)$$

corresponding to the mixture of ^2S and ^4S states, where states ^2S, ^2P, ... and ^4S, ^4P, ...

$$\begin{matrix}\alpha(\sigma)=1 & \text{or } 0\\ \beta(\sigma)=0 & \text{or } 1\end{matrix}$$

according as $\sigma = 1$ or -1.

If we insert (4) in the wave equation (3), multiply both sides by $\chi^*(s)$ and integrate with respect to $\vec{s}$, we obtain an integro-differential equation

$$\Delta\varphi + \frac{4M}{3\hbar^2}(E' + E_D)\varphi = \frac{4M}{3\hbar^2}\iiint \chi^*(s)\{(J(s')\pm J(s))\,\varphi(\vec{r}'')\chi(s'') + K(s'')(\varphi(\vec{r}'')\chi(s')\pm\varphi(\vec{r})\chi(s))\}d\vec{s}, \qquad (7)$$

the approximate orthogonality

$$\iiint \chi^*(s)\chi(s')\,d\vec{s} \simeq 0 \qquad (5)$$

being assumed. ~~This~~ (7) can be transformed into an integral equation with the asymptotic form

$$\varphi(\vec{r}) \simeq F(r,\theta) + \frac{e^{ikr}}{4\pi r}\iiint G(\vec{q})F(q,\pi-\varphi)\,d\vec{q} \qquad (8)$$

for large R,[1] where

$$G(R) = \frac{4M}{3\hbar^2}\iiint \chi^*(s)\{(J(s')\pm J(s))\,\varphi(r')\chi(s') + K(s')(\varphi(r)\chi(s')\pm\varphi(r)\chi(s))\}\,ds, \qquad (9)$$
$$+\,U_\pm(r)$$

~~and~~ θ, denotes ~~the inclination~~ the angle betwwen the vector R and the z-axis, i.e. the direction ~~of incidence~~ of incidence of the neutron. and $F(R,\theta)$ is an axially symmetric solution of the homogeneous equation φ the angle between R and $\vec{q}$.

$$\Delta F + \frac{4M}{3\hbar^2}(E' + E_D \mp U_\pm(r))F = 0 \qquad (10)$$

with the asymptotic form, ~~where~~

$$F(r,\theta) \sim e^{ikz} + \frac{e^{ikr}}{r}f(\theta) \qquad (11)$$

where $k = \frac{2}{\hbar}\sqrt{\frac{M(E'+E_D)}{3}}$ and $U_\pm(r)$ are any auxiliary potentials, which will be discussed in the following section.

2. Estimation of the Scattering Cross Section

In order to estimate the scattering cross section without solving the complicated integral equation, it is needed to choose the auxiliary potential $U_\pm(s)$ in (9) and (10) such that $F(s,\theta)$ ~~is~~ becomes already an approximate ~~solution of the~~ expression for the required function $\varphi(\vec{s},\theta)$. If we consider the deuteron as a sphere of diameter $\frac{1}{\alpha} = 4.36 \times 10^{-13}$ cm, and the nuclear force with a definite range $a = 2.32 \times 10^{-13}$ cm, the interaction between the neutron and the deuteron reduces to zero for s larger than $b = \frac{1}{2\alpha} + a = 4.5 \times 10^{-13}$ cm, so that we can put

$$U_\pm(s) = U_\pm = \text{const.} \quad \text{or} \quad 0$$

according as $s > b$ or $s < b$,

(1) Mott, Theory of Atomic Collisions, Oxford, 1933, Chapt. VI.

