Neutron の Electric Moment に就て.

June 22, 1937.

Neutron に magnetic moment があるらしいことは多くの実験と理論の比較から大体容認される.

然らば次に Electric Moment があるかどうか、それを如何にして測定し得るかを考へて見る. Dirac の Electron theory に於て el. moment が imaginary term として magnetic moment に伴ふ. これは然し charged particle の場合には之を測ることは不可能である. neutron の様な neutral particle に於ては、かゝる el. ~~moment~~ field に於ては el. mom. が効いて来るであらう.

~~velocity の大きい neutron~~
~~之れに直角な~~
~~inhomogeneous~~
~~field (a は大きい) 中で走らすと.~~

§1. Scattering of a Electric dipole in the Coulomb Field

その為に先づ charged particle と electric dipole の衝突の問題を考へて見る. charge を e, dipole moment を $\mathfrak{D}$ とすると interaction potential は (dipole の方向が一定に定まってて z-axis にとる)

$$-\frac{e[\mathfrak{D}\,\mathfrak{r}]}{r^3} = \frac{-eD\cos\theta}{r^2}$$

$$H = \frac{p^2}{2m} - \frac{eD\cos\theta}{r^2}$$

$$(\mathfrak{D}\,\mathrm{grad}\left(\frac{e}{r}\right))$$

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right)\psi \quad = -\frac{e[\mathfrak{D}\,\mathfrak{r}]}{r^3}$$

$$+ \frac{2m}{\hbar^2}\left(W + \frac{eD}{r^2}\cos\theta\right)\psi = 0$$

$$\left\{r^2\frac{\partial^2}{\partial r^2} + 2r\frac{\partial}{\partial r} + \frac{2mW}{\hbar^2}r^2\right\}\psi + \left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{2meD}{\hbar^2}\cos\theta\right\}\psi$$

$$+ \frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2} = 0 \qquad \psi = R\,\Theta\,\Phi.$$

$$R^{-1}\left\{r^2R''+2rR'+\frac{2mW}{\hbar^2}r^2R\right\}=\lambda$$

$$+\Theta^{-1}\left\{\frac{1}{\sin\theta}\frac{d}{d\theta}\sin\theta\frac{d\Theta}{d\theta}+\frac{2meD}{\hbar^2}\cos\theta\right\}\Theta$$

$$+\Phi^{-1}\frac{1}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}=0$$

$$\cancel{R^{-1}}\ r^2R''+2rR'+\left(\frac{2mW}{\hbar^2}r^2-\lambda\right)R=0$$

$$\sin\theta\frac{d}{d\theta}\sin\theta\frac{d\Theta}{d\theta}+\left(\frac{2meD}{\hbar^2}\frac{\cos\theta}{\sin\theta}+\lambda\sin^2\theta-\mu\right)\Theta=0$$

$$\frac{\partial^2\Phi}{\partial\phi^2}+\mu\Phi=0$$

μ

$\mu=m^2$,

$$\sin\theta\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\left(\frac{2meD}{\hbar^2}\sin^2\theta\cos\theta+\lambda\sin^2\theta-m^2\right)\Theta=0$$

$\cos\theta=x$

$$\frac{d}{d\theta}=-\sin\theta\frac{d}{dx}$$

$$(1-x^2)\frac{d}{dx}\left((1-x^2)\frac{d\Theta}{dx}\right)+\left\{\frac{2meD}{\hbar^2}(1-x^2)x+\lambda(1-x^2)-m^2\right\}\Theta=0$$

$$\frac{d}{dx}\left\{(1-x^2)\frac{d\Theta}{dx}\right\}+\left\{\frac{2meD}{\hbar^2}x+\lambda-\frac{m^2}{1-x^2}\right\}\Theta=0.$$

