§2. Estimation of the Scattering Cross Section

In order to estimate the scattering cross section without solving the complicated integral equation, it is needed to choose the auxiliary potentials $U_\pm(r)$ in (9) and (10) such that $F(r, \theta)$ becomes already an approximate expression for the required function $\varphi(\vec{r})$. 参 If we consider the deuteron as a sphere of diameter $\frac{1}{\alpha} = \frac{\hbar}{\sqrt{ME_D}} = 4.36 \times 10^{-13}$ cm, and the nuclear force with a definite range $a = 2.32 \times 10^{-13}$ cm, the interaction between the neutron and the deuteron reduces to zero for r larger than $b = \frac{1}{2\alpha} + a = 4.5 \times 10^{-13}$ cm, so that we can put

$$U_\pm(r) = U_\pm = \text{const.} \quad \text{or} = 0$$

according as $r > b$ or $r < b$.

A legitimate value for U_+ can be obtain by assuming that (10) has a solution with the energy

$$E' = -E_T = -8.3 \times 10^6 \text{ eV}$$

corresponding to the normal state of ^{3}H. Thus we find

$$-U_+ = 13.8 \times 10^6 \text{ eV} \quad \text{for} \quad b = 4.5 \times 10^{-13} \text{ cm}$$

and the cross section of the deuteron initially at rest becomes

$$\sigma_+ = 2.7 \times 10^{-24} \text{ cm}^2$$

for neutrons with the energy $E_0 = \frac{3}{2}(E' + E_D)$ small compared with E_D. The cross section decreases slowly with the energy. The above procedure is not sensitive to the simultaneous change of b and U_+, provided that the binding energy E_T of ^{3}H is not altered. The cross section for small energy becomes

$$\sigma_+ = 2.3 \times 10^{-24} \text{ cm}^2,$$

if we take, for instance, $b = \frac{1}{2\alpha} = 2.18 \times 10^{-13}$ cm and $-U_+ = 16.4 \times 10^6$ eV.

In order to obtain more accurate results, one has to insert the function $F(r, \theta)$ thus determined in place of $\varphi(\vec{r})$ in the expression for $G(\vec{r})$ and perform the integration of the second term of the right hand side of (8), which involves the complicated effect of the exchange of neutrons.

On the other hand, U_- can not be determined in the similar manner as U_+, since little is known of the excited states of ^3H and we can say only that U_- is much smaller in magnitude than U_+ and may be even positive. Thus, the cross section in this case can not be determined without making use of the experimental informations as follows.

Namely, the observed cross section of the deuteron, which is to correspond to the average

$$\sigma = \frac{1}{4}\sigma_+ + \frac{3}{4}\sigma_- \qquad (12)$$

of the above two cases, is about

$$\sigma_c = 4 \times 10^{-24}\ \mathrm{cm}^2$$

for slow neutrons of the C-group, according to the experiment of Dunning, ~~Fink,~~ Pegram, Fink and Mitchell.(1) If we consider the effect of chemical binding of deuterons, the cross section σ_f of the free deuteron for slow neutrons of several volts should be about half of that for neutrons of C-group,(2) so that we obtain a presumable value

$$\sigma_f = 2 \times 10^{-24}\ \mathrm{cm}^2.$$

Hence, the potential U_- can be determined so as to give this value as the cross section for slow neutrons by using the relation

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(1) Dunning, Pegram, Fink and Mitchell, Phys. Rev. 48, 265, 1935.

(2) Compare §3.