$= \frac{\hbar}{\sqrt{ME_D}} =$ which $\alpha = \frac{\sqrt{ME_D}}{\hbar}$

2.8 MEV
resonance

10^6 eV	0.075	0.15	0.75	1.5	7.5	75
E_0	$0.15 \times mc^2$					
mc^2	0.1	0.2	1	2	10	100
σ	2	1.9	1.4	1.0	0.34	0.01

2.32
1.16
2.18
―――
3.34

~~The angular distribution of the scattering neutrons in this case are approximately spherically symmetric in the coordinate, in which the centre of mass is at rest, so that σ becomes~~

(6)

A legitimate value for U_+ can be obtained by assuming that $(10)_+$ has a solution with the energy

$$E' = -E_T = -8.3\times10^6\,eV$$

corresponding to normal state ~~where E = 8.3 10 eV is the~~ the binding energy of the binding energy of ^3H. Thus we ~~obtain~~ find

$$-U_+ = 13.8\times10^6\,eV \quad \text{for} \quad b = 4.5\times10^{-13}\,cm$$

and the cross section ~~becomes~~ of the deuteron initially at rest becomes

$$\sigma_+(0) = 2.7\times10^{-24}\,cm^2$$

for neutrons with the energy $E_0 = \frac{3}{2}(E' + E_D)$ small compared with E_D. The cross section decreases slowly with the energy, and ~~becomes~~, for instance,

$$\sigma_+(E) = 2.3\times10^{-24}\,cm^2$$

for $E_0 = 0.75\times10^6$ eV. ~~These results are affected little by~~ Further the above procedure is not sensitive to the simultaneous change of b and U_+ and ~~the value~~ the cross section for small energy, for instance, take the value ~~becomes~~

$$\sigma_+(0) = 2.3\times10^{-24}\,cm^2$$

, provided that ~~the~~ the binding energy E_T of ^3H is not changed,

if we take for instance, $b = \frac{1}{2\alpha} = 3.8\times10^{-13}$ cm and $U_+ = 16.4\times10^6$ eV. In order to find more ~~decide the~~ accurate values ~~accuracy of these results~~, one has to insert the function $F(r,0)$ thus obtained in place of $\varphi(\vec{r})$ in the expression for $G(\vec{r})$ and per-form the integration of the second term of the right hand side of (8).

On the other hand, U_- can not be determined in a similar manner as U_+ and we can say only that it ~~will be~~ is much smaller than U_+ and may be even negative, as ~~we know little~~ little is known of the excited states of ^3H. Thus, if we take $U_- = \frac{U_+}{2}$, 0 or ~~$U_- =$~~ $-U_+$ with $b = 4.5\times10^{-13}$ cm, we obtain for small energies

$$\sigma_-(0) = 8.1, \quad 0, \quad \text{or} \quad 1.1\times10^{-24}\,cm^2$$

respectively

~~Thus, if~~ ~~so that~~ so that ~~the determination of~~ the cross section σ_- in this case ~~can not be determined without further experimental~~ can not be determined without ~~large uncertainty~~ making use of further experimental informations ~~as follow~~.

† These conclusions in turn result in another that the ~~capss~~ section of capture of slow neutrons by deutrons will be much smaller than that by protons, ~~which~~ This seems to be in agreement with the ~~result~~ recent experimental ~~value~~ of Kikuchi, Aoki and Takeda, ~~which show that~~ 0.3×10^{-25} cm² ~~give a indicates a very much~~ gives the value 0.3×10^{-25} cm² for the upper limit of the cross section ~~of~~ of ~~neutron-deuteron~~ the γ-ray emission by ~~collision~~ (conti collision)

in ~~comparison~~ contradistinction to the virtual ¹S ~~state~~ level of ²H, which ~~he~~ was ~~estimate~~ inferred to have the ~~value~~ energy about 12×10^4 eV. from the theory of the neutron-proton collision. ~~the~~ existence of ... of ³H (²S, ⁴S) of ³H

In this case, the virtual ¹S level of energy about 12×10^4 eV ~~seems~~ ~~has been considered~~ inferred from ~~the~~ theoretical ... to be ...