$m=0$:

$$\frac{d}{dx}\left\{(1-x^2)\frac{d\Theta}{dx}\right\}+\left(\frac{2meD}{\hbar^2}x+\lambda\right)\Theta=0.$$

$$(1-x^2)\Theta''-2x\Theta'+(\varkappa x+\lambda)\Theta=0.$$

$$\frac{2meD}{\hbar^2}=\varkappa$$

$$\frac{d\Theta}{dx}=\left\{\frac{1}{1-x}+\frac{1+x}{(1-x)^2}\right\}\frac{d}{dy}$$

$$=\frac{2}{(1-x)^2}\frac{d}{dy}=\frac{(y+1)^2}{2}\frac{d}{dy}$$

$$(1-x^2)\frac{d\Theta}{dx}=2\frac{1+x}{1-x}\frac{d\Theta}{dy}=2y\frac{d\Theta}{dy}$$

$$(y+1)^2\frac{d}{dy}\left(y\frac{d\Theta}{dy}\right)+\left\{\varkappa\frac{y-1}{(y+1)}+\lambda\right\}\Theta=0$$

$$(y+1)^3\frac{d}{dy}\left(y\frac{d\Theta}{dy}\right)+\{(\varkappa+\lambda)y+(\lambda-\varkappa)\}\Theta=0.$$

-1 　 0 　 $+1$

-1 — 0 — 1 — ∞

$$\frac{1+x}{1-x}=y\qquad 1-x^2=\frac{4y}{(y+1)^2}$$

$$1+x=y-yx\qquad 1-x=\frac{2}{y+1}$$

$$x=\frac{y-1}{y+1}$$

~~$\lambda = l(l+1)$~~

~~$\Theta = 1$~~

$\Theta = \sum_l c_l P_l(x)$

$x P_l = \frac{l+1}{2l+1} P_{l+1} + \frac{l}{2l+1} P_{l-1}$

$\sum_l \{\lambda - l(l+1)\} c_l P_l + x \sum_l c_l \left(\frac{l+1}{2l+1} P_{l+1} + \frac{l}{2l+1} P_{l-1}\right)$

$P_l(x) = \frac{1}{2^l l!} \frac{d^l (x^2-1)^l}{dx^l}$

$\{\lambda - l(l+1) + \frac{xl}{2l+1}\} c_l + \frac{xl}{2l-1} c_{l-1} = 0$

~~$\lambda = l(l+1)\ \frac{xl}{2l+1}$ for certain l~~ ~~$\ln s$~~ $c_{l+1} = c_{l+2} = \cdots = 0.$

$$\frac{d}{dx}\left[(1-x^2)\frac{d\Theta}{dx}\right] + \left(\kappa x + \lambda - \frac{m^2}{1-x^2}\right)\Theta = 0$$

$$\Theta = \sum_l c_l P_l^m(x)$$

$$\sum_l \{\lambda - l(l+1)\} c_l P_l^m + \kappa \sum_l c_l x P_l^m = 0$$

$$(2l+1) x P_l^m = (l+m) P_{l-1}^m + (l+1-m) P_{l+1}^m$$

$$\kappa \frac{l-m}{2l-1} c_{l-1} + (\lambda - l(l+1)) c_l + \kappa \frac{l+1-m}{2l+3} c_{l+1} = 0$$

$$\lambda = l(l+1) + \kappa\lambda_1 + \kappa^2\lambda_2 + \cdots$$

$$c_l = c_l^{(0)} + c_l^{(1)}\kappa + c_l^{(2)}\kappa^2 + \cdots$$

$$c_{l-1} = c_{l-1}^{(1)}\kappa + c_{l-1}^{(2)}\kappa^2 + \cdots$$

$$c_{l+1} = c_{l+1}^{(1)}\kappa + c_{l+1}^{(2)}\kappa^2 + \cdots$$

$$c_{l-2} = c_{l-2}^{(2)}\kappa^2 + \cdots$$

$$c_{l+2} = c_{l+2}^{(2)}\kappa^2 +$$

$$\lambda_1 = 0$$

$$\lambda_2 = \frac{(l+m)(l-m)}{2l(2l-1)(2l+1)} - \frac{(l+1+m)(l+1-m)}{2(l+1)(2l+1)(2l+3)}$$