σ_f

and the value for

the function of $-U$

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1.9×10^6 eV, which

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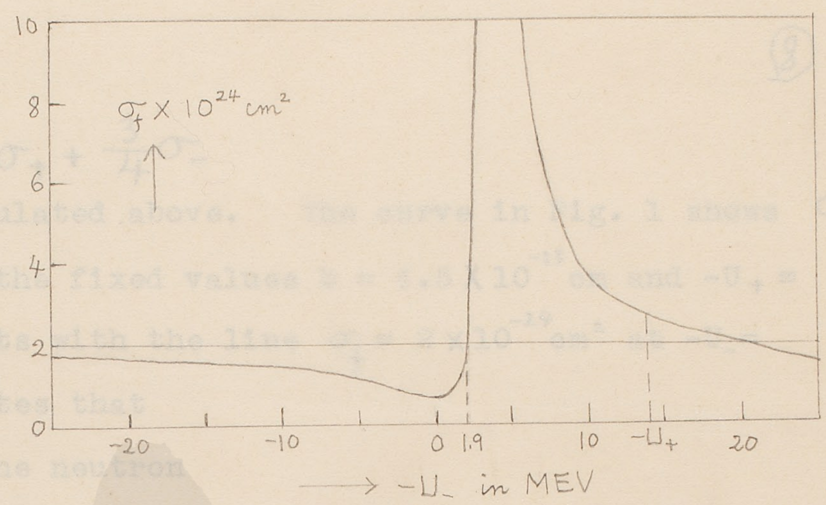
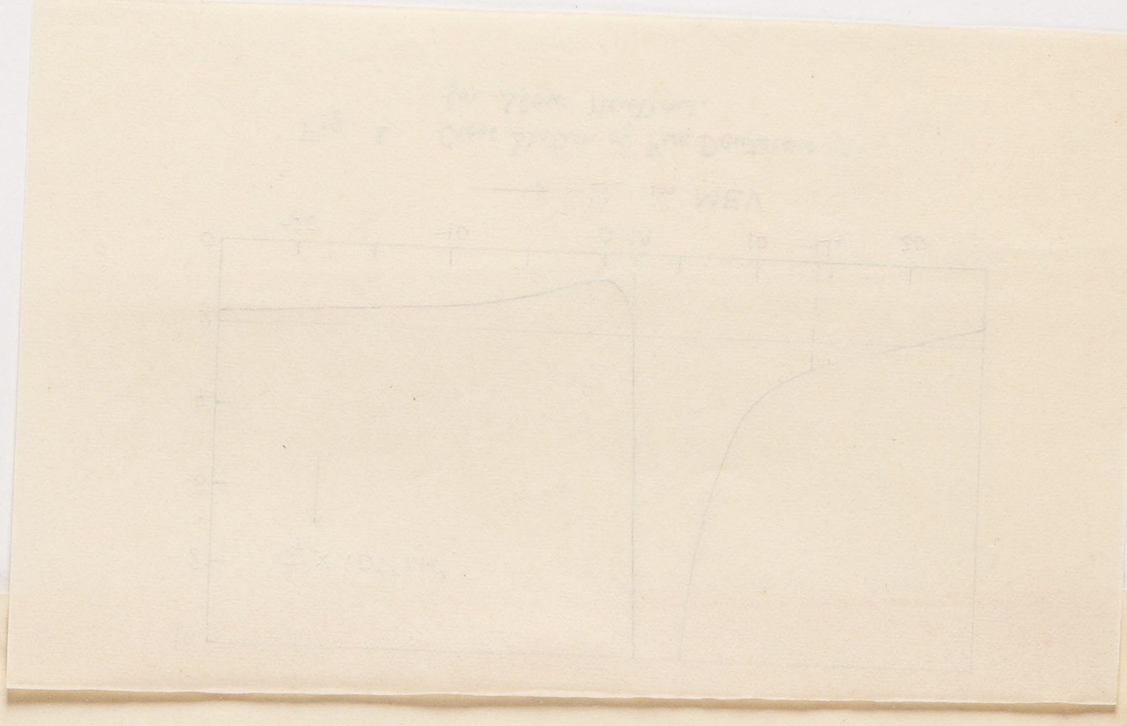


Fig. 1. Cross Section of Free Deuterons for Slow Neutrons.

system is antisymmetric with respect to the coordinates of two neutrons, as long as the relative velocity is not large enough for the disintegration of the deuteron. Such a shallow potential hole U_- has no true levels and the energy of the first virtual S-level is as high as 2.8×10^6 eV, and also U_+ has no true P-level, if we take the above value. These results in turn lead to further conclusion that the cross section of capture of slow neutrons by deuterons will be much smaller than that by protons, the virtual S-level of ^2H with energy about 12×10^4 eV ~~has~~ being considered to be effective for the capture in the latter case. This seems to be in good accord with the recent experimental results of Kikuchi, Aoki and Takeda,[1] which shows that the cross section of the γ-ray emission by collision of slow neutrons with deuterons is very small and has a value ~~an upper limit~~ 0.3×10^{-25} cm^2 as the upper limit.