* ~~expressed~~ represented by a small attractive force, ~~altho~~ when ~~they~~ two ~~The spin~~ the ~~total~~ wave function of the total system ~~has~~ is symmetric with respect to the spins of two neutrons, ~~provided that~~ as long as ~~the~~ the relative velocity is not large enough for disintegration the deuteron. ~~Although Further, the Thus, the~~ Further the ~~excited~~ Such a ~~shallow~~ potential hole has ~~no true~~ of course, excited energy level and ~~the~~ the first ~~virtual level will be high~~ and large can exist in the first energy ~~of~~

Now, the observed cross section of the deuteron, which ~~is~~ is to correspond to the average ~~value~~

$$\sigma = \frac{1}{4}\sigma_+ + \frac{3}{4}\sigma_-$$

of the above two cases, ~~and~~ is about ~~which becomes~~

~~$\sigma(0) = 6.8 \times 10^{-24}\ \mathrm{cm}^2$ or $1.5 \times 10^{-24}\ \mathrm{cm}^2$~~

~~for small energies, according as $U_- = \frac{U_+}{2}$ or $U_- = -U_+$.~~

$\sigma_c \doteqdot 4 \times 10^{-24}\ \mathrm{cm}^2$

according to the experiment of Dunning, Pegram, Fink and Mitchell(1) for slow neutrons of C-group. If we consider the effect of chemical binding of deuterons, the cross section σ_f of the ~~slow~~ free ~~neut~~ deuteron for slow neutron ~~should be of several volts~~ (of several volts) ~~not~~ should be about half of that for neutron of C-group,(2) so that we obtain the appropriate value

$$\sigma_f \approx 2 \times 10^{-24}\ \mathrm{cm}^2.$$

Hence, the ~~depth~~ potential U_- can be determined so as to give ~~the correct value~~ this value for the cross section ~~for observed cross section~~ σ_f for slow neutrons by using the formula

~~$\sigma_f = 2 \times 10$~~

$$\sigma_f = \frac{1}{4}\sigma_+(0) + \frac{3}{4}\sigma_-(0)$$

~~The ordinate~~ Fig. 1 shows ~~the values~~ σ_f ~~obtained by changing U_-~~ ~~the dependence of~~ as the function of U_- ~~for~~ with the fixed values ~~U_+~~ $\lambda = 4.5 \times 10^{-13}$ cm and $-U_+ = 13.8$. The curve ~~which~~ intersects with the ~~dotted~~ line $\sigma_f = 2 \times 10^{-24}\ \mathrm{cm}^2$ at ~~about~~ $-U_- = 1.9 \times 10^6$ eV. ~~with~~ This shows that the interaction ~~that which~~ between the neutron and the deuteron ~~we are~~ leads us to the conclusion ~~is small and can be~~ can be*

(1) Dunning, Pegram, Fink and Mitchell, Phys. Rev. 48, 265, 1935.

(2) Compare §3.

Further, we calculated the ~~average~~ cross ~~energy dependence~~ cross section $\frac{1}{4}\sigma_+ + \frac{3}{4}\sigma_-$ ~~as the function of the~~ for several values of the neutron energy E_0 by taking the above values for B.E., U_+ and U_-, the result being shown in Fig. 2. Thus, the cross section is nearly constant as long as E_0 is the order of 10^5 or smaller and ~~the~~ decreases steadily as ~~the energy~~ E_0 increases further. Although ~~these~~ ~~the above result can not be applicable immediately~~ to the various complications ~~are~~ will be superimposed on the simple S-scattering* (above considered) for E_0 larger than E_D, these results are ~~not at least~~ not contradictory to the experimental cross section ~~of the~~

1.71×10^{-24} cm^2

for fast neutrons, obtained by Dunning and others ~~by using the~~ from Ra-Be source.

Thus far, we ~~take~~ took only ~~the~~ Majorana forces into account. The ~~[illegible]~~ presence of ~~Wigner~~ small forces of Wigner ~~type does not causes any substantial~~ type ~~will~~ doesnot give rise to any substantial modifications, while ~~the superposition~~ Heisenberg or exchange forces ~~of Heisenberg type results~~* ~~will~~ ~~that with the~~ of ~~in~~ to the further splitting up of spin coupling of ^2S ~~[illegible]~~ $^2S_+^*$, $^4S_-$, $^2S_-$ states, but it seems improbable that the general feature of the above results ~~are~~ is essentially changed*.