$$\psi = \sum_l R_l P_l(\cos\theta)$$

$$\sum_l \left\{ r^2\frac{d^2}{dr^2} + 2r\frac{d}{dr} + \frac{2mW}{\hbar^2} r^2 - \cancel{l(l+1)}) + \frac{2meD}{\hbar^2}\cos\theta \right\} R_l P_l(\cos\theta) = 0$$

$$\cos\theta \cdot P_l(\cos\theta) = \frac{l+1}{2l+1} P_{l+1} + \frac{l}{2l+1} P_{l-1}$$

$$r^2\frac{d^2R_l}{dr^2} + 2r\frac{dR_l}{dr} + \left\{ \frac{2mW}{\hbar^2} r^2 \cancel{R_l} - l(l+1) \right\} R_l + \frac{2meD}{\hbar^2} \cancel{F(r)}$$

$$\left\{ \frac{l R_{l-1}}{2l-1} + \frac{(l+1)R_{l+1}}{2l+3} \right\} = 0$$

$$k = \frac{\sqrt{2mW}}{\hbar}$$

$$R_l = \left(\frac{\pi}{2kr}\right)^{\frac{1}{2}} J_{l+\frac{1}{2}}(kr) + \chi_l$$

$$r^2\frac{d^2\chi_l}{dr^2} + 2r\frac{d\chi_l}{dr} + \{k^2r^2 - l(l+1)\}\chi_l$$

$$+ \frac{2meD}{\hbar^2}\left(\frac{\pi}{2kr}\right)^{\frac{1}{2}} \left\{ \frac{l J_{l-\frac{1}{2}}(kr)}{2l-1} + \frac{(l+1) J_{l+\frac{3}{2}}(kr)}{2l+3} \right\}$$

$F(r)$

$l(l-1)$

$(l+1)(l+2)$

$$= 0$$

$$\chi_l = a_l \left(\frac{\pi}{2kr}\right)^{\frac{1}{2}} J_{l-\frac{1}{2}}(kr) + b_l \left(\frac{\pi}{2kr}\right)^{\frac{1}{2}} J_{l+\frac{3}{2}}(kr)$$

$$-2l\, a_l + \frac{2meD}{\hbar^2}\frac{l}{2l-1} = 0$$

$$2(l+1)\, b_l + \frac{2meD}{\hbar^2}\frac{l+1}{2l+3} = 0$$

$$\chi_l = \left(\frac{\pi}{2kr}\right)^{\frac{1}{2}} \left\{ \frac{meD}{\hbar^2}\frac{J_{l-\frac{1}{2}}}{(2l-1)} - \frac{meD}{\hbar^2}\frac{J_{l+\frac{3}{2}}}{(2l+3)} \right\}$$

$$J_{l+\frac{1}{2}}(kr) \sim (kr)^{-1}\sin\left(kr - \frac{1}{2}l\pi\right) \quad \sin\left(kr + \frac{\pi}{2}\right) \quad \sin\left(kr - \frac{\pi}{2}\right)$$

$$R_l \sim (kr)^{-1}\left\{ \sin\left(kr - \tfrac{1}{2}l\pi\right) + \frac{meD}{\hbar^2}\frac{\sin(kr - \frac{1}{2}(l-1)\pi)}{2l-1} - \frac{meD}{\hbar^2}\frac{\sin(kr - \frac{1}{2}(l+3)\pi)}{2l+3} \right\}$$

$$\cong (kr)^{-1}\left\{ \sin\left(kr-\tfrac{1}{2}l\pi\right) + \frac{meD}{\hbar^2}\left(\frac{1}{2l-1}+\frac{1}{2l+3}\right)\cos\left(kr-\tfrac{1}{2}l\pi\right)\right\}$$