(1) Kikuchi, Aoki and Takeda, Sci. Pap. Inst. Phys. Chem. Res. 31, 195, 1937.



$$\sigma_f = \frac{1}{4}\sigma_+ + \frac{3}{4}\sigma_-$$

and the value for σ_+ calculated above. The curve in Fig. 1 shows σ_f as the function of $-U_-$ with the fixed values $b = 4.5 \times 10^{-13}$ cm and $-U_+ = 13.8 \times 10^6$ eV. It intersects with the line $\sigma_f = 2 \times 10^{-24}$ cm^2 at $-U_- = 1.9 \times 10^6$ eV, which indicates that the interaction between the neutron and the deuteron can be represented roughly by a small attractive force, when the wave function of the total system is antisymmetric with respect to the coordinates of two neutrons, as long as the relative velocity is not large enough for the disintegration of the deuteron. Such a shallow potential hole U_- has no true levels and the energy of the first virtual S-level is as high as 2.8×10^6 eV, and also U_+ has no true P-level, if we take the above value. These results in turn lead to further conclusion that the cross section of capture of slow neutrons by deuterons will be much smaller than that by protons, the virtual S-level of ^{2}H with energy about 12×10^4 eV ~~was~~ being considered to be effective for the capture in the latter case. This seems to be in good accord with the recent experimental results of Kikuchi, Aoki and Takeda,[1] which shows that the cross section of the γ-ray emission by collision of slow neutrons with deuterons is very small and has a value ~~an upper limit~~ 0.3×10^{-25} cm^2 as the upper limit.

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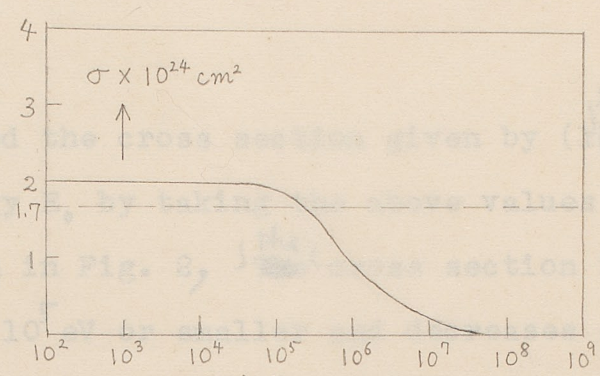


Fig. 2. Cross Section of Deuterons as Function of the Neutron Energy.

Further, we ca … veral values of the neutro … and U_-, ~~the results bei~~ A … it as long as E_0 is the or … as E_0 increases further. … erimental cross section 1.71 X … st neutrons from the R … effects, which may be superim … ed, for fast neutrons.

Fig. 2

For small values of E_0, the angular distribution of scattered neutrons are, of course, spherically symmetric in the coordinate system, in which the centre of mass is at rest, so that the probability that the neutron is scattered by an angle between θ and $\theta+d\theta$ in the ordinary coordinate system becomes

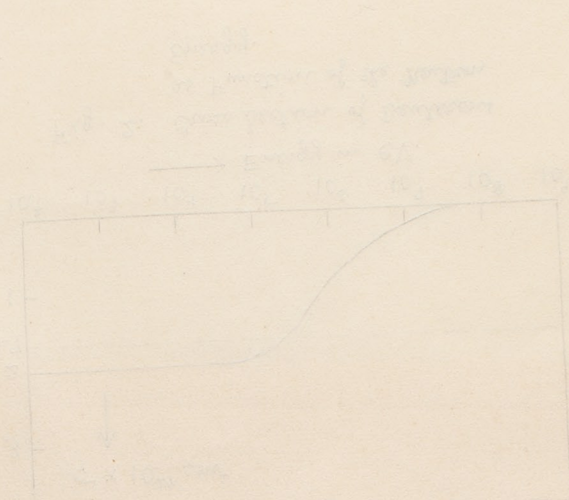
Fig. 3.

$$\frac{1}{2}\left\{\frac{3+2\cos^2\theta}{2\sqrt{3+\cos^2\theta}}+\cos\theta\right\}\sin\theta\, d\theta, \qquad (13)$$

as shown in ~~the~~ Fig. 3.

Thus far, we took only Majorana forces into account. The inclusion of small Wigner forces does not give rise to any substantial modification, while that of Heisenberg forces results in the further splitting up and coupling of $^2S_+$, $^2S_-$, $^4S_-$ states, but it seems improbable that the general feature of the above results is altered essentially.

(1) Loc. cit.



Further, we calculated the cross section given by (1~~1~~2) for several values of the neutron energy E_0 by taking the above values for b, U_+ and U_-. ~~the results being~~ As shown in Fig. 2, ~~The~~ the cross section is constant as long as E_0 is the order of 10^5 eV or smaller and decreases steadily as E_0 increases further. These results are not contradictory to the experimental cross section 1.71×10^{-24} cm^2 obtained by Dunning and others[1] for fast neutrons from the Rn-Be source, although we have ~~neglected~~ to take into account various effects, which may be superimposed on the simple S-scattering above considered, for fast neutrons.

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