(8)

§ 3. Effect of Chemical Binding of Deuterons

on their Collision With Slow Neutrons

In the above calculations, we assumed the ~~neutrons~~ deuterons to be free and initially at rest, but we have to consider the effect of chemical binding of heavy hydrogen atoms with other atoms, if the neutron energy becomes few volts or smaller. Such an effect in the case of ~~the~~ ordinary hydrogen atoms was dealed with in detail by Fermi[1], ~~and The~~ which calculation can be easily extended to a more general case, in which the scattering nuclei have the mass M' not equal to the mass M of the neutron. ~~although~~ the res limitation of his method namely, if the energy of the neutron is small compared with that of chemical binding of ~~heavy~~ hydrogen atoms in the scatterers.

In the limit of strong binding, the scattering cross section ~~tends~~ to the value ~~becomes~~ becomes

$$\sigma_b = \left(\frac{M+M'}{M'}\right)^2 \sigma_f$$

and the angular distribution of the scattered neutrons becomes is spherically symmetric in the ordinary coordinate system, as where σ_f is the cross section ~~for~~ of free deuterons~~, which~~ for slow neutrons was calculated in in the preceeding section. Hence, the ratio is

$$\frac{\sigma_b}{\sigma_f} = 2.25$$

for the deuteron~~, i.e.~~ $(M'=2M)$, which is midway between 4 for the proton $(M'=M)$ and 1 for heavy nuclei $(M' \gg M)$.

Thus, the ~~scattering~~ cross section for thermal of deuterons in the heavy water ~~of scattering by of thermal~~ neutrons ~~by deuterons in the heavy water~~ will be nearly twice as large as that ~~of~~ for neutrons with the energy of several volts.

(1) ~~Sci. Pap. Inst. Phys. Chem. Res. 31, 195~~

(1) l.c. loc. cit.

(9)

If we ~~consider~~ attribute (consider) the cross section ($4.0\times10^{-24}\,cm^2$) of ^{2}H ~~for slow neutrons~~ measured by Dunning and others[1] as ~~due to the~~ that for, to thermal neutrons, ~~the~~ the cross section that for neutrons of several volts (~~will be~~ becomes that for) about 2.0×10^{-24} cm^2, as already used in the preceding section, ~~which differs little from the measured value 1.71×10^{-24} cm^2 for fast neutrons. This result is good accord with the theoretical expectation that the cross section will be approximately constant throughout the wide energy range in the absence of the resonance.~~ The discussion of these energy dependence of measured by the same authors. Than the of ^{2}H seems to be in contrast to that of ^{1}H.

It should be noticed that the calculation in the limiting case of small binding can also be made by the method of Fermi, On the contrary, however, in the limit of the weak binding, but ~~it~~ the result which ~~does~~ ~~is not in agreement~~ consistent in general with that of the preceeding section. Namely, the cross section~~s~~ of free deuterons obtained by these two methods, differs by a factor

$$\frac{1}{2}\left(\frac{M+M'}{M}\right)^2\left(\frac{M'}{M+M'}+\frac{M'-M}{2M}\log\frac{M'+M}{M'-M}\right)$$

$$=\frac{\sigma\,(\text{Fermi})}{\sigma}$$

( ) Fermi's method ( ) 1.368 for obtained by §3 ~~Born~~ method, which is not equal to 1 except for the cases $M=M'$ and $M\ll M'$,

from each other. ~~Besides, the~~ ~~It becomes~~ Correspondingly, the angular distribution of scattered neutrons becomes ~~correspondingly~~

$$\left(\frac{M'}{M+M'}+\frac{M'-M}{2M}\log\frac{M'+M}{M'-M}\right)^{-1}\left(\frac{M\cos\theta+\sqrt{M'^2-M^2\sin^2\theta}}{M+M'}\right)\sin\theta\, d\theta$$

~~and~~ in contradiction to

, both in the ordinary coordinate system, which are also at variance with ~~different from each other~~ except for ~~the~~ $M=M'$ and $M\ll M'$.

(1) ~~Dunning and others,~~ loc.cit.