$$\eta_l = \frac{2meD}{\hbar^2}\,\frac{2l+1}{(2l-1)(2l+3)} \ll 1$$

$$\cong (kr)^{-1}\left\{ \sin\left(kr-\tfrac{1}{2}l\pi+\eta_l\right)\right.$$

$$f(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\left(e^{2i\eta_l}-1\right)P_l(\cos\theta)$$

$$Q = \frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\frac{2meD}{\hbar^2}\,\frac{2l+1}{(2l-1)(2l+3)}$$

$$\cong \frac{4\pi}{k^2}\left(\frac{2meD}{\hbar^2}\right)^2\sum_l\frac{(2l+1)^3}{(2l-1)^2(2l+3)^2}$$

この sequence は diverge する。従って cross section は ∞ である。

これは small angle scattering が大きいためである

$$f(\theta) \cong \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\;2i\left\{\frac{2meD}{\hbar^2}\,\frac{(2l+1)}{(2l-1)(2l+3)}\right\}P_l(\cos\theta)$$

$$\cancel{= \frac{1}{2ik}\sum_{l=0}^{\infty}}$$

$$= \frac{1}{k}\,\frac{2meD}{\hbar^2}\sum_l\frac{(2l+1)^2}{(2l-1)(2l+3)}P_l(\cos\theta)$$

$$\cancel{\sum_l\frac{(2l+1)^2(x^2-1)^l}{(2l-1)(2l+3)}} =$$

$l=0$ とすると $Q_0 \cong \dfrac{4\pi}{k^2}\left(\dfrac{2meD}{\hbar^2}\right)^2\dfrac{1}{9}$

$D \cong er_0$. $\left(\dfrac{2me^2r_0}{\hbar^2}\right)^2$

§2. Screening の影響.

$$a = \frac{\hbar^2}{2me^2}$$

$$Q_0 \cong \frac{4\pi}{k^2}\times\left(\frac{2Mr_0}{ma}\right)^2$$

proton mass

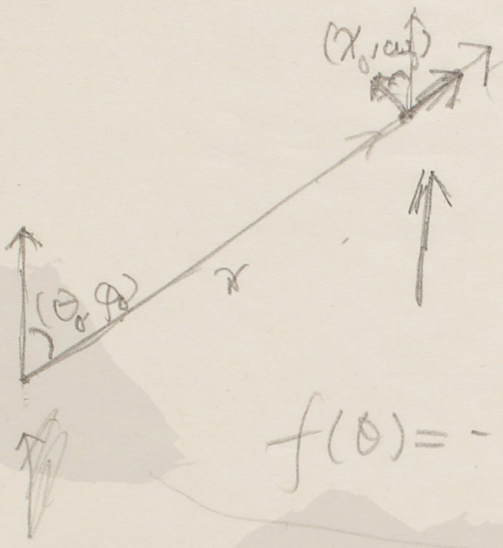
electron mass

§3. Born's Method in Non-Central Field

$$f(\theta,\varphi) = -\frac{1}{4\pi}\int e^{ik(\mathfrak{n}_0-\mathfrak{n})\mathfrak{r}} V(\mathfrak{r})\,dv$$

$$V(\mathfrak{r}) = (\mathbf{D}\,\mathrm{grad})\left\{\frac{Ze}{r} - e\int\frac{\rho(\mathfrak{r}')\,dv'}{|\mathfrak{r}-\mathfrak{r}'|}\right\}$$

dipole の式 $= (\mathbf{D}\,\mathrm{grad})\,U(r)$



$= D\cos\Phi\,\frac{dU}{dr}$

$$\cos\Phi = \cos\Theta\cos\chi + \sin\Theta\sin\chi\cos(\varphi-\omega)$$
$$\cos\Phi_0 = \cos\Theta\cos\Theta_0 + \sin\Theta\sin\Theta_0\cos(\varphi-\varphi_0)$$

$$f(\theta) = -\frac{D}{4\pi}\int e^{ikr(\cos\Theta-\cos\Phi_0)}\cos\Phi\,\frac{dU}{dr}\,\chi r^2\sin\Theta\,d\Theta\,d\varphi$$