It ~~becomes~~ ~~is largest~~ takes the largest value 1.368 for the case of the deuteron, ~~i.e.~~ ($M'=2M$), Closer examination shows that the origin of this discrepancy is the illegitimate use of Born's approximation on the part of Fermi's method in the ~~case~~ (limit) of ~~so~~ weak binding ~~in the limiting case of small binding~~, the agreement of the results of both methods in the case of the scattering of neutrons by free protons being only accidental. ~~On the contrary,~~ Nevertheless, the ~~only~~ result ~~method~~ of Fermi ~~seems to be~~ is only seems to be safely applicable ~~only one~~ practicable in the case of the strong binding. is true always in the ~~limit~~ case of the strong binding. In the case of the

§4. Slowing Down of Neutrons by Multiple Collision with Deuterons

If we assume the angular distribution of neutrons scattered by dueterons initially at rest to be given by ( ), the mean energy after a single collision will be only $\frac{5}{9}$ of the initail energy E_0, so that it will be reduced to $\left(\frac{5}{9}\right)^n E_0$ after n collisions, which does not differ very much from the corresponding value $\left(\frac{1}{2}\right)^n E_0$ in the case of collisions with protons.(1) Thus, the deuteron seems to have an ~~comparable~~ effect on the slowing of neutrons comparable with the proton.

There are, however, many reasons which lead us to the opposite conclusion. Firstly, the neutron energy can not be smaller than $\frac{1}{9}$ by a single collision, so that a neutron with the energy of the order of 10^6 eV should undergo at least 6 collisions with deuterons before it becomes a slow neutron of several volts, whereas this can be effected by a single collision with the proton. Secondly, the occurrence of the multiple collision with the former will be much rarer than that with the latter, under ~~in the~~ similar

(1) Condon and Breit, Phys. Rev. 49, 229, 1936.

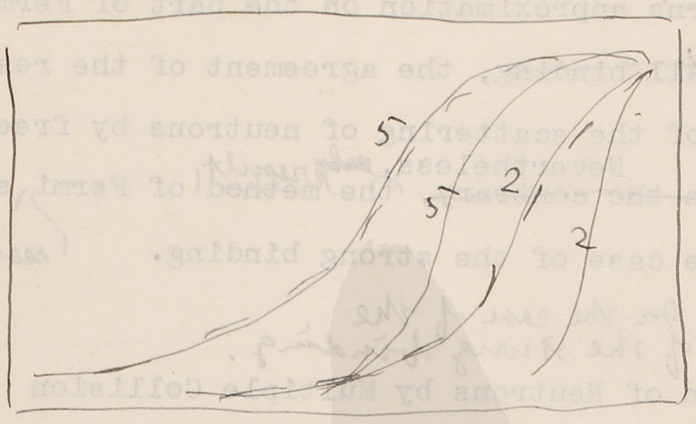


Fig. 2. The ordinate shows the probability that the neutron has a fraction of its initial energy less than the abscissa after a number of collisions marked on each curve. ~~The~~ Full lines refer to the collisions with deuterons and dashed lines to those with protons.

Fig.

(11)

conditions, because on account of the fact that the cross section of the former becomes much smaller than that of the latter as the energy decreases. Thirdly, the energy distribution function of neutrons after n collisions with the former ~~has~~, the general shape ~~of which are as illustrated in the Fig.~~ which can be evaluated by using the general formulae of Condon and Breit, and has has very small values in the neighborhood of the lower limit $\frac{E_0}{9^n}$. For example, the probabilitie~~y~~ that~~s~~ the neutron has an energy smaller than $\frac{E_0}{9^4}$ after 5 collisions with deuterons is only 1.15×10^{-4}, which is much smaller than the corresponding value 0.065 for protons. are 1.15×10^{-4} and 0.063 respectively. General shape of the distribution ~~curve is~~ curve is shown in Fig. 2. ~~Energy~~ Under these circumstances, we can not expect theoretically any appreciable contribution of the deuteron compound such as heavy water to the slowing down of neutrons in good agreement with the recent experimental results of Kikuchi, Aoki and Takeda,[1] which show~~s~~ that the number of neutrons of C-group produced in D_2O by using D-D source is at most 1/27 of that produced in equal amount ~~of~~ of H_2O, although quantitative comparison of the theory and the experiment is very difficult. Previous results of Dunning and others[2] indicating relative efficiencies about 1:5.5 of D_2O and H_2O for producing slow neutrons should be attributed to the contribution of B-goup already present in Be-Rn source as pointed out by Kikuchi and others.[1]

In conclusion, the authors are indebted to Prof. S. Kikuchi for valuable discussions.