$\mathbf{D} = D,\chi,\omega$

$\cos\chi = \frac{\mathbf{D}(\mathfrak{n}_0-\mathfrak{n})}{D}$

$(\mathfrak{n}_0-\mathfrak{n})\mathfrak{r} = r\cos\alpha$

$\mathbf{D}\,\mathrm{grad}\,U = \frac{(\mathbf{D}\mathfrak{r})}{r}\frac{dU}{dr}$

$\mathbf{D} = D'\mathfrak{n}' + D''\mathfrak{n}'' + D'''\mathfrak{n}'''$

$\mathfrak{r} = \cos\alpha\,\mathfrak{n}' + \sin\alpha\cos\beta\,\mathfrak{n}'' + \sin\alpha\sin\beta\,\mathfrak{n}'''$

$U(r) = \frac{(\mathbf{D}\mathfrak{r})}{r}\frac{dU}{dr}$

$$f(\theta,\varphi) = -\frac{1}{4\pi}\int e^{ik(\mathfrak{n}_0-\mathfrak{n})\mathfrak{r}}\frac{(\mathbf{D}\mathfrak{r})}{r}\frac{dU}{dr}\,dv$$

$$= -\frac{1}{4\pi}\iiint e^{iKr\cos\alpha}(D'\cos\alpha + D''\sin\alpha\cos\beta + D'''\sin\alpha\sin\beta)$$

$K = k|\mathfrak{n}_0-\mathfrak{n}|$

$$\times\frac{1}{r}\frac{dU}{dr}r^2\sin\alpha\,d\alpha\,d\beta$$

$$= \frac{1}{2}\int e^{iKrx}D'x\,dx\,\frac{dU}{dr}r\,dr$$

$$\int_{-1}^{1} e^{iKrx}x\,dx = \frac{e^{iKr}+e^{-iKr}}{iKr} - \frac{\int_{-1}^{+1}e^{iKr}dx}{iKr}$$

$$= \frac{e^{iKr}+e^{-iKr}}{iKr} - \frac{e^{iKr}-e^{-iKr}}{(iKr)^2} = \frac{2}{iKr}\left(\cos Kr - \frac{\sin Kr}{Kr}\right)$$

$$f(\theta\varphi) = -\frac{D'}{iK}\boxed{\int\left(\cos Kr - \frac{\sin Kr}{Kr}\right)\cdot\frac{dU}{dr}\,dr}$$

$$\cancel{\int \cos Kr - \frac{\sin Kr}{Kr} =}$$

$$\int \cos Kr\,\frac{dU}{dr}\,dr = \cos Kr\,U\Big|_0^\infty \cancel{= U(0)}$$

$$+\int U\,K\cdot\sin Kr\,dr \quad \cancel{= U(0) + K\int U\sin Kr\,dr}$$

$$\int\frac{\sin Kr}{Kr}\frac{dU}{dr}\,dr = \frac{\sin Kr}{Kr}U\Big|_0^\infty$$

$$-\int\left(\frac{\cos Kr}{r} - \frac{\sin Kr}{Kr^2}\right)U\,dr$$

$$K\int U\left\{\sin Kr\left(1+\frac{1}{K^2r^2}\right) - \frac{\cos Kr}{Kr}\right\}dr$$

$$\left(Kr - \frac{(Kr)^3}{3}\right)\left(1+\frac{1}{K^2r^2}\right) - \frac{1}{Kr}\left(1-\frac{(Kr)^2}{2}\right)$$

$$= Kr - \frac{(Kr)^3}{3} + \frac{1}{Kr} - \frac{Kr}{3} - \frac{1}{Kr} + \frac{Kr}{2}$$

$$\simeq \frac{7}{6}Kr + \cdots \qquad K^2\frac{7}{6}\int U(r + \cdots)\,dr$$

Kが大きければ， $K^2\frac{7}{6}\int U r\,dr = K^2\frac{7}{6}.$