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(1) loc. cit.

(2) loc. cit.

$$\sigma_f = \frac{1}{4}\sigma_+ + \frac{3}{4}\sigma_-$$

and the theoretical value for σ_+. The curve in ~~Fig. 1~~ Fig. 1 shows σ_f as the function of $-U_-$ with the fixed values $b = 4.5 \times 10^{-13}$ cm and $-U_+ = 13.8 \times 10^6$ eV. It intersect with the line $\sigma_f = 2 \times 10^{-24}$ cm^2 at $-U_- = 1.9 \times 10^6$ eV, which leads us to the conclusion that the interaction between the neutron and the deuteron can be represented by a small attractive force, when the wave function of the total system is symmetric with respect to the spins of two neutrons, as long as the relative velocity is not large enough for the disintegration of the deuteron. Such a shallow potential hole U_- has no ~~excited~~ true levels and the energy of the first virtual S-level is as high as 2.8×10^6 eV, and also U_+ has no true P-level. These results in turn lead to ~~which in turn results in~~ further conclusion that the cross section of capture of slow neutrons by deuterons will be much smaller than that by protons. This seems to be in good accord with the recent experimental value 0.3×10^{-25} cm^2 of Kikuchi, Aoki and Takeda[1] for the upper limit of the cross section of the γ-ray emission by collision of slow neutrons with deuterons.

~~(1) In this case, the existence of the virtual 'S level of energy about 12 × 10⁴ eV was inferred from theoretical considerations.~~

(1) Kikuchi, Aoki and Takeda, Sci. Pap. Inst. Phys. Chem. Res. 31, 195, 1937.

the virtual 'S level of ^{2}H of energy about 12×10^4 eV being considered to be effective for the capture in the latter case.

Further, we calculated the cross section given by (12) ~~$\sigma = \frac{1}{4}\sigma_+ + \frac{3}{4}\sigma_-$~~ for several values of the neutron energy E_0 by taking the above values for b, U_+ and U_-, the result being shown in Fig. 2. Thus, the cross section is constant as long as E_0 is the order of 10^5 eV or smaller and decreases steadily as E_0 increases further. These results are not contradictory to the experimental cross section 1.71×10^{-24} cm^2 obtained by Dunning and others for fast neutrons from the Rn-Be source,[1] although various complicated effects will be suerimposed on the simple S-scattering above considered for E_0 larger than E_D.[2] For small values of E_0, The angular distribution of scattered neutrons are, ~~of course~~, spherically symmetric in the coordinate system in which the centre of mass is at rest, so that ~~it becomes the~~ the prob. that it is ~~probability that it becomes~~ scattered by an angle between $\Theta, \Theta + d\Theta$ in the ordinary

$$\frac{1}{2}\left\{\frac{3+2\cos^2\theta}{2\sqrt{3+\cos^2\theta}} + \cos\theta\right\}\sin\theta\, d\theta,$$

~~which is~~ as shown in ~~the figure~~ Fig. 3.

Thus far, we took ~~of~~ only Majorana forces into account. The inclusion of small Wigner forces does not give rise to any substantial modifications, while that of Heisenberg forces results in the further splitting up and coupling of $^2S_+$, $^2S_-$, $^4S_-$ states, but it seems improbable that the general feature of the above results is altered essentially.

(1) Loc. cit.

(2) The first virtual level for the potential U_+ is ^2P with energy larger than 10^6 eV.

2. Estimation of the Scattering Cro~~s~~ss Section

In order to obtain an approximate value of the scattering cross section without solving the complicated integral equation, it is needed to choose the auxiliary potentials U (r) in (9) and (10) such that F (r,) become already approximate expressions for the required functions (r,).

If we consider the ~~deutr/~~ deuteron to be a sphere of diameter = 4.36 10 cm and the nuclear forces to have a definite range a = 2.32 10 cm the interaction between the neutron and the deuteron can be represented by a potential hole with the radius b, which may take a value between = 2.18 10 cm and = 4.5 10 cm. Thus we can put

U (r) = U = const. or 0

according as r b or r b.

A legitimate value for U can be obtained by assuming that (10) has a solution with the energy

E = -E = -8.3 10 eV

corresponding to the normal state of H. Such a proce~~s~~dure was already used by Massey and Mohr. Thus we find

-U = 13.8 10 eV or 32.5 10 eV

for the extreme case~~s~~ b = 4.5 10 cm or 2.18 10 cm. and consequently the cross section of the deuteron at rest for slow neutrons becomes

= 2.7 10 cm or 1.46 10 cm

(1) Massey and Mohr, Proc. Roy. Soc. A, 148, 206, 1935.

We have calculated further the cross section for several values of the neutron energy, the results being shown in Fig. 1. In the former case reaches to a large maximum and then decreases as the energy increases on account of the presence of the virtual P-level, while in the latter case it decreases steadily with the energy.

On the other hand, U can not be determined in like manner, as little is known of the excited states of H, and we can say only that -U is much smaller than -U or may even be negative. Thus the cross section in this case can be determined only by making use of further experimental information as follows. Namely, the observed cross section of the deuteron in heavy water is = 4 10 cm for slow neutrons of C-group according to Dunning, Pegram, Fink and Mitchell. If we consider the effect of chemical binding of deuterons, the cross section of y the free deuteron for slow neutrons should be about half of the above , so that we obtain an approximate value = 2 10 cm. This value is to correspond to the average

of the cross sections of the deuterons for slow neutrons in two cases above considered.

(1) Dunning, Pegram, Fink nd and Mitchell, Phys. Rev. 48, 265, 1935.

(2) Compare 3.

Hence, we can choose U with a given value of b so as to make satisfy the relation

for slow neutrons, where is has the value determined with the same b. We obtain in this manner the results

= 1.8 10 cm -U = 1.9 10 eV

for b = 4.5 10 cm and

= 2.18 10 cm -U = 11 10 eV

for b = 2.18 10 cm. As shown in Fig. 1, decreases with the energy in the latter case, while it has a small hump due to the virtual S-level in the former case.

The average cross section given by ( ) depends on the energy in the manner as indicated by the curves in Fig. 2. If we change b between the above two extreme values, the shape of the curve will be altered therewith, so that the correct value of b will be determined, when the detailed experimental results for the cross section will be obtained. The cross section 1.71 10 cm measured by Dunning and others for fast neutrons from the Rn-Be source is an average over a wide energy range and is insufficient for the determination ofb b, it seems to be in favour of the value of b na nearer to 2.18 10 cm than to 4.5 10 cm.

In any case, the cross section of capture of slow neutrons by deuterons

(1) Loc. cit.

will be much smaller than that by protons, as the above results shows that there is no true or virtual S or S -levels of small energy, while the existence of the virtual S-level of energy about 12 10 eV is expected from theoretical arguments. This seems to be in agreement with the recent experimental results of Kikichi, Aoki and Takeda, which indicate that the cross section of the -ray emission by collision of slow neutrons with deuterons is very small and has an upper limit 0.3 10 cm.

Thus far, we took only Majorana forces into account. The inclusion of small Wigner forces will not give rise to any substantial modification, while the presence of Heiseberge forces will result in the further splitting up and coupling of two set of states above considered, but our method of estimation is too crude to afford such discussions of finer details.

It shoud be noticed that the angular distribution of t scattered neutrons is, of course, spherically symmetric in the relative coordinate system for small energies, so that the probability of it being scattered into by an angle between and in the ordinary coordinate system becomes

Thus, most of the neutrons are scattered into the forward direction as shown in Fig. 3.

(1) Kikuchi, Aoki and Takeda, Sci. Pap. Inst. Phys. Chem. Res. 31, 195, 1937.

(2) Fermi, loc. cit.